Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.

2. You may use any class notes, books, or other written material.

3. You may use mathematica or any software or computational tools to assist you.

4. Feel free to ask me questions about the exam.

5. The final is due by 5:00 p.m. on the last day of the final period. If you need your grade turned in to the registrar for purposes of graduation, then the final is due at 9:00 a.m. on Monday June 6. Else, Friday June 10 is the due date.

6. The point values are listed for each problem to assist you in allocation of your time.

7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.
Problem #1 (15 Points): (force closure)

A 3-dimensional body (such as an ellipsoid) is grasped by two fingers in an antipodal point grasp (Figure 1). Let one of the contacts be modelled by the point contact with friction model. Let the other contact be modelled by a soft finger contact.

Part (a): Sketch the structure of the grasp map for this grasp

Part (b): Is this grasp force closure? Justify your answer using one of the force closure definitions.

Problem #2 (15 Points): (force closure)

Consider the grasp seen in Figure 2. You can assume that both contacts are frictional. Is this grasp force closure? You should choose a yes or no answer, and argue/analyze your answer. However, also argue why your answer may be wrong.

Problem #3 (10 Points): (force closure)

For grasp analysis which include the geometry of the finger tip, we must often parametrize the surface which models the finger tip surface. let’s consider an elliptical finger tip. The surface
of an ellipsoid can be parametrized in a variety of ways. One particular parametrization follows. For an ellipsoid with principle dimensions $2A, 2B, 2C$ (with $A > B > C$), the surface can be covered by the 8 orthogonal coordinate patches:

$$f(u, v) = \begin{bmatrix}
\pm A \sqrt{(A-u)(A-v)} \\
\pm B \sqrt{(B-u)(B-v)} \\
\pm C \sqrt{(C-u)(C-v)}
\end{bmatrix} \begin{bmatrix}
\frac{1}{(A-B)(A-C)} \\
\frac{1}{(B-A)(B-C)} \\
\frac{1}{(C-A)(C-B)}
\end{bmatrix}$$

where $u \in [C, B]$, $v \in [B, A]$.

**Part (a):** Verify that these coordinates are indeed orthogonal. 

**Part (b):** Over what domains (in $(u, v)$ space) are these parametrizations valid?

**Problem #4 (20 Points):** (force closure)

An important issue in grasping, which we discussed only briefly, is the minimal number of fingers necessary to grasp a given class of objects.

**Part (a):** True or False: For any convex 3-D object, a force-closure grasp can be constructed using at most three soft-finger contacts (at smooth point locations on the surface of the body).

**Part (b):** If the object is everywhere smooth, argue why a force closure grasp can be constructed using at most three frictional point contacts.

**Problem #5 (20 Points):** (force closure)

Consider the following grasping problem. A set of *planar, flat* fingers are to immobilize a planar ellipse. By a flat finger, we mean a finger tip which is not a point, but a line segment (e.g., imagine a square finger tip–one side of the square is the flat finger surface). How many flat fingers are necessary to immobilize the ellipse?