Chapter 1

Introduction and Overview

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1.1 Purpose of this book

This book is devoted to the kinematic and mechanical issues that arise when robotic mechanisms make multiple contacts with physical objects. Such situations occur in robotic grasping, workpiece fixturing, and the quasi-static locomotion of multilegged robots. Figures 1.1(a,b) show a many jointed multi-fingered robotic hand. Such a hand can both *grasp* and *manipulate* a wide variety of objects. A grasp is used to affix the object securely within the hand, which itself is typically attached to a robotic arm. Complex robotic hands can implement a wide variety of grasps, from *precision grasps*, where only only the finger tips touch the grasped object (Figure 1.1(a)), to *power grasps*, where the finger mechanisms may make multiple contacts with the grasped object (Figure 1.1(b)) in order to provide a highly secure grasp. Once grasped, the object can then be securely transported by the robotic system as part of a more complex manipulation operation. In the context of grasping, the joints in the finger mechanisms serve two purposes. First, the torques generated at each actuated finger joint allow the contact forces between the finger surface and the grasped object to be actively varied in order to maintain a secure grasp. Second, they allow the fingertips to be repositioned so that the hand can grasp a very wide variety of objects with a range of grasping postures. Beyond grasping, these fingers' degrees of freedom potentially allow the hand to manipulate, or reorient, the grasped object within the hand.

In most cases, the quality of the grasp is largely dictated by the location of the finger tip placements and the local geometry of the contacting bodies in the vicinity of the contacts. While the use of torques at the finger joints can actively change the finger tip contact forces, for purposes of grasp analysis, one can conceptually replace the finger mechanisms by the equivalent forces and torques, or *wrenches*, that are generated at the contacts. Thus, as depicted in Figure 1.1(c), the analysis of a multi-fingered grasp can be simplified to the study of a central *grasped object* in contact with multiple *finger bodies*. In some cases, we may

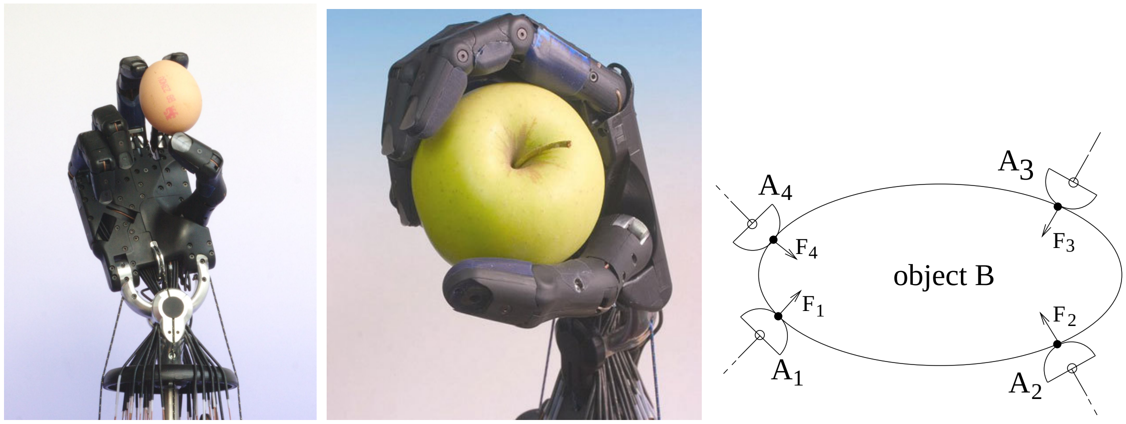

Figure 1.1: (a) Multi-fingered robotic hand demonstrating a *precision* grasp; (b) Multi-fingered robotic hand demonstrating a *power* grasp; (c) abstraction of a grasp to a collection of rigid bodies (a central grasped object interacting with multiple finger bodies).

wish to assume that the finger bodies can actively control the forces of contact (if the finger mechanism associated with that finger body has sufficiently many joints), while in other cases we may wish to assume that a finger body is an immobile rigid body, or a compliant body which will generate reaction forces at the contact in response to perturbations of the centrally grasped object's location

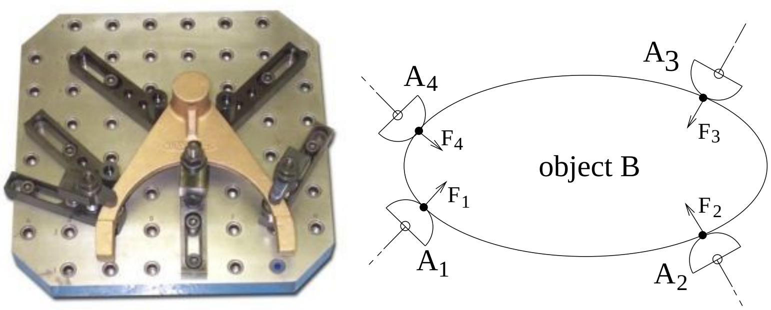

Figure 1.2: Photograph of a modular fixture, in which the curved brass workpiece is held by black fixels ; (b) abstraction of a fixture as a collection of rigid bodies (a central workpiece object interacting with multiple fixel bodies).

