

FIGURE 3.3 A simple link which supports two revolute axes.

3.3 Link connection description

The problem of connecting the links of a robot together is again one filled with many questions for the mechanical designer to resolve. These include the strength of the joint, lubrication, bearing and gearing mounting, etc. However, for the investigation of kinematics, we need only worry about two quantities which will completely specify the way in which links are connected together.

Intermediate links in the chain

Neighboring links have a common joint axis between them. One parameter of interconnection has to do with the distance along this common axis from one link to the next. This parameter is called the **link offset**. The offset at joint axis i is called d_i . The second parameter describes the amount of rotation about this common axis between one link and its neighbor. This is called the **joint angle**, θ_i .

Figure 3.4 shows the interconnection of link $i-1$ and link i . Recall that a_{i-1} is the mutual perpendicular between the two axes of link $i-1$. Likewise a_i is the mutual perpendicular defined for link i . The first parameter of interconnection is the link offset, d_i , which is the signed distance measured along the axis of joint i from the point where a_{i-1} intersects the axis to the point where a_i intersects the axis. The offset d_i is indicated in Fig. 3.4. The link offset d_i is variable if joint i is prismatic.

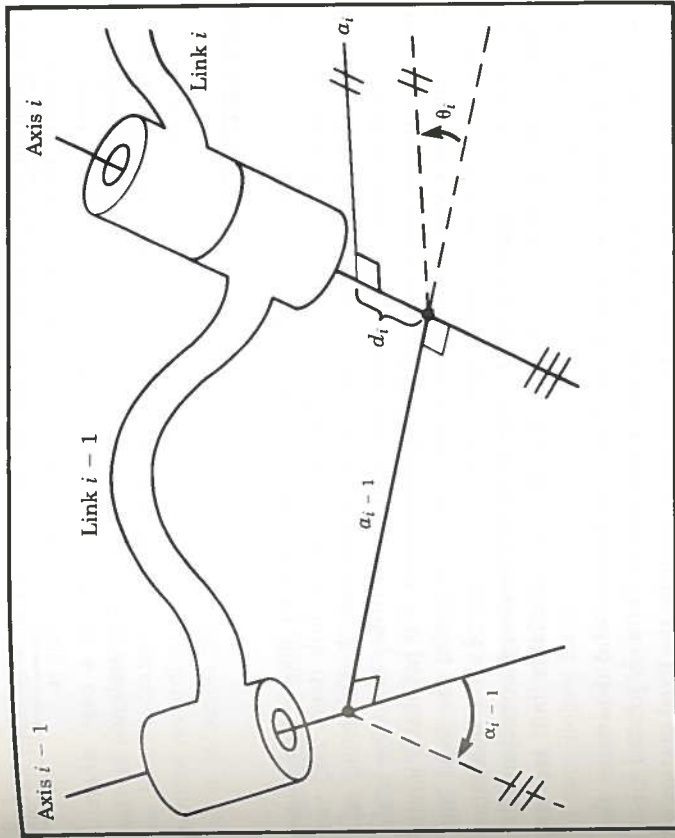


FIGURE 3.4 The link offset, d , and the joint angle, θ , are two parameters which may be used to describe the nature of the connection between neighboring links.

The second parameter of interconnection is the angle made between an extension of a_{i-1} and a_i measured about the axis of joint i . This is indicated in Fig. 3.4, where the lines with the double hash marks are parallel. This parameter is named θ_i , and is variable for a revolute joint.

First and last links in the chain

Link length, a_i , and link twist, α_i , depend on joint axes i and $i+1$. Hence a_1 through a_{n-1} and α_1 through α_{n-1} are defined as discussed above in this section. At the ends of the chain, it will be our convention to assign zero to these quantities. That is, $a_0 = a_n = 0.0$ and $\alpha_0 = \alpha_n = 0.0$.^{*} Link offset, d_i , and joint angle, θ_i , are well defined for joints 2 through $n-1$ according to the conventions discussed above in this section. If joint 1 is revolute, the zero position for θ_1 may be chosen arbitrarily and $d_1 = 0.0$ will be our convention. Similarly, if joint 1 is prismatic, the

^{*} In fact, a_n and α_n do not need to be defined at all.

zero position of d_1 may be chosen arbitrarily, and $\theta_1 = 0.0$ will be our convention. Exactly the same statements apply to joint n .

These conventions have been chosen so that in a case where a quantity could be assigned arbitrarily, a zero value is assigned so that later calculations will be as simple as possible.

Link parameters

Hence any robot can be described kinematically by giving the values of four quantities for each link. Two describe the link itself, and two describe the link's connection to a neighboring link. In the usual case of a revolute joint, θ_i is called the **joint variable**, and the other three quantities would be fixed **link parameters**. For prismatic joints, d_i is the joint variable and the other three quantities are fixed link parameters. The definition of mechanisms by means of these quantities is a convention usually called the **Denavit-Hartenberg notation** [1].* Other methods of describing mechanisms are available but are not presented here.

At this point we could inspect any mechanism and determine the Denavit-Hartenberg parameters which describe it. For a six-jointed robot 18 numbers would be required to completely describe the fixed portion of its kinematics. In the case of a six-jointed robot with all revolute joints, the 18 numbers are in the form of six sets of (a_i, α_i, d_i) .

EXAMPLE 3.2

Two links, as described in Fig. 3.3, are connected as links 1 and 2 of a robot. Joint 2 is composed of a "B" bearing of link 1 and an "A" bearing of link 2 arranged so that the flat surfaces of the "A" and "B" bearings lie flush against each other. What is d_2 ?

