Determination of Singularities in Delta-Like Manipulators

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Abstract

The DELTA robot and the manipulators derived from the DELTA robot are a relevant class of translational manipulators. In this paper, the mobility analysis of these parallel translational manipulators is developed in full. The manufacturing and mounting conditions that guarantee the pure translation of the platform are analytically derived. Moreover, it is demonstrated that these manipulators can assume singular configurations, called rotation singularities, in which they are no longer able to impose the platform translation. Finally, the geometric and analytic conditions that make it possible to find all the singular configurations of these manipulators are provided.

KEY WORDS—kinematics, parallel mechanism, mobility analysis, singularities

1. Introduction

Spatial parallel manipulators (SPMs) arouse great interest in the academic and industrial world. In fact, usually, high stiffness and good positioning precision in about all the workspace are features required for a manipulator and SPMs exhibit these features among their advantages.

A SPM is composed of an end-effector (platform) connected to the frame (base) by means of a number of kinematic chains (legs). In general, only one joint is actuated in each leg and the number of legs is equal to the degrees of freedom (DoF) of the manipulator. The platform position and orientation (pose) are controlled by acting on the actuated joints. Moreover, when the actuators are locked the SPM becomes a structure in which all the legs concur to stand the external loads applied to the platform.

Many types of SPM have been studied and designed. Among these manipulators, SPMs suitable to impose platform translation with respect to the base constitute an important subset. Two criteria have been employed to achieve the platform translation: (i) the use of isostatic structures, e.g., the prism-robot (Hervé 1995), or overconstrained structures, e.g., the Y-star (Hervé 1995) and the Tsai robot (Stamper, Tsai, and Walsh 1997), which obtain the three translational DoFs by using repeated constraints; (ii) the use of 3-DoF mechanisms (Clavel 1988; Tsai 1996; Di Gregorio and Parenti-Castelli 1998; Di Gregorio 2000) which match some manufacturing and mounting conditions.

The DELTA robot (Clavel 1988), see Figure 1, and the manipulators derived from the DELTA robot (Zobel, Di Stefano, and Raparelli 1996; Clavel et al. 1999) are a relevant class of translational manipulators using a 3-DoF mechanism. Henceforth, these manipulators will be called DELTA-like manipulators (DLMs). DLMs (Figure 2) feature three legs like the one shown in Figure 3, where a parallelogram ($A_{i1}A_{i2}B_{i1}B_{i2}$ in Figure 3), whose sides are rods connected to each other by means of spherical pairs, has a side ($B_{i1}B_{i2}$ in Figure 3) fixed in the platform, while the parallelogram's side not adjacent to the platform ($A_{i1}A_{i2}$ in Figure 3) is connected to the base via an actuated joint (T_i in Figure 3), imposing the translation of the side with respect to the base. In the DELTA robot (Figure 1) the actuated joint, T_i, is a revolute pair with the axis parallel to the parallelogram's side bound to translate. The leg of Figure 3 (DELTA-like leg) leaves five DoFs to the relative motion between platform and base.

The success of the DLMs is related to the fact that the DELTA-like leg permits us to design very fast manipulators (Clavel 1988).

DLMs (Figure 2) are special cases of the parallel manipulator shown in Figure 4, whose three DoFs are not necessarily translational. Hereafter, the manipulator of Figure 4 will be called a generalized DELTA-like manipulator (GDLM). In GDLMs, the platform is connected to the base by means of three legs, each one made of an actuated joint (T_i, $i = 1, 2, 3$, in Figure 4), working out the translation of the straight line through the centers (A_{i1} and A_{i2}, $i = 1, 2, 3$, in Figure 4) of two spherical pairs, and two rods ($A_{i1}B_{i1}$ and $A_{i2}B_{i2}$, $i = 1, 2, 3$, in Figure 4) joined to the platform via a spherical pair.

Recently, Di Gregorio and Parenti-Castelli (1998, 1999) and Di Gregorio (2000) have shown that some translational parallel manipulators with three equal legs, each one leaving five DoFs to the relative motion between platform and

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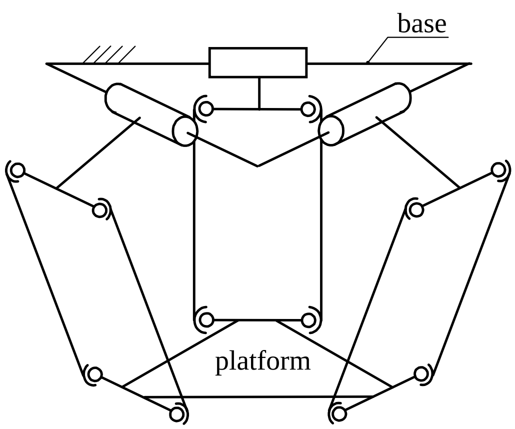

Fig. 1. DELTA robot.

