ME/CS 132(b)

Notes on the Visual Odometry Lab

This lab explores the concept of visual odometry, and how it can be combined with wheel odometry in an Extended Kalman Filter (EKF) framework. These notes review the basic EKF equations for this sensory combination.

1 The EKF structure

The EKF used to "fuse" or blend the wheel odometry uses the standard two-step process that alternates dynamic updates and measurement updates.

Dynamic Update. We assume that the discrete time motion of the robot can be modeled as:

$$\vec{p}^R_{k+1} \;=\; f(\vec{p}^R_k, \vec{u}_k) \;+\; \vec{\eta}_k \;.$$

where $\vec{p}^R$ is the position of the robot, and $\vec{\eta}$ is the disturbance in the vehicle's motion.

Assuming that an estimate $\vec{p}^R_{k|k}$ and its covariance $P_{k|k}$ are known at t_k, the dynamic update is:

$$\begin{aligned} \vec{p}^R_{k+1|k} &= f(\vec{p}^R_{k|k}, \vec{u}_k) \\ P_{k+1|k} &= F_k P_{k|k} F_k^T + V_k \\ F_k &= \left.\frac{\partial f}{\partial \vec{p}^R}\right|_{\hat{p}^T_{k|k}, \vec{u}_k} \end{aligned}$$

and V_k is the covariance of the process noise. For a wheeled mobile robot, the process noise captures some of the following effects:

- the unmodeled and unmeasured slipping of the drive wheels during the robot's movement,
- small errors in the modeling process (e.g., the kinematic model assumes that the wheel contacts the ground at a single point),
- disturbances when the vehicle rolls over non-planar floor features, such as cracks or or door jambs.

Measurement Update. We assume that the robot operates its on-board laswer scanner at time t_k to capture a "scan" of its nearby environment. The robot moves to a new location at t_{k+1}, and takes another scan. The scan matching process provides a noisy estimate of the robot's displacement between its location at time t_k and its location at t_{k+1}. Let $\Delta\vec{p}^T_{k+1} = \begin{pmatrix}\Delta x^T_{k+1} & \Delta y^T_{k+1} & \Delta\theta^T_{k+1}\end{pmatrix}$ denote this planar displacement. The measurement equation that relates this displacement measurement to the system state takes the form:

$$\Delta\vec{p}^R_{k+1} \;=\; \vec{p}^R_{k+1} \;-\; \vec{p}^R_k \;=\; h(\vec{p}^R_{k+1}, \vec{p}^R_k) + \vec{\omega}_{k+1}$$

where $\vec{\omega}_{k+1}$ is the noise on the measurement y_{k+1}.

Using this form of the measurement equation, the measurement update of the Extended Kalman Filter is:

$$\begin{aligned}
\vec{\nu}_{k+1} &= \vec{y}_{k+1} - h(\hat{p}^R_{k+1|k}, \hat{p}^R k|k) &(1)\\
S_{k+1} &= H_{k+1}P_{k+1|k}H^T_{k+1} + Q_k &(2)\\
K_{k+1} &= P_{k+1|k}H^T_{k+1}S^{-1}_{k+1} &(3)\\
\hat{p}^R_{k+1|k+1} &= \hat{p}^R_{k+1|k} + K_{k+1}\vec{\nu}_{k+1} &(4)\\
P_{k+1|k+1} &= (I - K_{k+1}H_{k+1})P_{k+1|k} &(5)
\end{aligned}$$

where Q_k is the covariance of the measurement noise, and H_{k+1} is the linearization of the meaasurement equation:

$$H_{k+1} \;=\; \left.\frac{\partial h(\vec{p}^R_{k+1}, \vec{p}^R_k)}{\partial \vec{p}^R_{k+1}}\right|_{\hat{p}_{k+1|k}} = I. \qquad (6)$$

NOTE: There is a subtle error in the above analysis that we will not concern ourselves about. The theory behind the derivation of the Kalman Filter (and by extension, the EKF) is based on an assumption about independence of noise across measurements. That is, the measurement noise $\vec{\omega}_{k+1}$ is assumed to be independent of the measurement noise $\vec{\omega}_k$. However, in the case of scan matching (which is a form of visual odometry), the noise at measurement noise at t_{k+1} is correlated with the measurement noise at t_k, since the measurements y_{k+1} and y_k use the laser scan at t_k in the displacement estimates at t_{k+1} and t_k. To properly account for those correlations, one can use the *stochastic cloning* version of the Kalman Filter–see the course web site for a reference.

