ME/CS 133(a): Lab #1

(Due Friday Nov. 17. 2017)

This lab is "computational" in nature. The goal is to get familiar with motion tracking, and in particular the OptiTrack motion tracking sytem in the CAST lab.

During class time in the CAST center, a group of students carried a "wand" around the motion tracking volume. The wand contained 5 markers, whose positions were tracked by the OptiTrack camera tracking system (at approximately 120 times per second). In addition to tracking the the marker positions, Optitrack also estimated the position of a reference frame attached to the wand, assuming that all 5 markers were attached to the same rigid body.

Data: A "snippet" of the data gathered during this experiment is the basis for this lab:

- Download the file "me133a_6nov_wand_250_270m.csv" from the course web site. This spreadsheet contains 20 seconds of data (sampled at 120 frames per second) from 250 to 270 seconds after the start of the data gathering process. Note that the first 10 seconds contains both data on the marker positions as well as OptiTrak's estimate of body configuration. The last 10 seconds contains only the data from the 5 markers (the body position estimate is removed).
- Download the file "optiTrack_matlab_template.m". This file allows you to read in the data from the .CSV file into MATLAB. You need not use MATLAB for this assignment.

The top rows of the .CSV file contain labels to tell you the source and nature of the data in each column. These columns have the following format:

- **Columns 1, 2:** The first column is an index of the "frame." The second column is the actual time at which the data is gathered.
- **Columns 3, 4, 5, and 6** (labeled X, Y, Z, and W). Each column is one of the four elements of a quaternion which quantifies the estimated orientation of the object (obtained by using all 5 markers). These columns correspond to the quaternion that represents rotation angle ϕ about rotation axis $\vec{\omega} = \begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}$ as:

$$q = \underbrace{\cos\left(\frac{\phi}{2}\right)}_{column\ W} + \underbrace{\omega_x \sin\left(\frac{\phi}{2}\right)}_{column\ X} i + \underbrace{\omega_y \sin\left(\frac{\phi}{2}\right)}_{column\ Y} j + \underbrace{\omega_z \sin\left(\frac{\phi}{2}\right)}_{column\ Z} k \ . \qquad (1)$$

- **Columns 7, 8 ,9:** These are OptiTrack's estimate of the rigid body's x, y, and z positions, as estimated by tracking the 5 markers.

- **Columns 10:** The "error per marker" column is an internal metric of the amount of error in OptiTrack's estimate of the rigid body's position.
- **Columns 11, 12, 13:** These are the x, y, and z measured positions of the first marker.
- **Column 14:** This is an internal OptiTrack estimate of the error in measuring the marker's location.
- **Remaining Columns:** The data from the other 4 markers is stored in the subsequent columns, using the same format: the x, y, and z positions, as well as an internal marker quality estimate.

Assignment: The goal of this assignment is to compare the use of the Rodriguez' spatial displacement equation to estimate body position from motion tracking markers, versus Optitrack's proprietary method to track position.

Recall that Rodriguez' spatial displacement equation, when applied to the successive positions of three Cartesian points (P, Q, R) yield an estimate for the screw displacement parameters $(\phi, d^{||}, \vec{\omega}, \rho_\perp)$ that are consistent with the displacement of the three points from their initial positions (P_1, Q_1, R_1) to the their subsequent positions (P_2, Q_2, R_2).

At a minimum, you should choose 3 of the 5 markers in the data set provided to you, and estimate the displacement between each frame using the Rodriguez equation. You should then convert the screw displacement parameters to position and quaternion parameters of displacement. Recall from Equation (2.40) of the Murray, Li, Sastry book that:

$$g_{AB} = \begin{bmatrix} R_{AB} & \vec{d}_{AB} \\ \vec{0}^T & 1 \end{bmatrix} = \begin{bmatrix} e^{\phi\hat{\omega}} & (I - e^{\phi\hat{\omega}})\rho_\perp + h\phi\vec{\omega} \\ \vec{0}^T & 1 \end{bmatrix}$$

where the pitch h is: $h = \frac{d^{||}}{\phi}$.

NOTE: The displacement, d_{AB}, computed by this method is **not** the same displacement calculated by OptiTrack. To interpret this displacement, imagine that the "wand" rigid body at frame i is extended as far as necessary so that there exists a reference frame rigidly attached to "wand" that is in coincidence with the origin of OptiTrack's world reference frame at i. Then the "wand" displaces to a new position at frame k where $k \geq (i + 1)$. The homogeneous transformation g_{AB} measures the displacement of this second body fixed reference frame when the "wand" body moves between frame i and frame k. Hence, if $g_{opt,i}$ is the homogeneous transformation

$$g_{opt,i} = \begin{bmatrix} R_{opt,i} & \vec{d}_{opt,i} \\ \vec{0}^T & 1 \end{bmatrix}$$

which describes the location of the body-fixed reference on the "wand" in frame i, and if $g_{opt,k}$ is the homogeneous matrix that describes the location of the wand's body fixed frame (the one tracked by OptiTrack) at frame k, then:

$$g_{opt,k} = g_{AB} g_{opt,i}$$

Part (a): The first part of your assignment is to compare your predicted positions of the "wand" with the positions estimated by OptiTrack during the first 10 seconds of the data set. Note, the OptiTrack estimates the absolute position and orientation, while the Rodriguez equation gives you the displacement between frames. To compare the two:

- To compare your estimate of the net change in orientation, you first must calculate from the OptiTrack data the change in orientation between frames. If the data were represented in terms of orientation matrices, then if R_i were the orientation of the "wand" at frame i, and if R_k were the orientation at frame k, then the net change of orientation, $R_{\delta,ik}$ is:

  $$R_{\delta,ik} = R_k R_i^{-1}.$$

  Because OptiTrack provides information in a quaternion format, then quaternion representation of the change in orientation between frame i and k is: $q_k q_i^{-1}$, where q_i and q_k are the OptiTrack quaternion estimates as frames i and k.

  You can plot your estimates versus OptiTrack's estimates in one of two ways. In the first procedure, you find the displacements between each adjacent pair of frames i, and $(i+1)$. Alternatively, you can estimate the change in orientation between frame 1 and frame k for $k = 2, 3, \ldots$. Plot your estimated change and OptiTrack's estimated change in orientation (versus frame index) on the same plot. You will have 4 such plots for each of the 4 quaternion parameters.

- Next visualize the displacement estimates of the Rodriguez' Equation, and the OptiTrack system. If $g_{AB,ik}$ is your estimated homogeneous displacement between frame i and frame k, then

  $$\hat{g}_{opt,k} = g_{AB,ik} g_{opt,i}$$

  where $\hat{g}_{opt,k}$ is the *estimated* position and orientation of the wand at frame k, where the estimate comes from the Rodriguez' equation. You have already compared the rotation estimates above. To compare the displacement estimate, plot $\hat{d}_{opt,k}$ and the measured $\vec{d}_{opt,k}$ from OptiTrack on the same plot. To most easily visualize the difference plot your displacement estimate versus frame number, and plot OptiTrack's displacement measurement versus frame number on the same plot. Because there are x, y, and z coordinates, you will have 3 such plots.

Part (b): In the second part of the assignment, plot your estimate of "wand" displacements (you need not plot the orientations) during the last 10 seconds of the data set.