ME/CS 132(a): Homework #3

(Due Friday, Feb. 17, 2017)

Consider a point robot located in the environment seen in the "sphere world" of Figure 1. The radius of the bounding sphere is $R_B = 10$. The two circular obstacles each have identical radius $R_O = 3$. The centers of both obstacles lie on the y-axis, and are each located a distance of 5 units from the bounding circle center. Assume that the point robot's initial position is located at a distance of 7 units from the origin of the bounding sphere along the negative x-axis. Consider two different possible goal positions, q_{f1} and q_{f2}. The first goal position is located along the positive x-axis a distance of 7 units from the center. The second goal position is located a distance of 7 units from the center, but is located along a line that makes a 60^o angle with the x-axis.

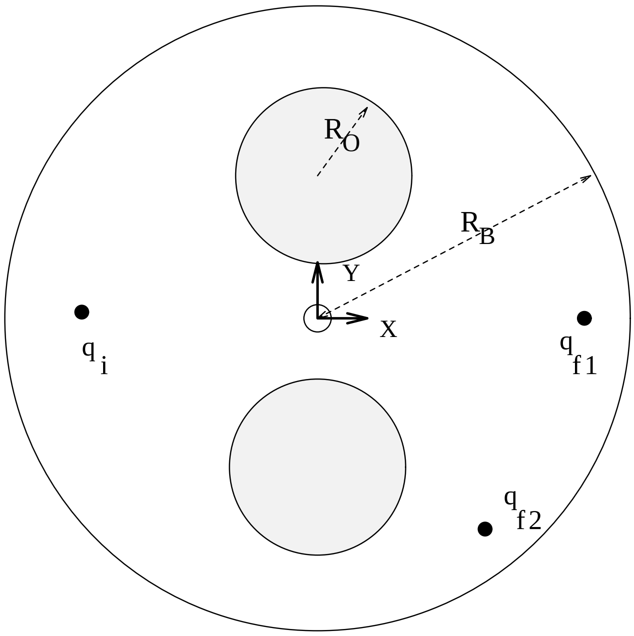

Figure 1: Schematic Diagram of Simplified Robot Environment

The following problems are a sequence of programming exercises related to the potential field method (the classical potential field consisting of a linear superposition of attractive and repulsive potentials).

Problem 1: (10 Points) Plot (using Mathematica, Matlab, or some other user-friendly plotting system) the attractive potential $U_{attr}(q) = \frac{\xi}{2} d^2_{goal}(q)$ for the two different goal positions.

Problem 2: (15 Points) consider the repelling potential function

$$U^i_{rep}(q) = \begin{cases} \frac{\eta}{2}(\frac{1}{d_i(q)} - \frac{1}{\rho_0})^2 & \text{for } d_i(q) \leq \rho_0 \\ 0 & \text{for } d_i(q) > \rho_0 \end{cases}$$

where $d_i(q)$ is the distance between q and the i^{th} obstacle. Recall that the distance between a point and the set of points defining obstacle $\mathcal{O}_i$ is defined as:

$$d_i(q) = \min_{y \in \ \mathcal{O}_i} ||q - y|| \ .$$

You will have to create a function which measures the distance between point q and a circle of known radius and known center.

Plot the repelling potential made up of the sum of the boundary and obstacle repelling potentials: $U_{rep}(q) = \sum_i U^i_{rep}$.

Problem 3: (10 Points) Plot the potential $U(q) = U_{attr}(q) + U_{rep}(q)$ for the two different goal positions. Choose different constants η and ξ.

Problem 4: (20 points) For a given choice of η and ξ and for each of the goal positions, plot the path that results from solving the equation:

$$\dot{q} = -\nabla U(q)$$

or from the equation:

$$m\ddot{q} = -\nabla U(q)$$

where m is the "mass" of the virtual particle. If you choose the later approach, you may wish to add some "damping" to the equations as a crude guard against certain numerical roundoff errors. This can be done by additing a damping term $-b\dot{q}$, where b is a constant.

Extra Credit: Repeat problems 3 and 4 above using the attractive and respulsive potentials of Rimon and Koditschek. Note, you may have to play with the value of the exponent of the attractive potential until you find a suitable number.