ME/CS 133(a): Lab #1

(Due Friday Nov. 9. 2018)

This lab is "computational" in nature. The goal is to get familiar with motion tracking, and in particular the OptiTrack motion tracking sytem in the CAST lab. You can perform this lab in groups of up to 3 people. Only one lab report need be submitted for the entire group.

During your lab time in the CAST center, a "wand" will be carried around in the tracking volume of the OptiTrack system. The wand contains 5 markers, whose positions were tracked by the OptiTrack camera tracking system (at approximately 120 times per second). In addition to tracking the marker positions, Optitrack also estimated the position of a reference frame attached to the wand, assuming that all 5 markers were attached to the same rigid body.

Data: A "snippet" of the data gathered during this experiment is the basis for this lab:

- Download the data file (in a "csv" format) from the course web site. This spreadsheet contains an interval of OptiTrack data (sampled at 120 frames per second) from a period after the start of the data gathering process. Note that the first part of the data contains both marker positions as well as OptiTrak's estimate of body configuration. The second part contains only the data from the markers (the body position estimate is removed).
- Download the file "optiTrack_matlab_template.m". This file allows you to read in the data from the .CSV file into MATLAB. You need not use MATLAB for this assignment.

The top rows of the .CSV file contain labels to tell you the source and nature of the data in each column. These columns have the following format:

- **Columns 1, 2:** The first column is an index of the "frame." The second column is the actual time at which the data is gathered.
- **Columns 3, 4, 5, and 6** (labeled X, Y, Z, and W). Each column is one of the four elements of a quaternion which quantifies the estimated orientation of the object (obtained by using all the markers in the field of view). These columns correspond to the quaternion that represents rotation angle ϕ about rotation axis $\vec{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}$ as:
$$q = \underbrace{\cos\left(\frac{\phi}{2}\right)}_{column\ W} + \underbrace{\omega_x \sin\left(\frac{\phi}{2}\right)}_{column\ X} i + \underbrace{\omega_y \sin\left(\frac{\phi}{2}\right)}_{column\ Y} j + \underbrace{\omega_z \sin\left(\frac{\phi}{2}\right)}_{column\ Z} k\ . \tag{1}$$
- **Columns 7, 8 ,9:** These are OptiTrack's estimate of the rigid body's x, y, and z positions, as estimated by tracking the 5 markers.

- **Columns 10:** The "error per marker" column is an internal metric of the amount of error in OptiTrack's estimate of the rigid body's position.
- **Columns 11, 12, 13:** These are the x, y, and z measured positions of the first marker.
- **Column 14:** This is an internal OptiTrack estimate of the error in measuring the marker's location.
- **Remaining Columns:** The data from the other 4 markers is stored in the subsequent columns, using the same format: the x, y, and z positions, as well as an internal marker quality estimate.

Assignment: The goal of this assignment is to show how your knowledge of different rigid body coordinate systems can be used to implement an important part of the motion capture process. Moreover, this lab will familiarize students with the CAST facility, and the OptiTrack system.

In summary, your goal is to use (1) the Rodriguez space displacement equation and (2) the quaternion-based Least Squares approach to estimate the the position of the moving want from the marker data. During the first interval of time, you will be able to compare your results against OptiTrack's estimates. During the second data interval, you are to calculate the wand's location.

Rodriguez Spatial Displacement Equation: Recall that Rodriguez' spatial displacement equation, when applied to the successive positions of three Cartesian points (P, Q, R) yield an estimate for the screw displacement parameters $(\phi, d^{||}, \vec{\omega}, \rho_{\perp})$ that are consistent with the displacement of the three points from their initial positions (P_1, Q_1, R_1) to the their subsequent positions (P_2, Q_2, R_2).

At a minimum, you should choose 3 of the 5 markers in the data set provided to you, and estimate the displacement between each frame using the Rodriguez equation. You should then convert the screw displacement parameters to position and quaternion parameters of displacement. Recall from Equation (2.40) of the Murray, Li, Sastry book that:

$$g_{AB} = \begin{bmatrix} R_{AB} & \vec{d}_{AB} \\ \vec{0}^T & 1 \end{bmatrix} = \begin{bmatrix} e^{\phi\hat{\omega}} & (I - e^{\phi\hat{\omega}})\rho_{\perp} + h\phi\vec{\omega} \\ \vec{0}^T & 1 \end{bmatrix}$$

where the pitch h is: $h = \frac{d^{||}}{\phi}$. Also recall that a unit quaterion that represents a rotation of angle ϕ around rotation axis $\vec{\omega}\, \begin{bmatrix} \omega_x & \omega_y \omega_z \end{bmatrix}^T$ is given by

$$q = \left(\cos\left(\frac{\phi}{2}\right), \omega_x \sin\left(\frac{\phi}{2}\right), \omega_y \sin\left(\frac{\phi}{2}\right), \omega_z \sin\left(\frac{\phi}{2}\right) \right) \qquad (2)$$