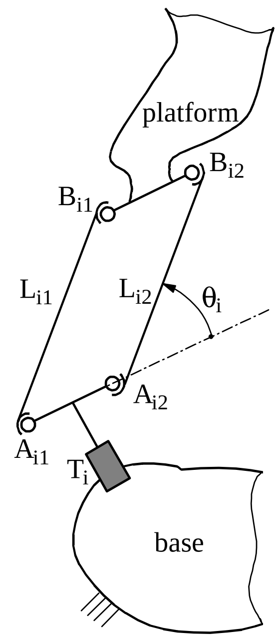

Fig. 3. DELTA-like leg.

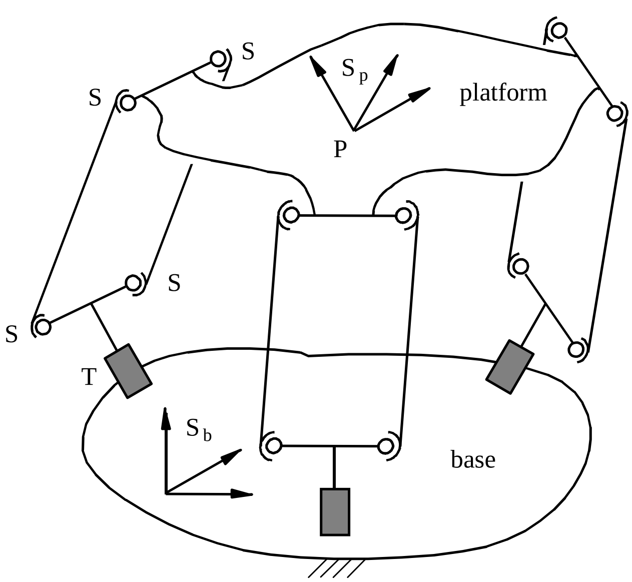

Fig. 2. DLM: *S* indicates a spherical pair and *T* indicates a joint that makes one straight line translate.

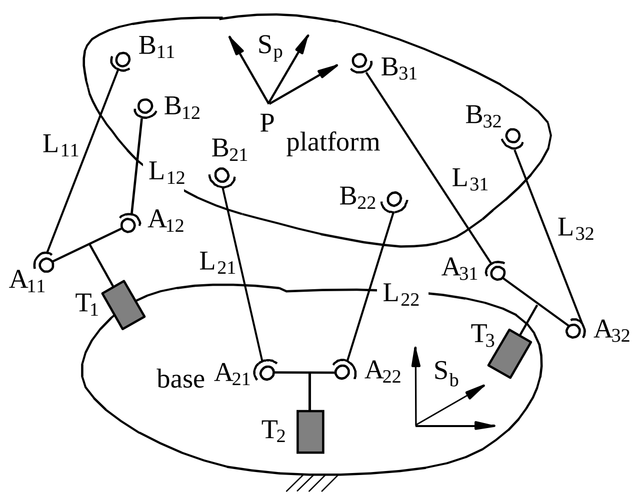

Fig. 4. A GDLM.

base, may reach singular configurations in which the angular velocity of the platform is not determined. These singular configurations are called rotation singularities. Starting from a rotation singularity, the platform motion may no longer be purely translational.

Finding all the singularities is a mandatory step in the design of these manipulators.

In this paper, the mobility analysis of the DLMs is presented on the whole and the ability of the DLMs to reach rotation singularities is demonstrated. Moreover, the equations which make it possible to draw the loci of all the DLM singularities in the platform's Cartesian space are provided. The presented equations contain the geometric parameters of the manipulator and can be employed to choose the manipulator's geometry with the goal of clearing the workspace from the singular configurations.

The following section demonstrates the skill of GDLMs to make the platform translate under some mounting and manufacturing conditions. Next, the DLM singularity conditions will be derived as a corollary of this demonstration.

2. Kinematic Analysis of the GDLM

With reference to Figure 4, the closure equations of the GDLM can be written as follows

$$(\mathbf{B}_{ij} - \mathbf{A}_{ij})^2 = L_{ij}^2 \quad i = 1, 2, 3;\ j = 1, 2 \tag{1}$$

where L_{ij} is the constant length of the rod $A_{ij}B_{ij}$.

Differentiating eq. (1) with respect to the time variable yields

$$(\dot{\mathbf{B}}_{ij} - \dot{\mathbf{A}}_{ij}) \cdot \mathbf{u}_{ij} = 0 \quad i = 1, 2, 3;\ j = 1, 2 \tag{2}$$

where

$$\mathbf{u}_{ij} = \frac{(\mathbf{B}_{ij} - \mathbf{A}_{ij})}{L_{ij}} \quad i = 1, 2, 3;\ j = 1, 2. \tag{3}$$

In eq. (2) $\dot{\mathbf{B}}_{ij}$ and $\dot{\mathbf{A}}_{ij}$ are the velocities of the points B_{ij} and A_{ij} (Figure 4), respectively.

