The “Star” Algorithm for Computing Fixed-Orientation C-Obstacle Slices of Convex Objects  
(ME/CS 132)

The star algorithm is based on a simple geometric construction. Let $A(q)$ the set of points comprising the robot when it lies at configuration $q$. Let $O$ denote an obstacle, and let $CO$ denote its associated configuration space obstacle. If the robot’s orientation is fixed at some angle, $\theta^*$, the boundary of $CO$ in this fixed orientation slice through configuration space corresponds to all physical configurations where $A$ is touching $O$. Practically, this boundary can be constructed by “sliding” the robot (in a fixed orientation $\theta^*$) all around $O$ while maintaining contact between $A$ and $O$. The path traced out be the origin of $A$’s body fixed frame is exactly the shape of the fixed-orientation c-obstacle slice.

While this construction is clear from a graphical point of view, one needs a systematic procedure in order to turn this concept into an implementable algorithm. The key to a straightforward implementation is the star diagram, which is now defined.

Let the vertices of $A$ be labeled successively $a_1$, $a_2$, ..., $a_{NA}$ where $NA$ is the number of $A$’s vertices. Similarly, let $O$’s vertices be labeled $o_1$, $o_2$, ..., $o_{NO}$. Let $E^A_i$ denote the $i^{th}$ edge of $A$, which connects $a_i$ to $a_{i+1}$. Similarly, let $E^O_i$ denote the $i^{th}$ edge of $O$. Let $\alpha_i$ denote the inward pointing normal to $E^A_i$, while the outward pointing normal to $E^O_i$ is denote $\beta_i$. These variables are depicted in the figure below.

The star diagram is constructed from a translation of all of the normal vectors to a common origin. For example, the star diagram of the figure above is shown below. It is the angle of the vectors (say with respect to the x-axis) that matters in this diagram.
The star diagram assists you in determining the type of sliding contact that holds around the perimeter of the fixed-orientation slice, and the sequence of sliding operations that will construct the c-obstacle boundary in a continuous fashion. To start the construction, select a start angle. Typically the start angle will be zero degrees. Next choose either a clockwise or counterclockwise direction to proceed around the star diagram. Either choice will work fine, as long as you consistently move in the chosen direction. In our example, the start angle is shown by the blue dashed line, and we will proceed in a clockwise fashion. Consequently, the c-obstacle boundary will be constructed in a clockwise fashion.

The first normal vector, \( n_1 \), reached by proceeding in the chosen direction around the star diagram, (normal vector \( \beta_2 \) in our example) defines the first segment of the c-obstacle boundary to be constructed. If \( n_1 \) is associated with the obstacle (as is the case in our example), then the first segment will be constructed by sliding a vertex of \( A \) along the associated edge of \( O \). If \( n_1 \) was derived from the robot, then the segment will be constructed by sliding the associated edge of \( A \) along a vertex of \( O \). The vertex can be determined as follows. Find the two closest normals from the opposite object (i.e., if \( n_1 \) is associated with \( O \), then these normals must be associated with \( A \), and vice-versa) that bound \( n_1 \) in the clockwise and counterclockwise directions. Let these normal vectors be denoted \( n_j \) and \( n_k \). In our example, normal vectors \( \alpha_2 \) and \( \alpha_3 \) are closest to \( \beta_2 \), and lie on either side of \( \beta_2 \). The edge associated with \( n_1 \) contacts the vertex that joins the edges associated with normals \( n_j \) and \( n_k \). In our example, vertex \( a_3 \) joins edges \( E^A_2 \) and \( E^A_3 \). Thus this segment physically corresponds to the sliding of vertex \( a_3 \) along \( E^O_2 \).

The construction of the fixed-orientation c-obstacle boundary then continues by proceeding around the diagram. In our example, the next normal vector reached by continuing in a clockwise direction is \( \alpha_3 \), which is bounded by normal vectors \( \beta_1 \) and \( \beta_2 \). Thus, the next contiguous segment of the c-obstacle boundary will physically correspond to sliding of \( E^A_3 \) (which is associated with \( \alpha_3 \)) along vertex \( o_2 \) (the vertex joining edges \( E^O_1 \) and \( E^O_2 \), which are the associated adjacent normal vectors \( \beta_1 \) and \( \beta_2 \)). Once all of the normal vectors in the star diagram have been processed, the boundary of the fixed-orientation c-obstacle “slice” is complete.