CDS 270: Solution to Problem Set #2

Solution #1: (10 points) Problem 2(a,c) in Chapt. 5 of MLS text.

- **(a):** the grasp is force closure because; (1) the grasp map is full rank (two or more non-coicident frictional point contacts in the plane); (2) there exists a null force in the interior of the fructions cones. The null force consists of equal normal forces on all three contacts
- **(c):** The grasp **is** force closure because; (1) the grasp map is full rank (3 non-colinear frictional 3-dimensional point contacts implies a full rank grasp map), and (2) there exists and internal force inside the friction cones.

Solution #2: (10 points). Problem 3(b) in Chapt. 5 of MLS text.

In a reference frame whose x-axis is collinear with the two contact normals and pointing to the right of the figure on page 259 of MLS, and whose z-axis is pointing upward, the grasp map is;

$$G = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ R & 0 & 0 & 0 & -R & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 & R & 0 & 0 \end{bmatrix} \tag{1}$$

where R is the half-width of the box. By Theorem 5.7 of the MLS text, this grasp is force closure. The theorem states that for spatial grasp with two soft- finger contacts, there is force closure if and only if the line connecting the contact points lies inside both friction cones.

Solution #3: (10 points) Problem 4(b) in Chapt. 5 of MLS text.

There are different solutions to this problem depending upon the assumptions that you make. Clearly, because the surface of the rectangular face makes a frictional "patch" contact with the plane, it can support a normal force, tangential forces in all directions, and a torque about the contact normal. Hence, at the minimum, this contact is just like a soft finger contact model, which has wrench basis:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution #4: Problem 5(a) in Chapt. 5 of MLS text. There are several different possible ways to approach this problem. One relatively easy idea is to choose the grasped object to be an equilateral triangle. You can then show geomeetrically that there is no way to construct a line between two point contacts that will like inside both friction cones in the case where $\mu = \tan(30^o)$.

Solution #5: Problem 8(a) in Chapt. 5 of MLS text.

This problem was meant to be a warm up for our future work on grasp planning. Figure 1(a) recalls the geometry of this problem. Let s denote an arc-length, or distance parameter, measuring distance around the periphery of the object in a clockwise direction. For this object, the perimeter has total length of 5 units. Figure 1(b) shows the "contact space" for all two fingered grasps. That is, s_1 denotes the distance around the perimeter to the location of finger #1, while s_2 denotes the distance around the perimeter to finger #2.

To define the regions on contact space where force closure exists, imagine sliding Finger #1 along the boundary, starting with $s_1 = 0$. For each placement of Finger #1, there will be a range of location(s) where Finger #2 can be placed so that force closure is achieved. The regions should be symmetrical about the diagonal in the contact space, as the symmetric comes from the exchange of the two fingers indices. Figure 1(b) shows the regions for the case where Finger #1 is varied around the perimeter.

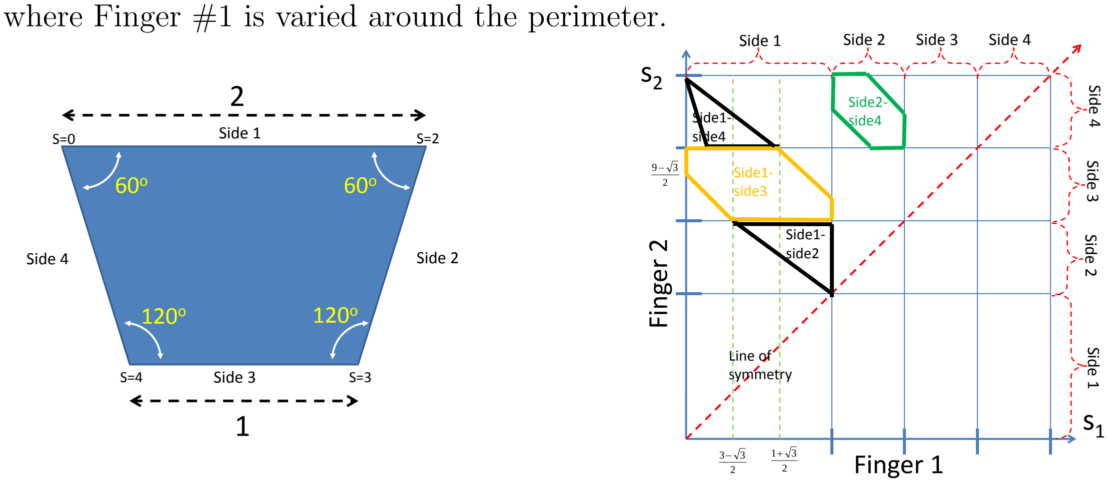

Figure 1: (a) Ellipse object and elliptical finger tip body; (b) Elliptical object in contact with two *concave* finger tips, each having the same radius of curvature at the contact points.