Similar situations involving multiple contacts also arise in *fixturing* or *workpiece holding* applications. Figures 1.2(a) shows a photograph of a *fixture*, which consists of *fixels* (or "clamps") that restrain a *workpiece*. Fixtures are commonly used to securely hold workpieces during machining and manufacturing processes. Fixtures are also used during assembly operations to hold parts in precise alignment. While some fixturing systems include actuation to actively control some of the clamping forces, most fixturing systems are unactuated. All of the contact forces between the fixels and the workpiece are generated during *preloading* of the fixtures, or arise from the compliant reactions of the fixel and workpiece materials as they are stressed by machining or assembly operations. Thus, as depicted in Figure 1.2(b), the mechanical analysis of fixturing systems can also be idealized to the study of a central object (the workpiece), in contact with multiple finger bodies (the fixels). Here, it is quite logical to assume (especially in the case where the forces experienced during the manufaturing proccess are small) that the finger bodies are rigid and immobile objects. When large manufacturing forces may be involved, then it is preferable to assume that the object and finger bodies are compliant objects, with the finger bodies fixed to a solid substrate.

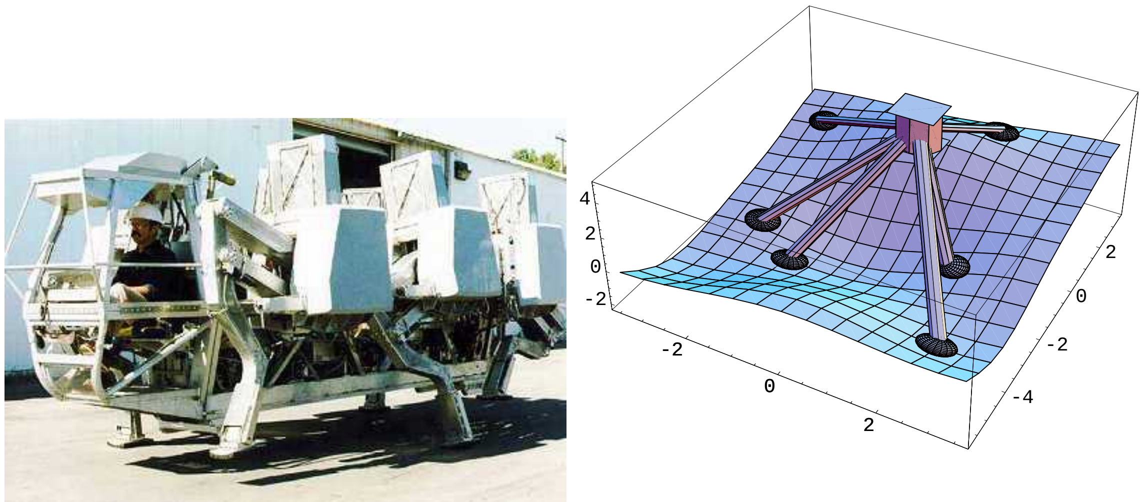

Figure 1.3: (a) Photo of the *Adaptive Suspension Vehicle*, a 7,000 lb. walking machine built at Ohio State in the 1980's; (b) abstraction of a quasi-static walking posture as a multi-contact configuration.

The physical arrangements seen in Figures 1.1(c) and 1.2(b) can also be applied to the study of some aspects of multi-legged robotic locomotion. Figure 1.3(a) shows a photograph of the Adaptive Suspension Vehicle (ASV), a 7,000 pound six-legged walking machine built at Ohio State in the 1980's. This amazing vehicle could walk over uneven and muddy terrain while towing a 2,000 pound load. In the mechanical process of locomotion, mulitple "finger bodies" (corresponding to those feet which contact the ground) interact with a central object (the terrain). Multi-legged vehicles such as the ASV are said to walk with a *quasi-static* gait if at least three legs maintain contact with the ground at all times, and the mechanism's center of mass always lies within the support of the leg contacts. If all of the leg joints were to be instantaneously fixed, then the robot's stance, or *posture*, would be *quasi-statically stable* if the rigidified mechanism were to be naturally stable with respect to the influence of gravity. Thus, as suggested by Figure 1.3(b), the question of quasi-static posture stability can be reduced to the study of the gravitational stability of a single body (whose geometry is defined by the locomotor's virtually frozen mechanism geometry) making multiple contacts with the terrain body. Such analyses are needed to design the leg placement algorithms of a quasi-static locomotion planner. We shall see that quasi-static posture stability is a function of the leg placement locations, the geometry of the contacts between the feet and the terrain, and the surface friction properties of each contact. The stability of a heavy object transported by the coordinated activity of mutliple robots can similarly be analyzed, with the analysis similarly serving as the basis for planners that will successfully coordinate the transport of the heavy object.

From these examples we can see that diverse problems in the subjects of robotic grasping, industrial fixturing, and quasi-static locomotion posture selection can be reduced to the study of the kinematic, mechanical, and stability properties of multiply-contacting bodies. While the majority of the results described in this book apply equally well to grasps, fixtures, and quasi-static multi-legged postures, for simplicity, the term "grasp" or "grasping" will often be used to refer to these related problems.