2 Iterated Closest Point (ICP) Algorithm

This section presents one of many possible approaches to derive a robot displacement estimate from a pair of laser scans. At time t_k, the robot takes a *scan*, $\mathcal{V}_k$, of its nearby environment. This scan consists of N_k measurements of the distance (or *range*) r to the nearest reflecting surface along N_k different measurement directions (or *bearings*), ϕ, in the *scanning plane*. For a planar robot, we generally assume that the scanning plane is a plane which is parallel to the plane upon which the robot moves, but at some fixed height above the floor.

Let $\{(r_k^i, \phi_k^i)\}$ for $i = 1, \ldots, N_k$ denote the N_k range and bearing measurements pairs obtained in scan $\mathcal{V}_k$. In the local body-fixed reference frame of the robot, these range and bearing data can be converted into the Cartesian location of the points where the laser beam intersects an obstacle in the scanning plane:

$$\vec{p}_{k,i} \;=\; \begin{bmatrix} r_k^i \; \cos(\phi_k^i) \\ r_k^i \; \sin(\phi_k^i) \end{bmatrix} \qquad \text{for } i = 1, \ldots, N_k.$$

Similarly, the range and bearing scans at time t_k can be converted into a set of points $\{\vec{p}_{k+1,i}\}$ for $i = 1, \ldots, N_{k+1}$, where N_{k+1} is the number of data points sampled at t_{k+1}. Generally, $N_k = N_{k+1}$, but the success of the following approaches doesn't require a uniform number of points to be found in each scan.

Due to occlusions and other geometric features of the robot's environment, not all of the points which are observed in t_k can be seen from the robot's vantage point at t_{k+1}. Similarly, some of the points observed at t_{k+1} cannot be seen by the robot at t_k. Let us assume that there is a subset of M points which can be viewed from both vantage points. Let us next assume that the points define M *matching pairs* of points. That is, for point $\vec{p}_{k,i}$ (where $i \in 1, \ldots, M$), there exist a point in $\mathcal{V}_{k+1}$ that is assumed to arise from the same reflection point in the environment. We will order the matching pairs in $\mathcal{V}_{k+1}$ so that this matching point is indexed as $\vec{p}_{k+1,i}$. In reality, due to the nature of the scanning process, the matching points are

To derive an estimate of the robot's displacement,

$$\Delta \vec{p}^{R}_{k+1} \;=\; \vec{p}^{R}_{k+1} - \vec{p}^{R}_{k} \;=\; \begin{bmatrix} \Delta x_{k+1} \\ \Delta y_{k+1} \\ \Delta \theta_{k+1} \end{bmatrix}$$

we will create a *matching error function* whose minimum is the "best" estimate of the robot's displacement:

$$E(\Delta x, \Delta y, \Delta\theta) = \sum_{i=1}^{M} \; ||R(\Delta\theta)\vec{p}_{k+1,k} \;+\; \vec{T} \;-\; \vec{p}_{k,i}||^2 \tag{7}$$

where $T = \begin{bmatrix} \Delta x & \Delta y \end{bmatrix}$ is the *translation* between the robot's pose at t_k and its subsequent pose at t_{k+1}, and $R(\Delta\theta)$ is the *rotation matrix* which represents the rotation from the robot's position at t_{k+1} to its position at t_k:

$$R(\Delta\theta) \;=\; \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \tag{8}$$

I.e., function (7) is the sum of the squares of the errors in the distance between the matching points in $\mathcal{V}_k$ and $\mathcal{V}_{k+1}$. Note that in order to measure the Cartesian distance between the matching points, they must be represented in a common coordinate system. In Equation (7), the data points from $\mathcal{V}_{k+1}$ are translated to the same coordinate system in which the measurements $\mathcal{V}_k$ are obtained. We could have alternatively translated the points in $\mathcal{V}_k$ into the robot's body-fixed coordinate system at t_{k+1}.

