Problem #1: Consider a fixed orientation slice through the configuration space obstacle associated with a convex polygon robot and a convex polygon obstacle. We know that the boundary of this slice is a polygon. Prove that the polygon is convex.

Problem #2: Consider the case of a 2-dimensional configuration space populated by planar polygonal c-obstacles. In such c-spaces, the arcs of the Voronoi diagram are locally of three types, depending on the two local polygon features which are equidistance from the Voronoi arc:

1. the two closest polygon features are an edge and an edge
2. the two closest polygon features are a vertex and an edge
3. the two closest polygon features are a vertex and vertex.

Derive formulas for the Voronoi arcs for the three cases described above, and qualitatively describe the geometry of each type of arc.

Background on Problems #3, 4, 5.

Consider the “workspace” shown in Figure 1. One might think of this diagram as being related to the following problem. A house must be painted, and in the process a bed must be moved from the bedroom to the living room. The two rooms are connected by a short corridor. The bed is first flipped on its side so it looks like a rectangle. The rectangle is the moving robot $A$. The outer wall is represented by union of four rectangular obstacles $B_{01}, B_{02}, B_{03}, B_{04}$. The interior walls are rectangular obstacles $B_1, ..., B_5$.

Problem #3: Sketch the Voronoi Diagram on a copy of this workspace, assuming that the robot is a point robot.

Problem #4: Assume that we create a “grid,” such as the one superimposed on Figure 1. Assume that for the purposes of the wavefront potential (or brushfire) method for constructing the Voronoi graph, you have a matrix whose individual entries are an integer that indicates the status of each cell. The status of each cell will be one of the following: (1) a cell is occupied by an obstacle; (2) a cell is freespace and has not been visited; and (3) a cell is freespace, but has been visited and has been assigned an integer.

In preparation for the next problem, write a program that will “print” the contents of each cell in an array structure. If the cell is an obstacle, print an “X.” If the cell is free space,
then print a “blank” if it has not been visited, or print the assigned integer if it has been visited.

Show a printout for the workspace in Figure 1 in the case that no cell has been visited.

**Problem #5:** Assume that the robot is a point robot. Assume that the robot’s initial configuration, $q_I$, is located in the cell that is 6 cells down and 5 cells to the right of the top left corner. Assume that the robot’s goal configuration, $q_F$, is located 4 cells to the right and 4 cells up from the bottom left corner. Implement the part of the wave front potential method that fills all of the grid cells with integers, thereby approximating a Voronoi Diagram. Print out your result, and then hand-draw the path from start to goal (using the approximate Voronoi Diagram as a roadmap) using your results.