Our first goal in writing this book is to assemble historical and recent results on these subjects into one volume. There are many compelling reasons to do so. While several issues in grasping and fixturing mechanics clearly merit further research, the basic framework of grasping, fixturing, and quasi-static locomotion mechanics has become well understood. The most recent book dedicated to grasping, Mason and Salisbury's *Robot hands and the mechanics of manipulation* [29], was published in 1985. In the intervening years, real progress has been made on the basic geometrical, mechanical, and modelling issues that underlie these processes. In particular, the notion of configuration space has emerged as the standard Some recent books (e.g.[28]), [38], and [56]) do discuss some important aspects of grasping. However, they do not provide a complete survey of the topics that we feel should be collected into one volume which would serve as both and educational and reference text.

A second goal of this book is to formulate the diversity of issues in grasp and fixture mechanics into a common mathematical language. We focus on a configuration space (c-space) formulation, as it provides an intuitively appealing viewpoint on many issues in grasp mechanics, and it is consistent with the main stream of robotics motion planning theory. This marriage of kinematics, mechanics, and c-space should simplify future efforts in algorithm development.

A third goal of this book is to highlight the impact of higher order kinematic and mechanical effects, such as surface curvature and non-linear compliance, on grasping performance. Such effects, which are often ignored in the grasping and fixturing literature, can be significant in some practical circumstances. We show by example when these effects can be practically important.

In developing our results, we rely upon a few mathematical tools, such as non-smooth analysis and stratified Morse theory, which are not standard in the grasping literature. We therefore provide reviews of these methods in appendices so that their use can be readily followed in the main body of text, and so that their effectiveness can be appreciated.

We have purposefully limited the scope of this book to the kinematics and mechanics underlying grasping, fixturing, and quasi-static postures. We do not consider in detail possible algorithms for hand manipulation, fixture design, or leg placement. Instead, our goal is to present material that would serve as a foundation for researchers or practioners interested in developing new algorithms, as any successful algorithm must incorporate the basic principles that are described in this book.

1.2 How to use this book

We have tried to organize this book so that it can serve different purposes. First, it could serve as a textbook for those portions of an introductory graduate robotics courses that touch upon grasping and manipulation. Consequently, we have added exercises and some worked out examples to help students master the key material through the problem solving process. Second, we hope that this book serves as a useful reference for robotics researchers and practitioners. Consequently, we have also included detailed proofs of some of the more critical and foundational results. Additionally, we have included more advanced reference material in some of the later chapters so that the issues addressed in this book, while limited in breadth, are treated in depth.

1.2.1 Organization of the book

The book is organized into five parts, whose contents and purposes are summarized as follows:

- **Part I: Introduction and Background.** The three chapters in this first part of the book review the essential geometrical and kinematic ideas behind the configuration space analysis of multiple contacting bodies. For students who are newly starting their study of grasp mechanics, Part I is essential background reading. Practioners already family with the field can skim most of these chapters to take note of the notational and kinematic conventions that we use. However, it should be noted that Chapter 2 presents a novel formulation of the geometry and kinematics of configuration-space obstacles using non-smooth distance functions and non-smooth analysis. We believe that this approach provides a more coherent and straightforward framework than the more practical methods which we used in our earliest papers on this subject [48, 49]. Similarly, Chapter 4 includes a relatively complete discussion and description of equilibrium grasps in terms of line geometry. This line geometry formulation has not previously appeared in a textbook. Similarly, the later half of Chapter 3 introduces what we term"graphical" parametrizations of the tangent spaces to a configuration space obstacle. These parametrizations, which are not standard in the literature, play an important role in subsequent chapters. Thus, even experienced readers may want to devote some attention to these portions of Part I.

- **Part II: Frictionless Rigid-Body Grasps and Stances.** This part of the book focuses on *frictionless* rigid-body grasps. While no grasp contacts are truly frictionless, nor are any physical objects truly rigid, such models constitute useful idealizations. First, they serve as a conservative approximation to many real grasps where friction is small, or where the grasps are lightly loaded. From the pedagogical point of view, because frictionless rigid-body grasps can be studied and analyzed using purely geometrical approaches, they serve as a useful bridge between the purely geometrical ideas of Part I and the mechanics-oriented methods of Parts III and IV. For students, Part II introduces the notions of form closure (or immobilization) and force closure, and how the geometric and kinematic ideas of Part I can be used to analyze these key issues.

Chapter 8 shows in detail how to use form closure analysis to bound the number of fingers that are necessary to grasp different classes of object in a form or force closure grasp. While we have found that students are particularly interested in the results of Chapter 8, this material is not essential reading for a first introduction to the subject. Chapter 9 also considers the gravitational stability of"heavy" frictionless grasps or quasis-static multi-legged robot postures. Experienced practioners should pay attention to the second order geometric analysis of Chapter 7, and the use of Stratified Morse theory in Chapter 9 (which is more fully summarized in Appendix E). Neither of these analyses have previously appeared in a textbook, and later chapters assume that the reader is knowledgeable on these topics. Chapter 8 also collects together results which previously have only appeared in the research literature.

- **Part III: Frictional Rigid-Body Grasps and Stances.** While Part III continues to rely upon the idealization of rigid bodies, the analyses and results of Part II are in extended in this part of the book to account for friction at the contacts. Both students and practioners may be interested in the portion of Chapter 10 which formulates the active force closure condition as a problem in linear matrix inequalities.