By calling P the origin of a Cartesian reference system, S_p, fixed in the platform (Figure 4), the $\dot{\mathbf{B}}_{ij}$ velocity can be written in the following way

$$\dot{\mathbf{B}}_{ij} = \dot{\mathbf{P}} + \omega \times (\mathbf{B}_{ij} - \mathbf{P}) \quad i = 1, 2, 3;\ j = 1, 2 \tag{4}$$

where $\boldsymbol{\omega}$ is the angular velocity of the platform and $\dot{\mathbf{P}}$ is the velocity of the point P.

By taking relationships (4) into account, eq. (2) becomes

$$\mathbf{u}_{ij} \cdot \dot{\mathbf{P}} + [(\mathbf{B}_{ij} - \mathbf{P}) \times \mathbf{u}_{ij}] \cdot \omega = \dot{\mathbf{A}}_{ij} \cdot \mathbf{u}_{ij} \qquad i = 1, 2, 3;\ j = 1, 2 \tag{5}$$

where the identity $[\boldsymbol{\omega} \times (\mathbf{B}_{ij} - \mathbf{P})] \cdot \mathbf{u}_{ij} = [(\mathbf{B}_{ij} - \mathbf{P}) \times \mathbf{u}_{ij}] \cdot \boldsymbol{\omega}$ has been used.

By using eq. (5) it is possible to demonstrate the following theorem:

THEOREM 1. If a GDLM performs an elementary motion starting from a non-singular configuration where all the legs satisfy the relationships

$$\mathbf{u}_{i1} = \mathbf{u}_{i2} \quad i = 1, 2, 3 \tag{6}$$

then the platform performs an elementary translation with respect to the base.

Proof. Subtracting eq. (5) with j=2 from eq. (5) with j=1 yields

$$(\mathbf{u}_{i1} - \mathbf{u}_{i2}) \cdot \dot{\mathbf{P}} + [(\mathbf{B}_{i1} - \mathbf{P}) \times \mathbf{u}_{i1} - (\mathbf{B}_{i2} - \mathbf{P}) \times \mathbf{u}_{i2}] \cdot \omega = \dot{\mathbf{A}}_i \cdot (\mathbf{u}_{i1} - \mathbf{u}_{i2}) \quad i = 1, 2, 3 \tag{7}$$

where $\dot{\mathbf{A}}_i$ indicates the velocity of the points A_{i1} and A_{i2} that translate together (which implies that they have same velocity and acceleration) because of the joint T_i (remember that the joint T_i is manufactured so as to make the segment $A_{i1}A_{i2}$ translate with respect to the base).

If relationships (6) are satisfied, eq. (7) becomes

$$\mathbf{n}_i \cdot \boldsymbol{\omega} = 0 \quad i = 1, 2, 3 \tag{8}$$

where, denoting $\mathbf{u}_i$ either of the unit vectors $\mathbf{u}_{i1}$ and $\mathbf{u}_{i2}$, the vectors $\mathbf{n}_i$, $i = 1, 2, 3$, are defined as follows:

$$\mathbf{n}_i = \mathbf{u}_i \times (\mathbf{B}_{i2} - \mathbf{B}_{i1}) \quad i = 1, 2, 3. \tag{9}$$

The vectors $\mathbf{u}_i$ and $\mathbf{n}_i$ depend only on the GDLM configuration. If the GDLM configuration is such as to make the $\mathbf{n}_i$ vectors linearly independent, i.e., the configuration is not singular, system (8) can be matched if and only if the platform angular velocity, $\boldsymbol{\omega}$, vanishes. □

Conditions (6) guarantee an infinitesimal translation of the platform.

A finite translation of the platform is an infinite sequence of infinitesimal translations which implies that the platform angular velocity, $\boldsymbol{\omega}$, and all the time derivatives of the platform angular velocity vanish at the starting configuration of the manipulator.

Differentiating eq. (5) with respect to the time variable yields

$$(\ddot{\mathbf{P}} - \ddot{\mathbf{A}}_{ij}) \cdot \mathbf{u}_{ij} + (\dot{\mathbf{P}} - \dot{\mathbf{A}}_{ij}) \cdot \dot{\mathbf{u}}_{ij} + [(\mathbf{B}_{ij} - \mathbf{P}) \times \mathbf{u}_{ij}] \cdot \dot{\omega} + [(\dot{\mathbf{B}}_{ij} - \dot{\mathbf{P}}) \times \mathbf{u}_{ij} + (\mathbf{B}_{ij} - \mathbf{P}) \times \dot{\mathbf{u}}_{ij}] \cdot \omega = 0 \quad i = 1, 2, 3;\ j = 1, 2. \tag{10}$$

