Laser Scan Matching

Range Scan Points
Lab Data:
53 Poses
32.8 meters
Raw Points

Kalman Filter
Based SLAM
Algorithm

Merged Line Map:
76 lines total
Derivation of Displacement Estimate

Let \( u_{i,k} = \) location of \( k^{th} \) scan point at configuration \( i \)

\[
  u_{i,k} = r_k \begin{bmatrix} \cos \beta_k \\ \sin \beta_k \end{bmatrix}
\]

Transform scan points in one local scan reference frame to the other reference frame which is to be matched.

- Transform points from the local frame of configuration \( q_{j+1} \) to the frame of \( q_j \)

\[
  u_{i+1,k}^i = \begin{bmatrix} \Delta x_{i+1} \\ \Delta y_{i+1} \end{bmatrix} + \left( \begin{array}{cc} \cos \Delta \theta_{i+1} & -\sin \Delta \theta_{i+1} \\ \sin \Delta \theta_{i+1} & \cos \Delta \theta_{i+1} \end{array} \right) u_{i+1,k}^{i+1} = T_{i+1} + R(\Delta \theta_{i+1})u_{i+1,k}^{i+1}
\]

Using an estimate of the displacement between frames:

\[
  \Delta q_{i+1} = [\Delta x_{i+1} \quad \Delta y_{i+1} \quad \Delta \theta_{i+1}]^T = q_{i+1} - q_i
\]

To improve the estimate of \( \Delta q_{i+1} \), define an Error Function that models errors in the matching of associated range points in the two sets

\[
  E(\Delta \theta_{i+1}, T_{i+1}) = \sum_{k=1}^{N_{\text{match}}} ||u_{i+1,k}^{i} - u_{i,k}||^2 = \sum_{k=1}^{N_{\text{match}}} ||T_{i+1} + R(\Delta \theta_{i+1})u_{i+1,k}^{i+1} - u_{i,k}||^2
\]
Derivation of Displacement Estimate

The matching error will be minimized when:

\[
\frac{\partial E}{\partial \Delta q_{i+1}} = 0 \implies \frac{\partial E}{\partial T_{i+1}} = 0; \quad \frac{\partial E}{\partial \Delta \theta_{i+1}} = 0
\]

Taking the derivatives and performing some algebra yields:

\[
\begin{bmatrix}
\Delta x_{i+1} \\
\Delta y_{i+1}
\end{bmatrix} = \begin{bmatrix} x' - (xcos \Delta \theta - ysin\Delta \theta) \\
y' - (xsin\Delta \theta + ycos\Delta \theta)
\end{bmatrix}; \quad \Delta \theta_{i+1} = \tan^{-1}(\tan \Delta \theta)
\]

Where

\[
\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{N_{\text{match}}} \sum_{k=1}^{N_{\text{match}}} u_{i+1,k}; \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{N_{\text{match}}} \sum_{k=1}^{N_{\text{match}}} u_{i,k}; \quad \tan \Delta \theta = \frac{\Delta xy' - \Delta yx'}{\Delta xx' - \Delta yy'}
\]

\[
\Delta xx' = \sum_{k=1}^{N_{\text{match}}} (x_i - x)(x_i' - x'); \quad \Delta yy' = \sum_{k=1}^{N_{\text{match}}} (y_i - y)(y_i' - y')
\]

\[
\Delta xy' = \sum_{k=1}^{N_{\text{match}}} (x_i - x)(y_i' - y'); \quad \Delta yx' = \sum_{k=1}^{N_{\text{match}}} (y_i - y)(x_i' - x')
\]
Robot Localization
(where am I?)

Indoors

Undersea

Space

Underground
Landmark–based Localization & Mapping

**Localization**: A robot explores an static environment where there are known *landmarks*.
- radio beacons (Lojack)
- infrared beacons (Northstar)
- bar-code decals

Estimate the robot’s position

**Mapping**: A robot explores an unknown static environment where there are identifiable landmarks. *E.g.*:
- doors, windows, light fixtures
- linoleum floor patterns

Build a *map* (estimate all landmark positions)
Estimation & Optimal (Kalman) Filtering

Observer

- Given \( \dot{x} = f(x, u) \)
- Calculate, infer, deduce the state \( x \) from measurements \( y \)
- E.g. the Luenberger Observer \( \dot{x} = Ax + Bu + L(y - Cx) \)

Estimator

- Given \( \dot{x} = f(x, u) + \xi \)
- \( \xi \) represents process noise/uncertainty (e.g., gust or unmodeled effects)
- \( \omega \) represents measurement noise/uncertainty
- Estimate (in an optimal) way the state \( x \) based on
  - measurements \( y \)
  - dynamic and measurement models
  - noise model(s).
Estimation Overview (continued)

Noise & Uncertainty Models for Estimation

\[ \dot{x} = f(x, u) + \xi \quad y = h(x) + \omega(t) \]

- Set-based: \( \xi \in \Xi \quad \omega \in \Omega \)
- Stochastic: \( \xi \) and \( \omega \) are random processes governed by \( p(\xi) \) and \( p(\omega) \)

Why Estimation?

- Enables state feedback control design (separation principle)
- **MANY** important problems can be posed as estimation problems. *E.g.*:
  - Inertial Navigation
  - Tracking and Prediction
  - Parameter Estimation
  - Sensor Processing and Fusion

Specialized estimation techniques & literatures
**SLAM: Simultaneous Localization & Mapping**

**Given:**
- Robot motion model: \( \dot{x} = f(x, u) + \xi \)
- The robot’s controls, \( u \)
- Measurements (e.g., range, bearing) of nearby features: \( y = h(x) + \omega \)

**Estimate:**
- Map of landmarks \( (x_L) \)
- Robot’s current pose, \( x_R \), & its path
- Uncertainties in estimated quantities

\[
\bar{x} = \begin{bmatrix} \bar{x}_R \\ \bar{x}_L \end{bmatrix} = \begin{bmatrix} x_R \\ x_{L_1} \\ \vdots \\ x_{L_N} \end{bmatrix}
\]

**Diagram:**
- Robot state \( x_R \)
- Landmark positions \( x_L \)