CDS 101/110: Homework #4

(Due Friday, October 28, 2016)

Problem 1 (CDS 101, CDS110): (35 points)

This problem considers the design of a state feedback controller to stabilize a simplified model of satellite (Figure 1). On the line connecting the center of the Earth to the center of the Earth's Moon, is a *Libration point*, L, where the gravitational pull of the Earth on a satellite is the same as the gravitational pull of the Moon plus the centrifugal force of the satellite's orbit (assuming that the satellite is orbiting the Earth with the same period as the Moon). However, this point is naturally unstable, and therefore a feedback controller is needed to stablize the satellite.

The dynamic equations for small deviations of the satellite away from the libration point can be described as:

$$\ddot{x} - 2\omega\dot{y} - 9\omega^2 x = 0 \quad (1)$$
$$\ddot{y} + 2\omega\dot{x} + 4\omega^2 y = u \quad (2)$$

where x is the radial perturbation (along the line connecting Earth and Moon), y is the azimuthal perturbation (transverse to the radial direction), m is the satellite mass, $\omega = 2\pi/29$ radians/day. The control $u = F/(m\omega^2)$, where F is the engine thrust in the y direction.

Part (a): With $u = 0$, show that the equilibrium point $x = y = 0$ is unstable.

Part (b): To stablize the position, use a state-feedback of the form

$$u = -k_1 x - k_2 \dot{x} - k_3 y - k_4 \dot{y} \ .$$

Choose the gains so that the poles are places at $s = -3\omega,\ s = -4\omega,\ s = (-3 \pm 3i)\omega$.

Part (c): Plot the (x, y) phase space of the feedback-controlled system in the vicinity of the equilibrium

Problem 2 (CDS 101, CDS 110): (5 points) Do problem 7.2 in Chapter 7 of FBS, 2^{nd} edition.

Problem 3 (CDS 110): (25 points) Do problem 7.13 in Chapter 7 of FBS-2e.

Problem 4 (CDS 110): (10 points) Do problem 7.10 in Chapter 7 of FBS-2e.

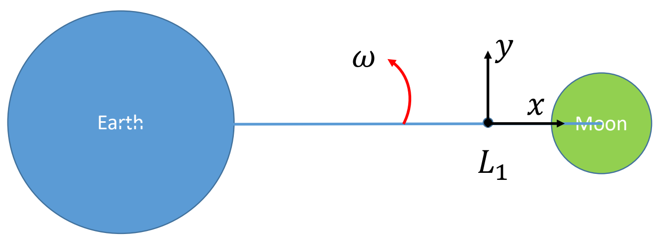

Figure 1: Schematic of Earth, Moon, and L_1 Libration point