By differentiating definition (3) the following expression for $\dot{\mathbf{u}}_{ij}$ results

$$\dot{\mathbf{u}}_{ij} = \frac{(\dot{\mathbf{B}}_{ij} - \dot{\mathbf{A}}_{ij})}{L_{ij}} \quad i = 1, 2, 3;\ j = 1, 2. \tag{11}$$

By taking relationships (4) into account, expression (11) becomes

$$\dot{\mathbf{u}}_{ij} = \frac{(\dot{\mathbf{P}} - \dot{\mathbf{A}}_{ij})}{L_{ij}} + \frac{\omega \times (\mathbf{B}_{ij} - \mathbf{P})}{L_{ij}} \quad i = 1, 2, 3;\ j = 1, 2. \tag{12}$$

Introducing relationships (12) and substituting $\dot{\mathbf{A}}_i$ for $\dot{\mathbf{A}}_{i1}$ and $\dot{\mathbf{A}}_{i2}$ into eq. (10) yields

$$\begin{aligned}(\ddot{\mathbf{P}} - \ddot{\mathbf{A}}_i) \cdot \mathbf{u}_{ij} + \frac{(\dot{\mathbf{P}} - \dot{\mathbf{A}}_i)^2}{L_{ij}} + [(\mathbf{B}_{ij} - \mathbf{P}) \times \mathbf{u}_{ij}] \\ \cdot \dot{\omega} + \{ \frac{2}{L_{ij}}\ (\mathbf{B}_{ij} - \mathbf{P}) \times (\dot{\mathbf{P}} - \dot{\mathbf{A}}_i) + \frac{1}{L_{ij}}(\mathbf{B}_{ij} - \mathbf{P}) \\ \times [\omega \times (\mathbf{B}_{ij} - \mathbf{P})] + (\dot{\mathbf{B}}_{ij} - \dot{\mathbf{P}}) \times \mathbf{u}_{ij}\} \cdot \omega = 0 \\ i = 1, 2, 3;\ j = 1, 2\end{aligned} \tag{13}$$

where $\ddot{\mathbf{A}}_i$ denotes either of the accelerations $\ddot{\mathbf{A}}_{i1}$ and $\ddot{\mathbf{A}}_{i2}$ that are equal to each other because of the joint T_i (Figure 4).

By using eq. (13), it is possible to demonstrate the following theorem:

THEOREM 2. If a GDLM meeting the following manufacturing conditions

$$L_{i1} = L_{i2} \quad i = 1, 2, 3 \tag{14a}$$

$$\|(\mathbf{A}_{i2} - \mathbf{A}_{i1})\| = \|(\mathbf{B}_{i2} - \mathbf{B}_{i1})\| \quad i = 1, 2, 3 \tag{14b}$$

is so mounted as to match conditions (6) in a non-singular configuration and starts moving from this configuration, then the platform performs a finite translation with respect to the base.

Proof. Since the starting configuration satisfies conditions (6), Theorem 1 holds and the platform angular velocity, $\boldsymbol{\omega}$, is equal to the null vector. Thus, in this case, eq. (13) becomes

$$\begin{aligned}(\ddot{\mathbf{P}} - \ddot{\mathbf{A}}_i) \cdot \mathbf{u}_i + \frac{(\dot{\mathbf{P}} - \dot{\mathbf{A}}_i)^2}{L_{ij}} + [(\mathbf{B}_{ij} - \mathbf{P}) \times \mathbf{u}_i] \cdot \dot{\omega} = 0 \\ i = 1, 2, 3;\ j = 1, 2\end{aligned} \tag{15}$$

where $\mathbf{u}_i$ has substituted either of the unit vectors $\mathbf{u}_{i1}$ and $\mathbf{u}_{i2}$.

Subtracting eq. (15) with j=2 from eq. (15) with j=1 yields

$$\mathbf{n}_i \cdot \dot{\omega} = (\dot{\mathbf{P}} - \dot{\mathbf{A}}_i)^2 \left(\frac{1}{L_{i2}} - \frac{1}{L_{i1}} \right) \quad i = 1, 2, 3 \tag{16}$$

where $\mathbf{n}_i$ is the vector defined by expression (9). If conditions (14a) are satisfied, eq. (16) becomes

$$\mathbf{n}_i \cdot \dot{\omega} = 0 \quad i = 1, 2, 3. \tag{17}$$

If the GDLM configuration is such as to make the $\mathbf{n}_i$ vectors linearly independent, i.e., the configuration is not singular, system (17) can be matched if and only if the platform angular acceleration, $\dot{\omega}$, vanishes. Since, in the starting configuration, $\boldsymbol{\omega}$ is the null vector and conditions (14a) and (14b) hold, the platform is constrained to perform an elementary translation at the end of which the GDLM reaches another configuration that still satisfies the conditions (6); moreover, since $\dot{\omega}$ is the null vector, $\boldsymbol{\omega}$ must still be the null vector in the configuration reached after the elementary translation. As a consequence, if the reached configuration is non-singular, the elementary motion that the platform can perform starting from the reached configuration must be an elementary translation with the same characteristics of the first elementary translation (i.e., the GDLM still satisfies conditions (6) and $\boldsymbol{\omega}$ must still be the null vector in the configuration reached after the elementary translation). By reiterating this reasoning, it can be concluded that starting from a non-singular configuration which satisfies conditions (6), a GDLM, which matches conditions (14), makes the platform perform an infinite sequence of elementary translations (i.e., a finite translation) until the GDLM reaches a singular configuration (rotation singularity) where the vectors $\mathbf{n}_i$, $i = 1, 2, 3$, are linearly dependent. □

