CDS 110: Homework #1
(Due Friday, October 14, 2016)

Problem 1 (CDS 101, CDS 110): (20 points for CDS 101, 25 Points for CDS 110)
The goal of this problem is to explore the behavior and stability of a hardening spring mass-spring-damper mechanical system, whose dynamics are given by:

\[ m\ddot{q} = -k(q + aq^3) - b\dot{q} \]

where \( m \) is the mass of the system, \( k \) is the “linear” spring constant, \( a \) is the hardening coefficient of the spring, and \( b \) is the “damping coefficient.” For this homework problem, choose \( m = 1000 \text{ kg} \), \( k = 250 \text{ kg/s}^2 \), \( a = 0.02 \), and \( b = 100 \text{ kg/s} \).

Part (a): Find the equilibria of this system.
Part (b): Using MATLAB, plot a representation of the phase space around an equilibrium point.
Part (c): Linearize the system about an equilibrium point.
Part (d): Determine the stability of this systems at an equilibrium (i.e., is it unstable, stable, or asymptotically stable).
Part (e), (CDS 110 only!): Is the system exponentially stable?

Problem 2 (CDS 101, 110): (15 points) Do problem 5.3 on page 5-34 of Chapter 5 in FBS-2e

Problem 3 (CDS 110): (15 points) Do problem 5.4 in page 5-34 in Chapter 5 of FBS-2e.

Problem 4 (CDS 110): (15 points) Do problem 5.10 in page 5-34 in Chapter 5 of FBS-2e.