A necessary condition to minimize this matching error with respect to the displacement variables is

$$\begin{pmatrix} \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \Delta x} \\ \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \Delta y} \\ \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \Delta\theta} \end{pmatrix} = \begin{pmatrix} \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial T} \\ \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \Delta\theta} \end{pmatrix} = \begin{pmatrix} \vec{0} \\ 0 \end{pmatrix}$$

Displacement Estimate. To estimate the displacement $\vec{T} = \begin{pmatrix}\Delta x & \Delta y\end{pmatrix}^T$,

$$\begin{aligned} \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \vec{T}} &= \left[\frac{\partial}{\partial \vec{T}}\left(\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right)\right]^T \left[\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right] \\ &= I\left[\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right] = MR(\Delta\theta)\begin{pmatrix} x \\ y \end{pmatrix} + M\vec{T} - M\begin{pmatrix} x' \\ y' \end{pmatrix} \quad (9) \\ &= 0 \end{aligned}$$

where:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{M}\sum_{i=1}^{M} \vec{p}_{k+1,i} \qquad \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{M}\sum_{i=1}^{M} \vec{p}_{k,i} \; .$$

From equation (9) we can see that

$$\vec{T} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} - R(\Delta\theta)\begin{bmatrix} x \\ y \end{bmatrix}. \tag{10}$$

Rotation Estimate. To estimate the rotation $\Delta\theta$

$$\begin{aligned} \frac{\partial E(\Delta x,\Delta y,\Delta\theta)}{\partial \Delta\theta} &= \left[\frac{\partial}{\partial \Delta\theta}\left(\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right)\right]^T \left[\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right] \\ &= \left[\sum_{i=1}^{M}\vec{p}_{k+1,i}\right]^T \left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T \left[\sum_{i=1}^{M} R(\Delta\Theta)\vec{p}_{k+1,i} + \vec{T} - \vec{p}_{k,i}\right] \\ &= M\begin{bmatrix} x \\ y \end{bmatrix}^T \left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T \left[MR(\Delta\theta)\begin{bmatrix} x \\ y \end{bmatrix} + M\vec{T} - M\begin{bmatrix} x' \\ y' \end{bmatrix}\right] \\ &= M^2\begin{bmatrix} x \\ y \end{bmatrix}^T \left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T R(\Delta\theta)\begin{bmatrix} x \\ y \end{bmatrix} + M^2\begin{bmatrix} x \\ y \end{bmatrix}\left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T \left[\vec{T} - \begin{bmatrix} x' \\ y' \end{bmatrix}\right] \\ &= 0 \end{aligned} \tag{11}$$

A tedious calculation shows that:

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T R(\Delta\theta)\begin{bmatrix} x \\ y \end{bmatrix} = 0.$$

Hence, Equation (11) reduces to:

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \left(\frac{\partial R(\Delta\theta)}{\partial\Delta\theta}\right)^T \left[\vec{T} - \begin{bmatrix} x' \\ y' \end{bmatrix}\right] = 0 \; . \tag{12}$$

To simplify this equation, let:

$$\Delta_{x,x'} \;=\; \Delta x - x' \qquad \Delta_{y,y'} \;=\; \Delta y - y' \;.$$

With this notation, Equation (12) becomes:

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \begin{bmatrix} \Delta_{x,x'} \\ \Delta_{y,y'} \end{bmatrix} = 0. \tag{13}$$

The terms in this equation can be expanded and rearranged to yield:

$$\tan(\Delta\theta) = \frac{y\Delta_{x,x'} - x\Delta_{y,y'}}{y\Delta_{y,y'} + x\Delta_{x,x'}} = \frac{y\Delta x - x\Delta y + xy' - yx'}{y\Delta y + x\Delta x - (yy' + xx')} \;. \tag{14}$$

ICP scan Matching Algorithm. Based on these results, one iterates the following algorithm to yield an estimate $(\Delta x, \Delta y, \Delta\theta)$

1. Choose an intial estimate of $(\Delta x, \Delta y, \Delta\theta)$. Typically, this estimate is provided by wheel odometry.
2. Transform the points in $\mathcal{V}_{k+1}$ to the coordinate system of $\mathcal{V}_k$ using the displacement estimate.
3. Find the M points which can be seen in both scans, and find the matching pairs of points.
4. Compute and estimate of $\vec{T}$ using Equation (10).
5. Using the translation estimates from the previous step, compute the rotation estimate using Equation (14).
6. If the error (7) is below a threshold, then stop and return the translation and rotation displacement estimates. Else, go to step 2.