- **Part IV: Compliant Grasping.** Part IV moves away from the rigid body assumption used in Parts II and III, and considers *quasi-rigid-body* models for the deformability of the grasped object and the finger bodies. The quasi-rigid body model assumption is consistent with the rich geometry introduced in Parts I and II, but is also consistent with many types of nonlinear compliance models found in the mechanics literature, including the classical Hertz contact model. In moving from the rigid body model to compliant body models, the key notions of form and force closure are now replaced by the dynamic notion of stability. The study of compliant objects is divided into cases involing frictionless (Chapter 12) and frictional (Chapter 13) contacts. Students will be most interested in the detailed formulation of the grasp *stiffness matrix* developed in Chapter 12, while practioners may wish to familiarize themselves with the more advanced models of Chapter 13 that combine both nonlinear compliance effects and frictional effects into the grasp analysis framework. The inclusion of all of these mechanical effects into a single framework has only previously appeared in the research literature.

- **Part V: Grasp Quality Measures.** This part of the book considers metrics that can assess the quality of a given grasp or fixture. Chapter 15 summarizes the mathematical formulation of grasp metrics, and offers some practical examples of grasp metrics. Chapter 16, is solely devoted to the topic of *frame-invariant* grasp metrics. A grasp metric is said to be frame-invariant if the value of the metric does not depend upon the selection of the reference frames which are naturally associated to grasp metrics. This chapter provides a new derivation of the conditions for metric frame invariance that were originally introduced by Lin and Burdick [25]. This more accessible analysis of the frame invariance conditions should interest both students and practitioners.

As part of an introductory robotics course, Part I and Chapters 5, 6, 10, and 12 would give students a working knowledge of key issues in grasp/fixture modeling and analysis. Armed with this knowledge, students could pursue a self-study of many other of this book's chapters, as well as much of the historical research literature. A longer introduction would also include Chapters 7 and 8, and possibly Chapter 9 for courses that also include some study of robotic locomotion. The remaining chapters of this book primarily serve as reference material. The portions of the text which are more apt to be used as part of a course also have an increased number of exercises and worked examples.

1.2.2 Prerequisites

Obviously, it is difficult to write a book that can be both accessible and engaging to readers with a vast range of backgrounds. In writing this book, we have assumed that the reader has the following preparation:

- A basic understanding of smooth manifolds, including the methods to define coordinates on a manifold.
- Working knowledge of rigid body kinematics and the kinematics of serial chain robotic linkages, such as can be found in Chapters 2 and 3 of [38]. While a knowledge of the Lie algebraic view of rigid body kinematics is helpful, we do not strongly stress or require knowledge of this approach.

The more advanced mathematical methods, including non-smooth analysis, stratified Morse theory, and relevant aspects of the Lie Algebraic theory of kinematics, are reviewed in Appendices A-E, and references to the relevant textbooks on these subjects are also given. By using this organizational approach for the book's material, we hope that the text is relatively self-contained for readers who are new to this field, and that this format allows experienced practitioners to quickly pinpoint topics of interest.

1.3 A brief history of grasping and fixturing mechanics

What follows is an abbreviated and admittedly incomplete history of the relevant developments in grasping mechanics, including connections to the contents of this book. Rather than giving a detailed list of references, this review emphasizes some of the main historical trends and their influence on the organization of this book.

1.3.1 The Early Influences

Modern grasping and fixturing theory is built upon three classical foundations: kinematics, mechanics, and mechanism theory. Before summarizing more modern developments, let us first review some of the classical developments that underlie the theories described in this book.

All practical grasps and fixtures involve the contact of multiple bodies. Depending upon the application, one may model these bodies as rigid, or compliant. To a very good approximation, in many practical situations one can analyze multi-body grasps and fixture arrangements using a quasi-static approach, where the inertial forces are ignored because they are insignificant with respect to the forces of contact and the forces arise from elastic storage elements. The subject of statics has been around for centuries. However, Louis Poinsot (working in the early 1800's) can be identified as one of the first to geometrize statics and mechanics principles. Poinsot's most useful contribution in the context of rigid body mechanics is his observation that any system of forces acting on a rigid body can be resolved into a single force and a couple. In the context of grasping, his works allows a system of finger forces applied to a grasped object to be replaced with its equivalent "net wrench."[1] Coupled with the use of line geometry, these ideas are the basis of the equilibrium grasp conditions that are reviewed in Chapter 4.

The detailed mechanics of the contact between fingers (or fixels) and a grasped object can have a significant influence upon the properties of the grasp. A *contact model* describes the types of forces that can be transmitted by or generated by a finger-object contact. Throughout the modern history of grasp analysis, contact models have often included the effects of surface friction. In 1773 Charles Augustin de Coulomb introduced the concept that friction and cohesion can have an influence on problems of statics. In his *Théorie des machines simple* (1781), Coulomb fully develops what we now call the "Couloumb friction model." Since Coulomb's time, research in the field of tribology has continually improved our understanding of surface frictional effects. The Coulomb friction model may be criticized as begin too simplistic and inaccurate since it does not capture many known tribological factors. However, experience has shown that the Coulomb model provides a surprisingly effective approximation for grasp analysis, and consequently it has been widely used in the study of grasping, fixturing, and legged locomotion. We review rigid body Coulomb friction contact models in Chapter 10, and introduce contact models that combine nonlinear compliance and Coulomb friction effects in Chapter 13.

