CDS 110: Homework #1

(Due Friday, October 7, 2016)

Problem 1 (CDS 101, CDS 110): (5 points)

Search the web for an article in the popular press about a feedback and control system. Describe the feedback system using the terminology given in the article. In particular, identify the control system and describe (a) the underlying process or system being controlled, along with the (b) sensor, (c) actuator and (d) computational element. If the some of the information is not available in the article, indicate this and take a guess at what might have been used.

Problem 2 (CDS 101, CDS 110): (20 points)

The goal of this problem is to see, in the context of a problem, how feedback can modify a system's performance. Consider a simplified model of a cruise control for a car

$$m\dot{v} = -av + u + d$$

where m is the mass of the car, v is the car's forward velocity, a is a "drag coefficient" which models the effects of air drag, u is the control input (a lumped model which accounts for the effect of the motor's output on the traction force at the wheel-to-ground contact), and d is a disturbance. Let's ignore the disturbance for now.

Part (a): Consider an *open loop* control strategy defined by

$$u = \hat{a}v_{ref}$$

where $\hat{a}$ is an *estimate* of the drag coefficient and v_{ref} is the desired reference speed. Note that the estimate $\hat{a}$ may not be accurate. Compute the steady state response for this strategy as a function of $\beta = a/\hat{a}$. You should plot v_{ss}/v_{ref} as a function of β, where v_{ss} is your steady state solution

Part (b): Consider an *proportional closed loop* control strategy defined by

$$u = -k_p(v - v_{ref})$$

where v_{ref} is the desired reference speed and k_p is the *proportional gain.* Compute the steady state response for this strategy when $kp = 10\hat{a}$, and again plot the result as a function of β.

j **Part (c):** Next consider a *proportional-integal* (PI) feedback control strategy.

$$u = -k_p(v - v_{ref}) - k_i \int_0^t (v - v_{ref})dt.$$

Again, compute the steady state response, and compare the result to your answers found in Parts (a) and (b). *Hint:* in one way of solving this problem, introduce $q = \int_0^t (v - v_{ref})dt$, and then $\dot{q} = (v - v_{ref})$

Problem 3 (CDS 110): (15 points) Consider the differential equations:

$$\frac{dy}{dt} + 3y = 4u, \tag{1}$$

$$\frac{d^2}{dt^2} + 2\frac{dy}{dt} + y = 2\frac{du}{dt} + u \tag{2}$$

where u is the system input, and y is the system output.

Part (a): Solve each of these equations.

Part (b): Find the response of each system to: (1) a unit step input, $u(t) = 1$; (2) an exponential signal $u(t) = e^{st}$.

Part (c): Derive the transfor functions for each of these systems.