As a final remark, it has to be observed that, if conditions (6) and (14) hold, the following relationships will also hold:

$$(\mathbf{B}_{i2} - \mathbf{B}_{i1}) = (\mathbf{A}_{i2} - \mathbf{A}_{i1}) \quad i = 1, 2, 3. \tag{18}$$

By taking relationship (18) into account, definition (9) becomes

$$\mathbf{n}_i = \mathbf{u}_i \times (\mathbf{A}_{i2} - \mathbf{A}_{i1}) \quad i = 1, 2, 3 \tag{19}$$

where $(\mathbf{A}_{i2} - \mathbf{A}_{i1})$ is a constant vector in a reference system, S_b, fixed in the base and $\mathbf{u}_i$ depends only on the GDLM configuration.

3. Mobility Analysis of the DLM

A GDLM satisfying the manufacturing conditions (14) is a DLM and, if it is so mounted as to match the mounting conditions (6), it makes the platform translate, provided that the singular configurations are not touched. The DLM mobility analysis, focused on the translational configurations, can be performed by taking the equation system constituted of the three eqs. (5) with $j = 1$ and the three eqs. (8) for reference. This system is as follows:

$$\mathbf{u}_i \cdot \dot{\mathbf{P}} + [(\mathbf{B}_{i1} - \mathbf{P}) \times \mathbf{u}_i] \cdot \omega = \dot{\mathbf{A}}_i \cdot \mathbf{u}_i \quad i = 1, 2, 3 \tag{20a}$$

$$\mathbf{n}_i \cdot \boldsymbol{\omega} = 0 \quad i = 1, 2, 3. \tag{20b}$$

System (20) can be written in matrix form as follows

$$\mathbf{J} \begin{Bmatrix} \dot{\mathbf{P}} \\ \omega \end{Bmatrix} = \mathbf{b} \tag{21}$$

with

$$\mathbf{J} = \begin{bmatrix} \mathbf{U} & \mathbf{M} \\ \mathbf{0} & \mathbf{N} \end{bmatrix} \tag{22.1}$$

$$\mathbf{b} = (\dot{\mathbf{A}}_1 \cdot \mathbf{u}_1,\ \dot{\mathbf{A}}_2 \cdot \mathbf{u}_2,\ \dot{\mathbf{A}}_3 \cdot \mathbf{u}_3,\ 0,\ 0,\ 0)^T \tag{22.2}$$

where $\mathbf{U}$, $\mathbf{N}$ and $\mathbf{M}$ are 3×3 matrices defined as follows

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1^T \\ \mathbf{u}_2^T \\ \mathbf{u}_3^T \end{bmatrix}; \quad \mathbf{N} = \begin{bmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \\ \mathbf{n}_3^T \end{bmatrix};$$

$$\mathbf{M} = \begin{bmatrix} [(\mathbf{B}_{11} - \mathbf{P}) \times \mathbf{u}_1]^T \\ [(\mathbf{B}_{21} - \mathbf{P}) \times \mathbf{u}_2]^T \\ [(\mathbf{B}_{31} - \mathbf{P}) \times \mathbf{u}_3]^T \end{bmatrix}; \qquad (22.3)$$

$\mathbf{0}$ is the 3×3 null matrix and $(\cdot)^T$ denotes the transpose of $(\cdot)$.

The mobility analysis consists of solving two problems: direct velocity analysis and inverse velocity analysis. Direct velocity analysis is the determination of the platform velocities $\dot{\mathbf{P}}$ and $\boldsymbol{\omega}$ when the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, imposed by the actuators, are known. Inverse velocity analysis is the determination of the signed intensities of the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, (note that the directions of the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, are always assigned by the structure of the actuators T_i, $i = 1, 2, 3$, (Figure 3)) to be imposed by the actuators in order to obtain an assigned value of the linear platform velocity $\dot{\mathbf{P}}$.

3.1 Direct Problem

If the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, are known and the platform velocities $\dot{\mathbf{P}}$ and $\boldsymbol{\omega}$ have to be computed, system (20) will be singular when the determinant of matrix $\mathbf{J}$ vanishes.