It is often convenient or expedient to model the bodies comprising a grasp as rigid bodies. However, no real material is perfectly rigid. For some applications, the bodies' compliance must be taken into account. Compliant modelling of grasps has its roots in elasticity theory. The key aspects of linear elasticity theory were largely established during the first half of the nineteenth century by Cauchy, Navier, Poisson, and Green. Building upon these earlier results, in the 1890's Hertz developed models for the deformation of elastic solids brought into forceful contact. Chapter 12 shows how to incorporate Hertz's theory into the analysis of multi-contact grasps and fixtures. The "linear spring" compliance models models that have been commonly used in robotics practice are also reviewed. A comparison of the two approaches in Chapter 12 shows that for some applications the use of the more sophisticated

[1] While Poinsot can be credited with showing that the concept of a wrench is the canonical way to describe a system of forces and torques, he did not actually coin the word "wrench" to describe this concept. The term "wrench" was introduced by Ball in his treatise on screw theory [3], though Ball's original use of the word differs form the currently accepted practice, though Ball's original notion of a wrench is what we now call the *magnitude* of the wrench [53].

Hertz model leads to more accurate performance predictions. Hertz' original model assumed that the contacts were frictionless. Mindlin [31], Deresiewicz [33, 32], Johnson [20], Walton [64], and others have extended the Hertz model to include Coulomb friction. We build upon their work in Chapter 13 to develop grasp compliance models that incorporate both Hertzian contact compliance models and Coulomb surface friction.

Many treatments of the history of grasping start with Reuleaux's 1875 book, *The Kinematics of Machinery* [45]. This book was one of the first to introduce a systematic theory for categorizing and analyzing mechanisms and machines. In this work, Reuleaux established the basic concept of *force closure*, which is reviewed in Chapter 5. Intuitively, a grasp is said to be force closure if any force and torque applied to the grasped object can be resisted by feasible finger contact forces. The force closure concept has been a central issue in grasping research.

A grasp is said to be in *form closure* if the contacting fingers provide sufficient constraint on the grasped object's motion so that the object is immobilized relative to the fingers. In 1900 Somoff provided the first systematic analysis of the form closure concept, showing that 4 point contacts are necessary to establish form closure on a planar object, and 7 contacts in the case of spatial objects. We provide a thorough review of form closure in Chapters 5, 6, 7, and 10. A key contribution of this book is the extension of form closure concepts to include surface curvature effects. By including such effects, we extend and update the classical results on form closure. Based on surface curvature effects, we introduce the concepts of higher order form closure, or immobilization. We show that by including curvature effects, one can immobilize objects with fewer contacts than is predicted by classical point contact models (Chapter 8).

The next important events concern the development of mathematical techniques needed to represent and analyze grasp kinematics. A formalism for analyzing the motions of rigid or quasi-rigid bodies is a fundamental requirement for grasp analysis. Two different schools of analysis emerged independently in the mathematics and engineering literatures of the late 19^{th} and early 20^{th} centuries. In the mid 1870's, Lie developed his notion of "infinitesimal groups," which we know today as Lie algebras. Between 1888 and 1893, Lie and Engel published their three volume work *Theorie der Transformationsgruppen*, which contained the foundational concepts of Lie groups. For the most part, this text takes a Lie Algebraic approach when warranted.

In 1900 Ball published *A Treatise on the Theory of Screws* [3] (in 1924, Richard von Mises introduced the related concept of "motors" [61]). While Ball was primarily motivated to reformulate the dynamics of rigid bodies in terms of "screws," the engineering kinematics field adopted his "screw theory" conceptualization and analysis of rigid bodies kinematics. From the viewpoint of physical intuition, several notions of screw theory are appealing. The engineering kinematics community largely relied upon screw theory until the 1980's, when the Lie group approaches were reintroduced to the engineering kinematics community by control theory researchers (e.g., Roger Brockett and Shankar Sastry) working in the newly energized field of robotics. Because of the prevalence of screw theory notions in the traditional grasping literature, we review the relevant aspects of Screw Theory in Chapters 2-4, and relate the screw theory and Lie approaches where appropriate.

In the late 1970's Lozano-Perez, Mason, and Taylor introduced the configuration space (c-space) concept for robotic motion planning. All of the analyses in this book are developed in the c-space framework, as it is compatible with modern robotic planning concepts. Moreover, this formulation also allows for the rich body of analytical methods from differential geometry to be applied to the analysis of multi-contact systems.