NOTE: The displacement, d_{AB}, computed by this method is **not** the same displacement calculated by OptiTrack. To interpret this displacement, imagine that the "wand" rigid body at frame i is extended as far as necessary so that there exists a reference frame rigidly attached to "wand" that is in coincidence with the origin of OptiTrack's world reference frame at i. Then the "wand" displaces to a new position at frame k where $k \geq (i+1)$. The homogeneous transformation g_{AB} measures the displacement of this second body fixed reference frame when the "wand" body moves between frame i and frame k. Hence, if $g_{opt,i}$ is the homogeneous transformation

$$g_{opt,i} = \begin{bmatrix} R_{opt,i} & \vec{d}_{opt,i} \\ \vec{0}^T & 1 \end{bmatrix}$$

which describes the location of the body-fixed reference on the "wand" in frame i, and if $g_{opt,k}$ is the homogeneous matrix that describes the location of the wand's body fixed frame (the one tracked by OptiTrack) at frame k, then:

$$g_{opt,k} = g_{AB} g_{opt,i}$$

Quaternion-Based Least Squares Approach: In the class notes, we showed that if N markers are located at positions $P_1, \ldots, P_N$ before displacement, and they move to new locations $Q_1, \ldots, Q_N$ after displacement, then the quaternion q_{12} that represents the rotation of the object between the two positions is given by the quaternion that maximizes the cost function

$$q_{12}^T N q_{12}$$

where matrix N is a function of all of the data points. The wand translation, $\vec{d}_{12}$, is computed by

$$\vec{d}_{12} = \bar{Q} - R_{12}\bar{P} \qquad (3)$$

where $\bar{P}$ and $\bar{Q}$ are the *centroids* of the data points. Hence, in this approach, one computes matrix N from the data points, and then estimates q_{12} as the largest eigenvector of N. From Equation (2) one can find the rotation angle ϕ and rotation axis $\vec{\omega}$. Then compute $R_{12}(\phi, \vec{\omega})$, which allows the translation d_{12} to be computed from Equation (3).

Part (a): The first part of your assignment is to compare your predicted positions of the "wand" with the positions estimated by OptiTrack during the first 10 seconds of the data set. Note, the OptiTrack estimates the absolute position and orientation, while the Rodriguez or quaternion approaches gives you the displacement between frames. To compare the two:

- To compare your estimate of the net change in orientation, you first must calculate from the OptiTrack data the change in orientation between frames. If the data were represented in terms of orientation matrices, then if R_i were the orientation of the "wand" at frame i, and if R_k were the orientation at frame k, then the net change of orientation, $R_{\delta,ik}$ is:

  $$R_{\delta,ik} = R_k R_i^{-1}.$$

  Because OptiTrack provides information in a quaternion format, then quaternion representation of the change in orientation between frame i and k is: $q_k q_i^{-1}$, where q_i and q_k are the OptiTrack quaternion estimates as frames i and k.

You can plot your estimates versus OptiTrack's estimates in one of two ways. In the first procedure, you find the displacements between each adjacent pair of frames i, and $(i+1)$. Alternatively, you can estimate the change in orientation between frame 1 and frame k for $k = 2, 3, \ldots$. Plot your estimated change and OptiTrack's estimated change in orientation (versus frame index) on the same plot. You will have 4 such plots for each of the 4 quaternion parameters.

- Next visualize the displacement estimates of the Rodriguez approach, the quaternion-based approach, and the OptiTrack system. If $g_{AB,ik}$ is your estimated homogeneous displacement between frame i and frame k, then

$$\hat{g}_{opt,k} = g_{AB,ik} g_{opt,i}$$

where $\hat{g}_{opt,k}$ is the *estimated* position and orientation of the wand at frame k. You have already compared the rotation estimates above. To compare the displacement estimate, plot $\hat{d}_{opt,k}$ and the measured $\vec{d}_{opt,k}$ from OptiTrack on the same plot. To most easily visualize the difference plot your displacement estimate versus frame number, and plot OptiTrack's displacement measurement versus frame number on the same plot. Because there are x, y, and z coordinates, you will have 3 such plots. Of course, they can be plotted on a common coordinate system.

Part (b): In the second part of the assignment, plot your estimate of the "wand" translations (you need not plot the orientations) during the last 10 seconds of the data set.