Definitions (22) give the following expression of the determinant of matrix $\mathbf{J}$

$$\det(\mathbf{J}) = \det(\mathbf{U}) \det(\mathbf{N}) \qquad (23)$$

where the determinants of the $\mathbf{U}$ and $\mathbf{N}$ matrices are

$$\det(\mathbf{U}) = \mathbf{u}_1 \cdot \mathbf{u}_2 \times \mathbf{u}_3 \qquad (24a)$$

$$\det(\mathbf{N}) = \mathbf{n}_1 \cdot \mathbf{n}_2 \times \mathbf{n}_3. \qquad (24b)$$

Taking expressions (23) and (24) into account, the singularity condition can be analytically expressed as follows:

$$(\mathbf{u}_1 \cdot \mathbf{u}_2 \times \mathbf{u}_3)(\mathbf{n}_1 \cdot \mathbf{n}_2 \times \mathbf{n}_3) = 0. \qquad (25)$$

Condition (25) is satisfied when at least one of the following conditions is matched:

$$\mathbf{n}_1 \cdot \mathbf{n}_2 \times \mathbf{n}_3 = 0 \qquad (26a)$$

$$\mathbf{u}_1 \cdot \mathbf{u}_2 \times \mathbf{u}_3 = 0. \qquad (26b)$$

When condition (26a) is satisfied, eq. (20b) is linearly dependent and the angular velocity, $\boldsymbol{\omega}$, of the platform is not determined (rotation singularity). It is worth noting that condition (26a) also makes system (17) singular and the angular acceleration, $\dot{\omega}$, not determined.

On the other hand, when condition (26b) is satisfied, the coefficient matrix of the platform translation velocity, $\dot{\mathbf{P}}$, in eq. (20a) is singular and $\dot{\mathbf{P}}$ is not determined (translation singularity).

To better understand when these singularities occur, in the following subsections, the actuators will be considered locked, while the DLM assumes a configuration matching either of conditions (26), and the additional DoF, introduced by the singularity, will be investigated.

3.1.1. Rotation Singularities

The rotation singularities are the configurations that match condition (26a).

From a kinematic point of view, the vectors $\mathbf{n}_i$, $i = 1, 2, 3$, are parallel to axes the platform cannot rotate around because eq. (20b) states that the dot products $\mathbf{n}_i \cdot \boldsymbol{\omega}$, $i = 1, 2, 3$, must be equal to zero. This result can also be deduced by considering that, if the two rods $A_{i1}B_{i1}$ and $A_{i2}B_{i2}$ (Figure 3) are parallel to each other, the segment $B_{i1}B_{i2}$, fixed in the platform, cannot rotate in the plane of the parallelogram $A_{i1}A_{i2}B_{i2}B_{i1}$ (Pierrot and Company 1999).

From a geometric point of view, by analyzing relationships (19) it comes out that the $\mathbf{n}_i$ direction is perpendicular to the plane which the parallelogram $A_{i1}A_{i2}B_{i2}B_{i1}$ (Figure 3) lies on and that the $\mathbf{n}_i$ intensity is proportional to the sine of the angle between $(\mathbf{B}_{i2}-\mathbf{A}_{i2})$ and $(\mathbf{A}_{i2}-\mathbf{A}_{i1})$, angle θ_i in Figure 3. Thus condition (26a) is verified, when one of the following geometric conditions occurs.

(R1) One out of the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, degenerates into a segment (Figure 5(a)). When this geometric condition occurs, the platform can perform infinitesimal rotations around an axis parallel to the intersection line between the planes of the two parallelograms which do not degenerate into a segment.

(R2) Two out of the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on the same plane or on parallel planes (Figure 5(b)). When this geometric condition occurs, the platform can perform infinitesimal rotations around an axis parallel to the planes of all three parallelograms.

(R3) The intersections of the planes, the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on, are three parallel lines (Figure 5(c)). When this geometric condition occurs, the platform can perform infinitesimal rotations around an axis parallel to the intersection lines between the planes of the parallelograms.

The actual occurrence of the additional rotational DoF, in some of the above listed geometric conditions, has been observed in a prototype of the DELTA robot (Figure 1) by Clavel (1988) and justified by static reasoning.

The vectors $\mathbf{n}_i$, $i = 1, 2, 3$, can be written as explicit functions of the coordinates of a platform point, e.g., P (Figure 2), and of the geometric parameters of the mechanism by solving