1.3.2 The Modern Robotics Era

While the field of engineering kinematics was active researched in the first half of the twentieth century [2], significant research of immediate relevance to the theory of robotic grasping and fixturing did not emerge again until the 1970's. The birth of grasp research in the 1970's was driven by three different lines of practical activity in the 1950's and 1960's: industrial robotics, telerobotics, and human prosthetics. The first industrial robot was installed in a factory during 1961. The earliest factory robots relied upon special purpose grippers to accomplish their grasping and part manipulation tasks. It was an obvious next step for robotic engineers and researchers to imagine general purpose dextrous hands modelled after the human hand. The hope was that anthropomorphic robotic hands could eliminate the needed to develop special purpose tooling and robot grippers, thereby enabling robots to be easily reprogrammed for different assembly operations. Examples of early hand designs and investigations along these lines include the Waseda WAM-1 arm of the late 1960's (which included a 4 degree-of-freedom gripper). Similarly, the rise of nuclear power generators in the 1950's motivated significant work in teleoperated robotic mechanisms. Telerobots could act as mechanical proxies, thereby removing humans from highly radioactive and dangerous environments. While simple parallel jaw grippers sufficed for most teleoperation tasks, some tasks demanded dextrous end-effectors that could mimic human hand motions. Additionally, electromechanical technology had sufficiently advanced by the early 1960's to the point where dextrous prosthetic hands were worthy of serious research endeavors. The most notable early work in this area is the Tomovic hand of 1965. In addition to questions about how to design such grippers was the challenging problem of how to coordinate the hands' multiple degrees of freedom.

The first steps toward a grasping mechanics framework

Kinematicians and theoreticians were challenged by these practical developments to develop more systematic and comprehensive approaches to the analysis of multi-fingered grasps and dextrous manipulation.

In his 1978 paper entitled "Mechanics of Form Closure,", Lakshminarayana updated Somoff's classical form closure results to estabilish necessary conditions on the number of fingers needed to establish a form closure grasp. Subsequent work over the following 15 years was dominated by the force closure approach to grasp analysis, instead of the form closure

[2] Chapter 1 of the kinematics text of Hartenberg and Denavit [17] has an excellent review of the history of mechanisms and kinematics up until 1950.

approach. In this book we try to give a balanced view between the force closure and form closure approaches to grasp analysis. From the viewpoint of geometric analysis, form closure is actually a more natural framework for rigid body grasp analysis.

In the late 1970's and through most of the 1980's, Bernie Roth and his students at Stanford University systematically analyzed many fundamental issues in grasping and dextrous manipulation. Much of their work was based on classical screw theory and its extensions [41]. We only list here a small sample of this line of work that is relevant to this book. In his Ph.D. work [54], Salisbury formalized models for frictional finger contacts—these contact models are reviewed in Chapter **??**. He also gave one of the first general formulations of stiffness in closed chain multi-body robotic systems [55]. Jeff Kerr showed that the force closure problem can be formulated as a problem in linear programming [21].

In 1986-87 Cai and Roth [10] determined how the point of contact between two bodies moves as a function of the bodies' relative motions and their surface curvatures. Until that time, essentially all grasp analyses assumed that the finger-to-object contact could be approximated by a point. Their analysis, which allows for the curvature of the contacting bodies to be taken into account, relied upon screw theory. Simultaneously, David Montana in his Ph.D. work at Harvard [] developed what are now known as the *Montana Contact Equations* to describe this same effect. Montana relied upon differential geometric means to obtain results analogous to that of Cai and Roth. The influence of surface curvature on the mechanics of grasping is a common theme in many chapter of this book. Our presentation tends more towards Montana's geometric formulation, as that approach is consistent with a configuration space framework. However, one can obtain these results with either approach.

The early 1980's also saw a new generation of multi-fingered hand prototypes. Salisbury completed the first prototype of his JPL/Stanford in in 1982, while Jacobsen, Hollerbach, and coworkers completed the first Utah/MIT hand prototype in 1985. These devices represented a new level of technical sophistication and performance. While these devices were still somewhat impractical because of the bulky drive systems, their successful demonstration of basic grasping operations motivated a new generation of researchers to tackle grasping and dextrous manipulation issues.

Grasping theory grows up

After this early and influential period, the field of grasp analysis expanded substantially in the late 1980's and early 1990's. Additionally, the areas of grasping and fixturing, which previously had attracted separate groups of researchers, were considered jointly. This section summarizes some of the main trends and themes of this era.

Rigid Body Models and Their Analysis. A majority of the work in this era focused on rigid body mechanical models of the fingers and object. The central issues of rigid grasp analysis of this era included:

- *How many contacts are needed to establish force (or form) closure?* For example, Markenscoff and Papadimitriou [26, 27] showed the sufficiency of Somoff's earlier necessary conditions that four (seven) contacts are required to establish form closure on a planar (spatial) object.

- *Closure Tests and Conditions.* Clearly, for force and form closure to be practically useful, one needs basic computational procedures to test if a given grasp does indeed satisfy a closure condition. One can then construct grasp planning algorithms with these tests as a basis. Trinkle used linear complementarity theory to formulate a computational condition for form closure [60], while Ponce and Merlet [44] use projective geometry to develop a force closure test for polyhedral objects. Bicchi [**?**] also considered force closure definitions that included the properties of the grasp mechanism's geometry.

- *Grasp synthesis.* Both Nguyen [39] and Mishra, Schwartz, and Sharir [36] presented algorithms for constructing grasps on rigid polyhedral objects. Several authors also presented algorithms to construct grasps that optimize a specific criteria. See e.g., the optimal 3-fingered grasp synthesis procedure of Mukherjee and Waldron [37]. An extensive survey of algorithms for grasp synthesis is beyond the scope of this text, which focuses on grasping mechanics. However, a procedure to synthesize minimum deflection grasps or fixtures is presented in **??** to illustrate how compliant mechanical models can serve as the basis for grasp synthesis.

