ME 132(a): Final Exam, Winter 2016-17

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use any material on the course website, but not other internet sites.
4. You may use mathematica or any software or computational tools to assist you.
5. Feel free to ask me or the T.A.s questions about the exam.
6. The final is due by 5:00 p.m. on the last day of the final period. If you need your grade turned in to the registrar for purposes of graduation, then the final is due at 5:00 p.m. on Friday, June 3.
7. The point values are listed for each problem to assist you in allocation of your time.
8. Please put all of your answers in a blue book, or carefully staple the pages of your work together in the proper order (adding page numbers and/or problem numbers to the pages will during the grading process).

Problem #1 (25 points): Vorononi Graph

Consider a planar workspace populated with circular obstacles.

Part (a): (10 points) If all of the circular obstacles have the same radii, prove that the Vorononi graph edges are always straight lines.

Part (b): (15 Points) What other shape edges can occur when the obstacles do not have the same radii? For concreteness, determine the shape of the Voronoi graph edge when one circular obstacle has radius R_1, while another circular obstacle has radius R_2, where $R_1 \neq R_2$.

Problem #2 (20 Points): Configuration-Space Obstacles.

Consider the convex polygonal robot, $\mathcal{A}$, and obstacle, $\mathcal{B}$, shown in Figure 1. The obstacle is a rectangle with side dimensions of 5 and 10 units, whose center is coincident with the origin of the fixed workspace observing reference frame (whose axes are denoted by X_R and Y_R). The rectangle faces are parallel to the workspace reference frame axes. The robot is an isoceles triangle whose base dimension is 4 and whose height is 6. Its body fixed reference frame is located so that its x-axis is aligned with the triangle's centerline, and its origin is located at the vertex bounded by the two equal sides.

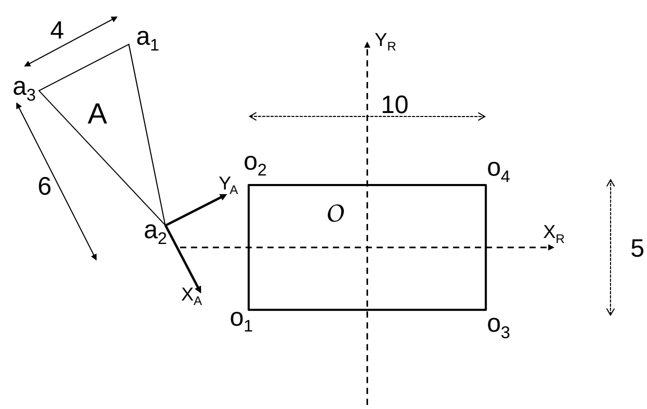

Figure 1: Triangular robot body and rectangular obstacle

Sketch the boundary of the configuration space obstacle when the orientation of the robot is fixed at $+45^o$.

Problem #3 (15 Points): Bug Algorithm Sensor-Based Motion Planning

Figure 2 has two copies of a simple environment. On the first environment model, sketch the operation of the Bug I algorithm (i.e., sketch the path that "bug" robot would take while executing the algorithm) from the start position to the goal position. On the second copy of the environment, sketch the operation of the Bug II algorithm.

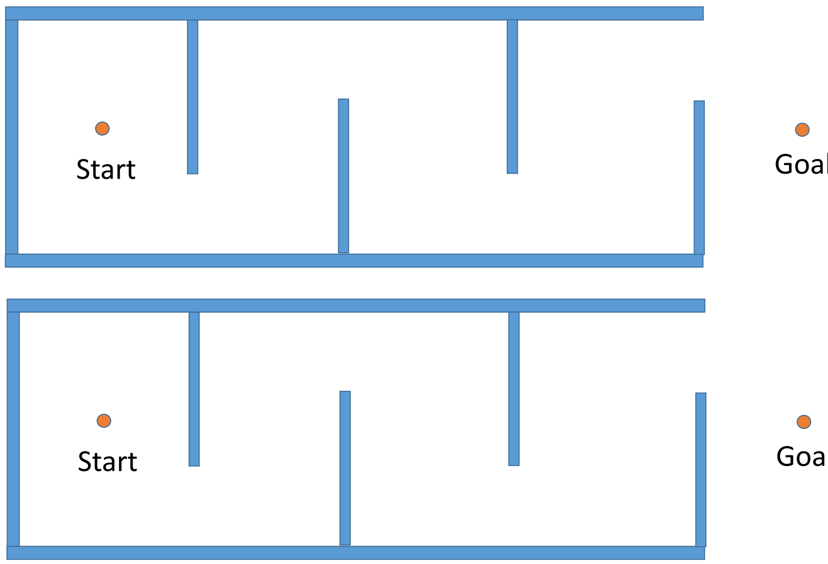

Figure 2: Workspace(s) for Bug I and Bug II Algorithm

Problem #4 (25 Points): C-space and motion planning of a planar linkage

Figure 3 shows a 2-link robot manipulator which moves in the plane. The configuration space (c-space) of this manipulator is formally a 2-torus, where θ_1 and θ_2 are two angles associated with the circular generators of the torus. For practical visualiation, we can represent the c-space as a square, with sides of length 2π, and with θ_1 and θ_2 coordinates.

Part (a): (17 points) *Sketch* the configuration space obstacle associated with the circular obstacle in the workspace.

Part (b): (8 points) *Sketch* on your c-space the *shortest path* in configuration space which connects an initial arm configuration (where $\theta_{1,start} = +45^o$, $\theta_{2,start} = -45^o$) with a final arm configuration ($\theta_{1,final} = -45^o$, $\theta_{2,start} = +45^o$). physically describe the motion of the robot arm along this c-space path.

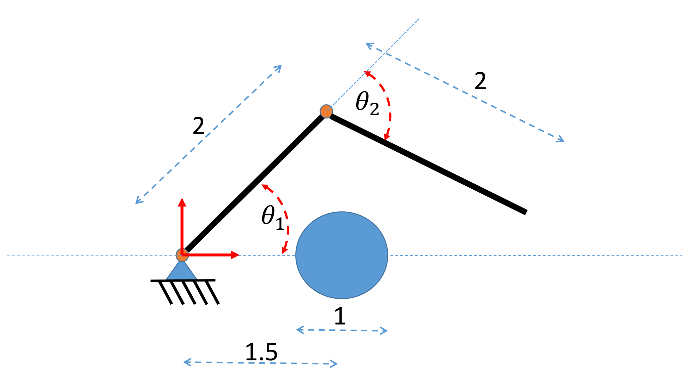

Figure 3: Two link robot arm with circular obstacle (diameter=1, center location at 1.5 on x-axis)