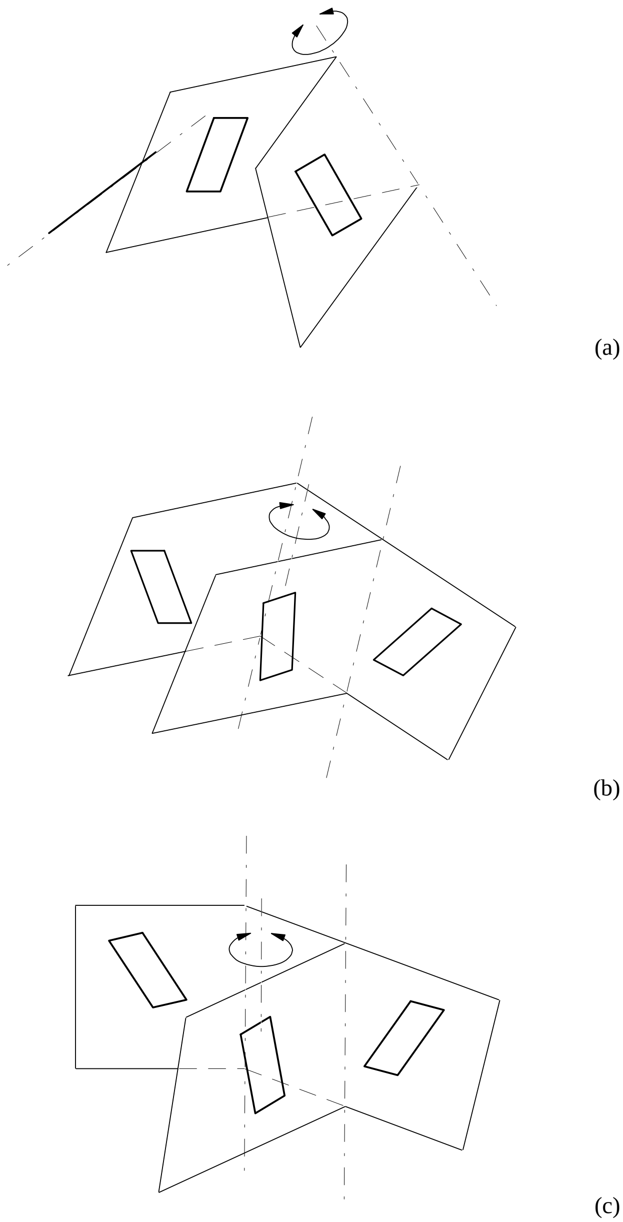

Fig. 5. Rotation singularities: (a) one out of the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, degenerates into a segment; (b) two out of the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on parallel plane; (c) the intersections of the planes, which the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on, are parallel lines.

the inverse position analysis of the DLM. Thus, for a given geometry eq. (26a) is a scalar equation in three unknowns: the coordinates of the platform point P. In the Cartesian space of the platform positions, the geometric representation of eq. (26a) is a fixed surface that is the geometric locus of the P point's positions, to which the DLM rotation singularities correspond.

3.1.2. Translation Singularities

The translation singularities are the configurations that match condition (26b).

From a kinematic point of view, the $\mathbf{u}_i$ unit vector directions are lines along which the platform translation is controlled by the actuators, and is forbidden when the actuators are locked, because the dot products $\mathbf{u}_i \cdot \dot{\mathbf{P}}$ are assigned by eq. (20a), where $\boldsymbol{\omega}$ is equal to the null vector out of singular configurations.

From a geometric point of view, condition (26b) is verified, when one of the following geometric conditions is verified.

(T1) The three parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on three parallel planes (Figure 6(a)). When this geometric condition occurs, the platform can perform infinitesimal displacement along a direction perpendicular to planes of the parallelograms. Configurations matching this condition also match condition (R2) and are both rotation and translation singularities.

(T2) Two out of the three parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, have the sides $A_{ij}B_{ij}$, $j = 1, 2$, parallel to the line which is the intersection of the two planes of the parallelograms. In this case, the four rods $A_{ij}B_{ij}$ of the two parallelograms are all parallel (Figure 6(b)) and all the rods $A_{ij}B_{ij}$ of the DLM are parallel to a single plane (plane π in Figure 6(b)). When this condition occurs, the platform can perform infinitesimal displacement along the direction perpendicular to the plane the rods $A_{ij}B_{ij}$ are parallel to.

The actual occurrence of the additional translational DoF, in some of the above listed geometric conditions, has been observed in a prototype of the DELTA robot (Figure 1) by Clavel (1988) and justified by static reasoning.

The unit vectors $\mathbf{u}_i$, $i = 1, 2, 3$, can be written as explicit functions of the coordinates of a platform point, e.g., P (Figure 2), and of the geometric parameters of the mechanism by solving the inverse position analysis of the DLM. Thus, for a given geometry eq. (26b) is a scalar equation in three unknowns: the coordinates of the platform point P. In the Cartesian space of the platform positions, the geometric representation of eq. (26b) is a fixed surface that is the geometric locus of the P point's positions, to which the DLM's translation singularities correspond.