- *Properties of Grasps.* In addition to the properties of force and form closure, researchers looked at other grasp properties. Bicchi [5] showed how to usefully decompose internal grasp forces into sets with different properties.

While the focus of this book is grasp mechanics, it is worth pointing out that there was a parallel growth in efforts aimed at understanding the basic issues of dextrous multi-contact manipulation. E.g., Trinkle [59] analyzed the motions of objects contacted at multiple points, while Li and Canny [**?**] and Murray [**?**] considered manipulation via nonholonomic finger rolling.

Up until the mid-1990's, nearly all grasp analysis was based on what we term in this book *first order* concepts. Starting in 1994, Rimon and Burdick introduced *higher order* form and force closure concepts that take surface curvatures into account [46, 47, 48, 49, 50, 51, 52]. The techniques developed in those papers are the basis for portions of this book. Using examples, we consider when such effects can be practically important in grasp analysis and syntheses.

Compliant Grasps. Compliance was recognized as a potentially important issue in grasping from the earliest days. The first systematic study of compliant effects in grasping dates to the work of Hanafusa and Asada (1977), who analyzed the stability of a planar object contacted by three linear springs [15]. Compliance can arise from the material deformations of the finger tips and object. Until quite recently, most models of such deformations used a simple "linear spring law" to model the material deformations. We discuss this simple model and its limitations in Chapter ?. Howe, Cutkosky, and Prasad [**?**] were among the first to suggest that nonlinear compliance models may be useful in grasp analysis. In 1997, Lin, Burdick, and Rimon [24] showed how nonlinear Hertzian contact can be useful for the analysis of grasp system compliance. More recently, Kao [65] has undertaken an extensive systematic of highly nonlinear soft finger materials.

In many cases, the compliance of the finger mechanisms dominates overall grasping system compliance. For these cases, Salisbury [54] and Cutkosky [12] developed expressions (i.e., stiffness matrices) for the compliance of the grasping system. Moreover, these stiffness models could be used as the basis for "stiffness control" of the hand, wherein specific mechanical stiffnesses can be commanded in Cartesian coordinates. Such behaviors are useful when a multi-fingered hand is used to assist assembly and mating operations. We briefly review these concepts in Chapter **??**.

Force and form closure are key qualities of useful rigid body grasps. For compliant systems, stability is a key notion that defines a good grasp. Intuitively, compliant grasp stability implies that the grasped object will return to its equilibrium after it experiences small perturbations (see Chapter ? for a more thorough discussion of stability concepts). The linear stability of a grasp can be directly determined from its stiffness matrix. Nguyen was among the first to give a formula for the stiffness matrix of a grasp [40]. He assumed that the finger and object compliances can be modelled by linear springs and that the fingers maintain a point contact. Howard and Kumar [19] developed a general stiffness matrix formula that still used a linear spring model, but that included the finger and object surface curvatures and the possibility of rolling contact. Lin, Rimon, and Burdick developed the analogous formulas for general nonlinear compliance laws, including the Hertz compliance model [24]. In his Ph.D. work, Lin also extensively analyzed the effects of contact curvature and compliance model selection on grasp stability [23]. These results are reviewed in Chapter **??**.

Grasp Metrics and Frame Invariance. Force and form closure in the case of rigid grasps, and stability in the case of compliant grasps, are basic criteria for establishing a quality grasp. For a given object and set of fingers (or fixels), there generally will exist more than one closure or stable grasp. To differentiate among these grasps, one resorts to a *grasp metric* or *grasp quality measure* to measure the effectiveness of a given grasp. The best grasp is the one that optimizes the grasp metric while also satisfying closure or stability conditions. Grasp metrics can include physically meaningful quantities such as the maximum deflection of the grasped object under a given load or the worst case off-normal finger reaction force. The latter metric quantifies how much the grasp relies upon friction for stability. Grasp metrics have been implicitly or explicitly described since the earliest days of grasp analysis. The first comprehensive study was given by Mishra [35]. The first detailed study of metrics for compliant grasps was presented by Lin in his Ph.D. thesis [23]. Grasp metrics are reviewed in Chapter **??**.

A key issue to consider in the definition of a grasp metric is its *frame-invariance*. I.e., is the value of the metric invariant with respect to changes in location of both fixed and moving body reference frames? Useful grasp metrics should should be frame-invariant so that the nature of the optimal grasp does not depend upon the choice of reference frames. Chapter considers the frame-invariance issue in detail, summarizing and reformulating the work of Lin and Burdick [25], who posed the issue of frame-invariance in terms of the *objectivity* of the grasp metric. Starting with the notion of objectivity, necessary and sufficient conditions for objectivity are developed in this chapter.

Fixturing and Industrial Applications

A *fixture* or *fixturing system* locates and holds a workpiece using a set of *fixture elements* or *fixels*. Fixturing is an essential part of flexible manufacturing systems. According to Ref. [57], the cost of fixture design and fabrication for a flexible manufacturing system can amount to 10–20% of total system cost. Fixtures are also extensively used in lighter-duty applications, such as robot assembly [7, 8] and automated part delivery systems [58].

