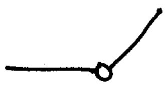
Primary Topics:

- Joints
- Degrees of freedom
- Workspace
- Inverse Kinematics
- Forward Kinematics (secondary topic)

- ground link
- end effector

Joints



revolute joint
(hinge)

prismatic joint
(extension)

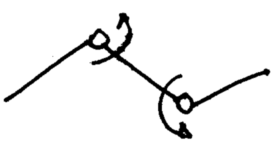
→ these joints are typical of "planar" motion (i.e. motion confined to a plane).

* we won't consider 3-D non-planar motion but other joints do exist for that space as well.

→ the above listed "planar" joints exhibit 1 Degree of freedom:

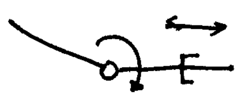
Defn: the set of independent displacements and/or rotations that completely specify the displaced or deformed position of the members of a rigid object.

ex



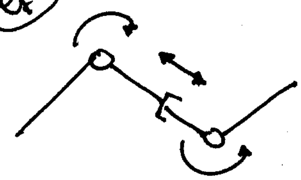
2 degrees of freedom

ex



2 degrees of freedom

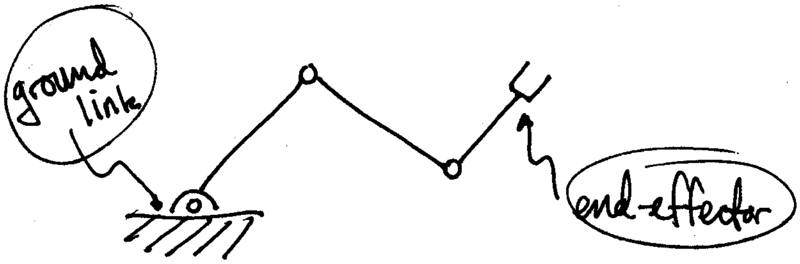
ex



3 degrees of freedom

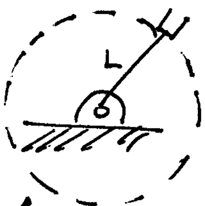
Often when dealing with robot kinematics (or, specifically, robot arms for that matter), there exists a "ground link" and an "end effector", and any which number of degrees of freedom in between. The "ground link" is that part of the robot arm which is rigidly attached to the ground. The "end-effector" is that part of the robot arm which is the very tip of the last link in the arm.

Typically, the end-effector contains some tool or object for which the robot arm is manipulating to touch some point in space



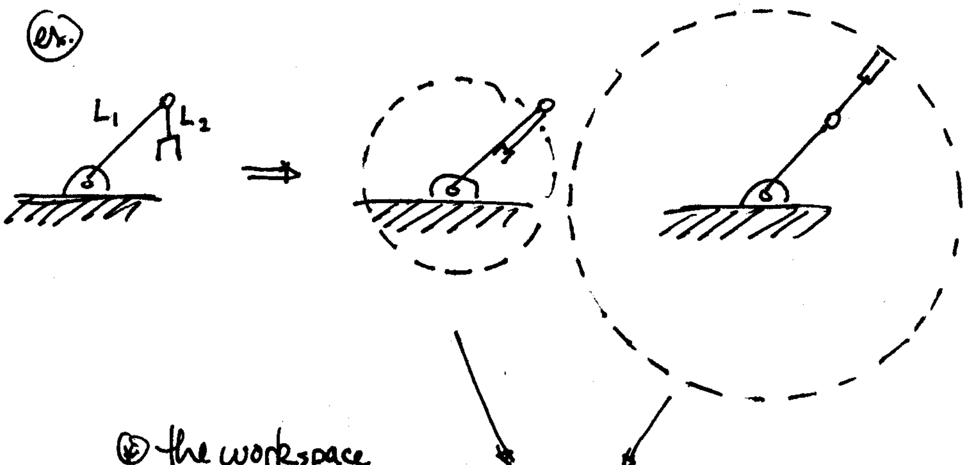
Now the set of all possible locations in space for which an end effector can reach is called the "workspace" of the robot arm. Consider the following example:

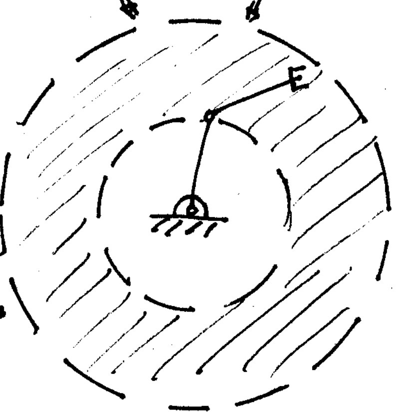
ex.



the workspace is only a circle of radius L

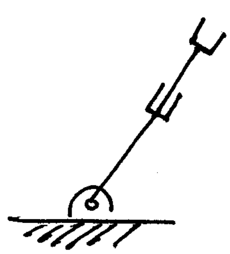
ex.