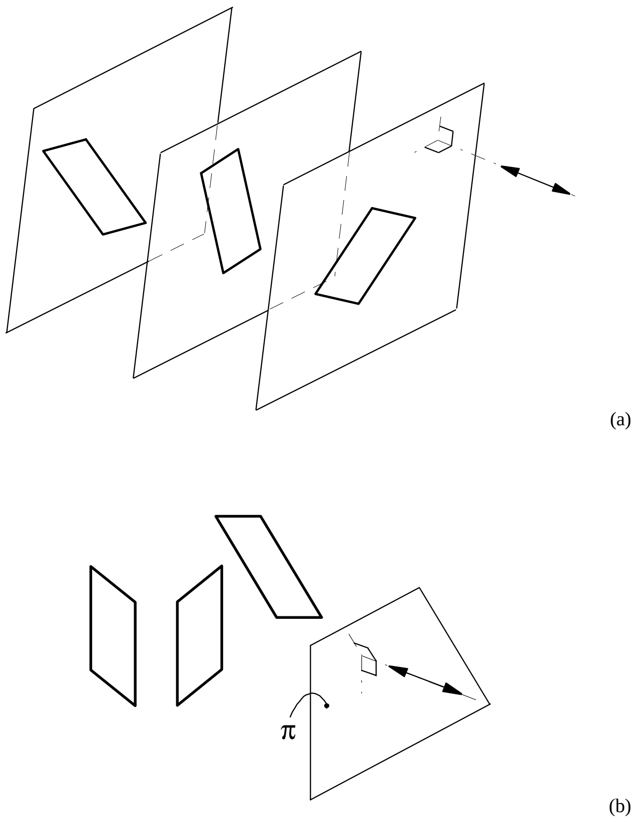

Fig. 6. Translation singularities: (a) the three parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, lie on three parallel planes; (b) two out of the three parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, have the sides $A_{ij}B_{ij}$, j=1, 2, parallel to the line which is the intersection of the two planes of the parallelograms.

3.2 Inverse Problem

If the platform linear velocity $\dot{\mathbf{P}}$ is known, the signed intensities of the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, can be computed by using eq. (20a), which is decoupled, where the solution of eq. (20b) (i.e., $\boldsymbol{\omega} = 0$ out of rotation singularities) is substituted for $\boldsymbol{\omega}$.

The analysis of the ith eq. (20a) shows that the signed intensity of the velocity $\dot{\mathbf{A}}_i$ cannot be determined if and only if the direction of the unit vector $\mathbf{u}_i$ is perpendicular to the direction of the velocity $\dot{\mathbf{A}}_i$. This geometric condition identifies the inverse problem singularities. In the DELTA robot (Figure 1) the inverse problem singularities occur when one out of the parallelograms $A_{i1}A_{i2}B_{i2}B_{i1}$, $i = 1, 2, 3$, degenerates into a segment (geometric condition (R1)). In general, if the directions of the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, are perpendicular to the segments $A_{i1}A_{i2}$, $i = 1, 2, 3$, (Figure 3) respectively, the inverse problem singularities will occur when geometric condition (R1) is matched.

3.3. Relationships to be Used out of Singularity

If conditions (26) are not satisfied, systems (17) and (20) are not singular, and the angular velocity, $\boldsymbol{\omega}$, and the angular acceleration, $\dot{\omega}$, must be equal to zero. Therefore, system (20) reduces to the following three equations ($\boldsymbol{\omega} = 0$)

$$\mathbf{u}_i \cdot \dot{\mathbf{P}} = \dot{\mathbf{A}}_i \cdot \mathbf{u}_i \quad i = 1, 2, 3 \tag{27}$$

and system (15) reduces to the following three equations ($\dot{\omega} = 0$)

$$\mathbf{u}_i \cdot \ddot{\mathbf{P}} = \ddot{\mathbf{A}}_i \cdot \mathbf{u}_i - \frac{(\dot{\mathbf{P}} - \dot{\mathbf{A}}_i)^2}{L_i} \quad i = 1, 2, 3 \tag{28}$$

where L_i indicates either of the lengths L_{i1} and L_{i2}.

Equation (27) makes it possible to calculate the platform translation velocity, $\dot{\mathbf{P}}$, as a function of the manipulator configuration, that appears in the vectors $\mathbf{u}_i$, $i = 1, 2, 3$, and of the velocities $\dot{\mathbf{A}}_i$, $i = 1, 2, 3$, that are imposed by the actuators.

On the other hand, eq. (28) makes it possible to compute the platform acceleration, $\ddot{\mathbf{P}}$, as a function of the manipulator configuration, of the platform translation velocity, $\dot{\mathbf{P}}$, and the inputs $\dot{\mathbf{A}}_i$ and $\ddot{\mathbf{A}}_i$, $i = 1, 2, 3$, that are imposed by the actuators.

4. Conclusions

In this paper the mobility analysis of a class of parallel translational manipulators derived from the DELTA robot has been developed in full. The manufacturing and mounting conditions that guarantee the pure translation of the platform have been derived analytically. Moreover, it has been demonstrated that these manipulators can assume singular configurations, called rotation singularities, in which they are not able to impose the platform translation any longer.

Finally, the geometric and analytic conditions that make it possible to find all the singular configurations of these manipulators have been provided.

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