The duality between grasping and fixturing has long been observed: the role of the grasped object is played by the workpiece, and the fingers are replaced by fixels (or "clamps"). From an abstract point of view, the mechanics of fixturing and grasping are essentially the same. Practically, there are some subtle differences between the two situations. Fixturing can often involve significantly higher contact forces than are typically found in grasps. Additionally, fixturing applications often place a higher emphasis on precision of the fixtured object's location, as compared to a grasp, where stability or closure is often the primary consideration. Consequently, there should be a greater emphasis on the role of mechanics in fixturing, as precision is highly influenced by mechanical issues. Moreover, most fixtures do not involve actively articulated linkages, or active control over the fixel forces.

The study of fixturing has a long history in the manufacturing community (e.g. see [58, 18] and references therein). Starting with the work of Asada [1, 2], the "robotics" community became active in fixturing research during the 1980's [34]. Based on rigid body modeling principles, Early research focused on fixture synthesis algorithms [4], rather than detailed models of fixturing mechanics. A number of researchers studied the synthesis of "modular" fixture arrangements [6, 63]. We show in Chapter **??** that the incorporation of the nonlinear mechanics models described in Chapter **??** can have a significant effect on the precise analysis of fixturing arrangements when the fixturing or manufacturing forces are large.

During this same period, work on *sensorless manipulation* of industrial parts blossomed [13, 14]. The sensorless manipulation approach seeks to reposition small parts by simple manuevers carried out in an open loop fashion without the benefit of sensory feedback. The part maneuvering plans are based on quasi-static mechanics models of part movement. Because the key issues in such systems is the synthesis of the part manipulation sequence, Such systems are beyond the scope of this book.

Quasi-static locomotion

A multi-legged walking robot moves in a *quasi-static* fashion if at least 3 of its legs maintain ground contact, and its center of mass lies over the support region of the legs. We conclude this brief and incomplete historical review with the observation that the mechanics of grasping and the mechanics of multi-legged quasi-static locomotion have much in common. In the case of a quasi-static walker, the ground plays the equivalent role of the grasped object, while the legs are analogous to fingers. Quasi-static multi-legged walking and grasping have several issues in common: the distribution of forces within the multi-body mechanism, the selection of contact forces, and the system's stability in the presence of gravity.

In 1968, McGhee and Frank developed their well known tripod stability criterion for a quasi-static legged vehicle operating under the influence of gravity [30]. Their original stability analysis assumed point feet placed on flat ground. Using a novel stability result obtained by application of stratified Morse theory, in Chapter **??** we develop a stability criteria and an efficient computational stability test that incorporates the effects of curved feet and uneven curvilinear terrains. In effect, this result extends the classical result of McGhee and Frank to more general contact and terrain models.

During the 1980's, a number of researchers such as Waldron, Orin, and Kumar looked at issues of force distribution in multi-legged walking vehicles [62, 43]. These researchers, primarily centered at Ohio State, were engaged in the development of the impressive six-legged Adaptive Suspension Vehicle (ASV) walking machine. These same researchers also studied the analogous issue of force distribution within a multifingered grasp [22] or general closed chain mechanisms [11]. They were among the first to recognize the duality between walking machines and grasping hands. Because we focus on grasp mechanics, we do not seriously review their work in this book. However, it is worth pointing out the the issues of force distribution in these mechanisms have much in common with our study of indeterminate contact reaction forces in Chapter ?.

At several points throughout the text, we try to point out where the discussion and analysis has applicability to quasi-static locomotion.

1.3.3 Other selected trends

From the mid-1990's onward there has been a significant expansion in research activity related to grasping, fixturing, and quasi-static posturing. Some examples of recent activity that are relevant to this book include:

- *Improved Computational Techniques.* Han, Li, and Trinkle [16] showed how to formulate force closure in terms of linear matrix inequalities, while Buss, Hashimoto, and Moore showed how to efficiently pose the force optimization problem in terms of positive definite matrices [9].

- *Experimental systems.* Many new experimental grasping and manipulation systems with nontrivial capabilities have been developed in recent year. Examples include the rolling manipulation systems of Antonio Bicchi, the multifingered hand of Omata, Kaneko's multifingered hand and power grasping system, work by Martin Buss and Thomas Schlegl, as well as demonstrations of very large scale grasping systems by by Zexiang Li and Bill Goodwine. The increasing number, sophistication, and success of these systems suggests that grasping theory is beginning to pay off in the area of enhanced experimental performance.

- *New closure definitions.* Yoshikawa introduced a "passive force closure" definition [66] that we extend and formalize in Chapter **??**. This definition is appropriate for many fixturing and industrial part manipulation applications, where some of the contacts are not actively controlled by servo motors. Instead, the reaction force at the contact can arise from the natural mechanical compliance of the object and finger, or due to a very simple joint stiffness feedback systems that does not actively sense and control the contact forces.

- *Indeterminate reaction force models.* It is well known that rigid body models can be ill posed in the sense that they do not always uniquely determine the finger tip reaction forces. Omata has introduced a bound on the indeterminate reaction forces of a rigid body power grasp [42]. In deriving this bound, Omata introduces the principle of micro-slip. In Chapter ? we extend his work to derive a bound on the reaction force that takes both micro-slip and other phenomena into account.

With this brief review in mind, the next section summarizes the structure of the book's chapters.

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