ME/CS 133(a): Final Exam
(Fall Quarter 2017/2018)

Instructions

1. Limit your total time to 5 hours. You can take a break in the middle of the exam if you need to ask a question, or go to dinner, etc. That break time does not count toward the total exam time, as long as you are not working on the exam during the break.

2. The time that it takes to read these instructions is not included in the 5 hour total.

3. You may use (1) your class notes, (2) the course textbook, (3) any material handed out in class, or on the class web site. The internet is off-limits to solve these problems, except for material on the course web site. You may not discuss this final with other class students or other people except the class instructor or the class Teaching Assistants.

4. You may use Mathematica, MATLAB, or any software or computational tools to assist you. However, if you find that your solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.

5. The final is due by 5:00 p.m. on the last day of finals.

6. The point values are listed for each problem to assist you in the allocation of your time.

7. Please put all of your work in a blue book, or carefully staple the pages of your solution in the proper order.

8. You will receive *one extra bonus point* if you scan your solutions and email the scanned document to the course Teaching Assistants: *me133tas@robotics.caltech.edu*. The time that it takes to scan and mail your final exam *does not* count against the 5 hour total. If you cannot scan your final, please turn in your exam to Sonya Lincoln, whose office is located at room 250 of the Gates-Thomas building. We will scan your final, which assists us with the grading process.

Problem 1: (25 Points)

The *Armatron* was a toy robot distributed by the Radio Shak company for many years. Figure 1 shows the first three

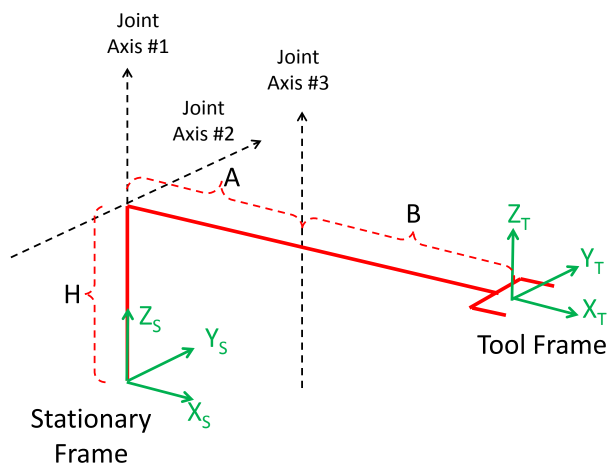

Figure 1: Schematic of Armatron Manipulator Geometry

Part (a): (5 points) Determine the Denavit-Hartenberg parameters of this manipulator

Part (b): (10 points) Using either the Denavit-Hartenberg approach or the product-of-exponentials approach, determine the forward kinematics.

Part (c): (10 points) Develop an expression for the spatial Jacobian matrix for this manipulator.

Problem 2: (15 Points)

We discovered numerous ways to represent and manipulate spatial displacements. Those crazy kinematicians have yet another variation on the same theme using something called "dual numbers." A dual number, $\tilde{a}$, takes the form:

$$\tilde{a} = a_r + \epsilon\ a_d$$

where a_r is the "real" part of the dual number and a_d is the "dual" or "pure" part of the dual number. The bases for the dual numbers are 1 and ϵ, and they obey the rules:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ 1 \cdot \epsilon &= \epsilon \cdot 1 = \epsilon \\ \epsilon^2 &= 0 \end{aligned}$$

Dual numbers have many interesting properties, though we will only explore one aspect of their characteristics in this problem.

Part (a): (10 points). We can represent spatial displacments as "dual rotation matrices." That is, if a spatial displacement has the form:

$$g = \begin{bmatrix} R & \overline{p} \\ \overline{0}^T & 1 \end{bmatrix}$$

where $R \in SO(3)$ and $\overline{p} \in \mathbb{R}^3$, then the dual representation of the spatial displacement is:

$$\tilde{g} = R + \epsilon(\hat{p}R)$$

1. Show that $\tilde{g}$ is an orthogonal matrix.

2. If g_1 and g_2 are spatial displacements, and $\tilde{g}_1$ and $\tilde{g}_2$ there dual equivalents, then show that $g_1\ g_2$ and $\tilde{g}_1\ \tilde{g}_2$ are equivalent.

Hint: in some ways of solving this problem, it might be useful to recall that if $A \in SO(3)$ and $\overline{v} \in \mathbb{R}^3$, then $\widehat{(A\overline{v})} = A\hat{v}A^T$).

