

CDS 110: Homework #1

(Due Friday, October 14, 2016)

Problem 1 (CDS 101, CDS 110): (20 points for CDS 101, 25 Points for CDS 110)

The goal of this problem is to explore the behavior and stability of a *hardening spring* mass-spring-damper mechanical system, whose dynamics are given by:

$$m\ddot{q} = -k(q + aq^3) - b\dot{q}$$

where m is the mass of the system, k is the “linear” spring constant, a is the hardening coefficient of the spring, and b is the “damping coefficient.” For this homework problem, choose $m = 1000$ kg, $k = 250$ kg/s², $a = 0.02$, and $b = 100$ kg/s.

Part (a): Find the equilibria of this system.

Part (b): Using MATLAB, plot a representation of the phase space around an equilibrium point.

Part (c): Linearize the system about an equilibrium point.

Part (d): Determine the stability of this systems at an equilibrium (i.e., is it unstable, stable, or asymptotically stable).

Part (e), (CDS 110 only!): Is the system exponentially stable?

Problem 2 (CDS 101, 110): (15 points) Do problem 5.3 on page 5-34 of Chapter 5 in FBS-2e

Problem 3 (CDS 110): (15 points) Do problem 5.4 in page 5-34 in Chapter 5 of FBS-2e.

Problem 4 (CDS 110): (15 points) Do problem 5.10 in page 5-34 in Chapter 5 of FBS-2e.