* the workspace in this example is a donut, w/ the inner radius being L_1 and the outer radius being L_1+L_2.

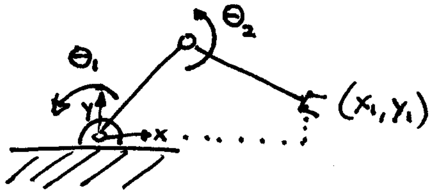
ex.



what would the workspace look like for this robotic arm?

So given that we understand what a workspace is, how do we make use of it? Often in robot kinematics we are interested in moving our robot arm (or more specifically, our end effector) to some point in the workspace ⇒ so how do we do that?

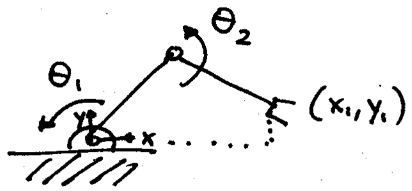
Forward Kinematics



given θ_1 & θ_2, where does the end effector end up?

⇒ $(x_1, y_1) = ??$

Inverse Kinematics

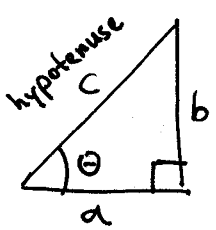


given a desired location (x_1, y_1), what should the motor commands θ_1 and θ_2 be?

⇒ $(\theta_1 = ?\ ,\ \theta_2 = ?)$

INTRODUCE ROBOTIC ARM CHALLENGE.

MATH REVIEW



← this is a right triangle.

some cool properties of right triangles:

- $a^2 + b^2 = c^2$ (PYTHAGOREAN THEOREM)
- $\sin\theta = \frac{b}{c}$
- $\cos\theta = \frac{a}{c}$
- $\tan\theta = \frac{b}{a}$

in general for RIGHT Δs: "SOH – CAH – TOA"

$\sin(\ ) = \frac{opp}{hyp}$, $\cos(\ ) = \frac{adj}{hyp}$, $\tan(\ ) = \frac{opp}{adj}$

angles

degrees ⟷ radians

360° ⟺ 2π

180° ⟺ π

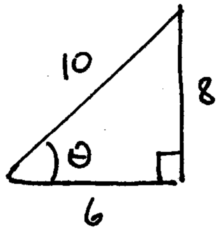
- convert "deg" to "rad"

$(\ \ ^\circ) \times \frac{2\pi}{360} = (\ \ )$ rad.

- convert "rad" to "deg"

$(\ \ ) \times \frac{360}{2\pi} = (\ \ )$ deg.

ex



$\underline{\underline{\Theta = ?}}$

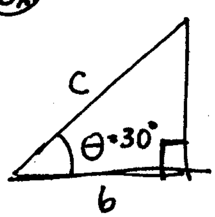
↳ $\sin(\Theta) = \dfrac{8}{10}$

$$\boxed{\Theta = \sin^{-1}\left(\frac{8}{10}\right)}$$

[the "$\sin^{-1}$" command exists on most scientific calculators; if you cannot find it, use google!]

NOTE: most calculators will by default give you an answer in radians. Know which units your answer is in.

ex



$\underline{\underline{c = ?}}$

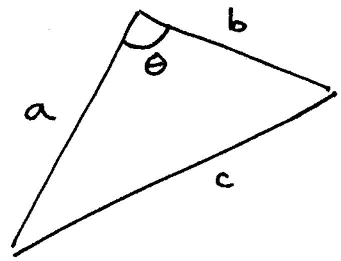
↳ $\cos(30°) = \dfrac{6}{c}$

$$\boxed{c = \frac{6}{\cos(30°)}}$$

NOTE: most calculators expect radians so you may need to convert degrees to radians before using "trig" functions.

what if we don't have a right triangle?

LAW OF COSINES



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

this equation may not seem useful now, but when you do the challenge, trust me, it will.