Part (b): (5 Points). We can also use dual numbers to represent twist coordinates. Let $\xi = [\overline{V}, \overline{\omega}]^T$ be a vector twist coordinates. Its dual representation is $\tilde{\xi} = \overline{\omega} + \epsilon\overline{V}$. Show that

1. if g is a spatial displacement, and ξ is a twist, then $Ad_g\xi$ is equivalent to $\tilde{g}\tilde{\xi}$.

2. If ξ_1 and ξ_2 are two twists, then the dual part of dual dot product $\tilde{\xi}_1 \cdot \tilde{\xi}_2$ is equivalent to the reciprocal product of ξ_1 and ξ_2. (Note, the real part of this product is called the "Klein product.").

Problem 3: (15 Points)

For the rotation matrix given below

$$R = \begin{bmatrix} 0.833333 & -0.186887 & 0.52022 \\ 0.52022 & 0.583333 & -0.623773 \\ -0.186887 & 0.79044 & 0.583333 \end{bmatrix}$$

Part (a): (5 points) Compute the axis of rotation and angle of rotation.

Part (b): (5 points) Determine the unit quaternion that is equivalent to this rotation.

Part (c): (5 points) What are the z-y-z Euler angles of this rotation?

Problem 4: (10 points) Assume that the orientation of a rigid body is described by z-y-z Euler angles, where the angles of rotation are respectively ψ, ϕ, and γ. Further assume that the body is spinning with rotation rates of $\dot{\psi}$, $\dot{\phi}$, and $\dot{\gamma}$ about the respective z, y, and z axes. Show that the spatial angular velocity of the body is:

$$\overline{\omega}^s = \begin{bmatrix} -\dot{\phi}\sin\psi + \dot{\gamma}\cos\psi\sin\phi \\ \dot{\phi}\cos\psi + \dot{\gamma}\sin\psi\sin\phi \\ \dot{\psi} + \dot{\gamma}\cos\phi \end{bmatrix}$$

Note that the solution to this problem is useful for the study of gyroscopes.

Problem 5: (15 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \overline{d} \\ \overline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and d respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. (5 points) Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (3 points) describe the other two eigenvectors of g.

c. (7 points) Every planar displacement is equivalent to a rotation about a "pole." Let a body-fixed reference frame attached to a planar rigid body be initially in coincidence with the origin of a fixed reference observing frame. Let this body undergo a planar displacement by rotation of angle ϕ about a pole located at a distance $\overline{p} = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ from the origin of the reference frame. Compute the 3×3 homogeneous transformation matrix that describes this displacment (in terms of ϕ, p_x, and p_y).

Extra Credit: (10 points)

Figure 2 shows a "PRR" robot. The first joint is prismatic, while the next two revolute joints intersect each other. Find the inverse kinematics of this manipulator, given that the goal is to place the origin of the tool frame at a particular point, $\begin{bmatrix} x_D & y_D & z_D \end{bmatrix}$.

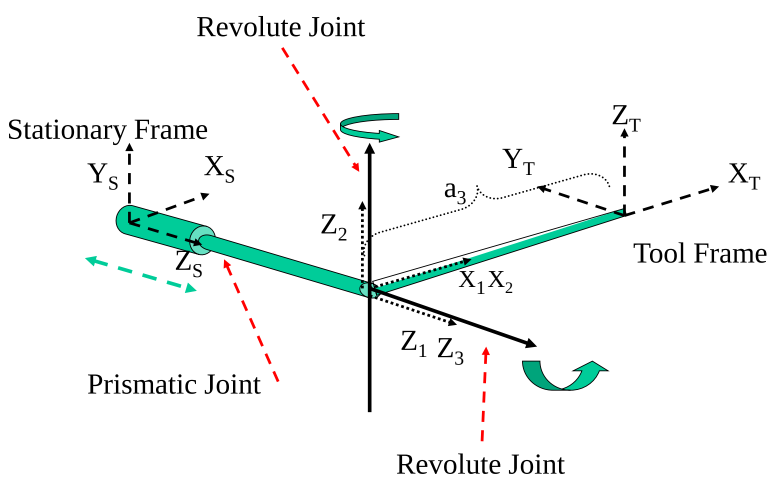

Figure 2: Schematic of a PRR manipulator