The link offset d_2 is the offset at joint 2, which is the distance, measured along the joint 2 axis, between the mutual perpendicular of link 1 and that of link 2. From the drawings in Fig. 3.3, this is 2.5 inches. ■

Before introducing more examples we will define a convention for attaching a frame to each link of the manipulator.

* Note that many related conventions go by the name of Denavit-Hartenberg but differ in a few details. For example, the version used in this book differs from some of the robotic literature in the manner of frame numbering. Unlike some other conventions, in this book frame $\{i\}$ is attached to link i and has its origin lying on joint axis i .

3.4 Convention for affixing frames to links

In order to describe the location of each link relative to its neighbors we define a frame attached to each link. The link frames are named by number according to the link to which they are attached. That is, frame $\{i\}$ is attached rigidly to link i .

Intermediate links in the chain

The convention we will use to locate frames on the links is as follows: The \hat{Z} -axis of frame $\{i\}$, called \hat{Z}_i , is coincident with the joint axis i . The origin of frame $\{i\}$ is located where the a_i perpendicular intersects the joint i axis. \hat{X}_i points along a_i in the direction from joint i to joint $i+1$.

In the case of $a_i = 0$, \hat{X}_i is normal to the plane of \hat{Z}_i and \hat{Z}_{i+1} . We define α_i as being measured in the right-hand sense about \hat{X}_i , and so we see that the freedom of choosing the sign of α_i in this case corresponds to two choices for the direction of \hat{X}_i . \hat{Y}_i is formed by the right-hand rule to complete the i th frame. Figure 3.5 shows the location of frames $\{i-1\}$ and $\{i\}$ for a general manipulator.

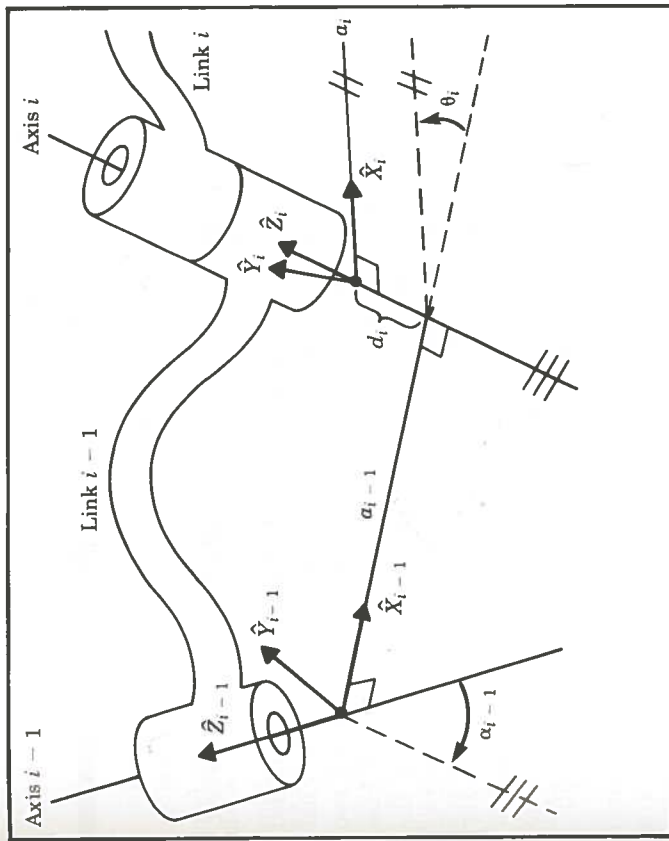


FIGURE 3.5 Link frames are attached so that frame $\{i\}$ is attached rigidly to link i .

First and last links in the chain

We attach a frame to the base of the robot, or link 0, called frame $\{0\}$. This frame does not move and for the problem of arm kinematics can be considered the reference frame. We may describe the position of all other link frames in terms of this frame.

Since frame $\{0\}$ is arbitrary, it always simplifies matters to choose \hat{Z}_0 along axis 1 and to locate frame $\{0\}$ so that it coincides with frame $\{1\}$ when joint variable 1 is zero. Using this convention we will always have $a_0 = 0.0$, $\alpha_0 = 0.0$. Additionally, this ensures that $d_1 = 0.0$ if joint 1 is revolute, or $\theta_1 = 0.0$ if joint 1 is prismatic.

For joint n revolute, the direction of \hat{X}_N is chosen so that it aligns with \hat{X}_{N-1} when $\theta_n = 0.0$, and the origin of frame $\{N\}$ is chosen so that $d_n = 0.0$. For joint n prismatic, the direction of \hat{X}_N is chosen so that $\theta_n = 0.0$, and the origin of frame $\{N\}$ is chosen at the intersection of \hat{X}_{N-1} and joint axis n when $d_n = 0.0$.

Summary of the link parameters in terms of the link frames

If the link frames have been attached to the links according to our convention, the following definitions of the link parameters are valid:

$$\begin{aligned} a_i &= \text{the distance from } \hat{Z}_i \text{ to } \hat{Z}_{i+1} \text{ measured along } \hat{X}_i; \\ \alpha_i &= \text{the angle between } \hat{Z}_i \text{ and } \hat{Z}_{i+1} \text{ measured about } \hat{X}_i; \\ d_i &= \text{the distance from } \hat{X}_{i-1} \text{ to } \hat{X}_i \text{ measured along } \hat{Z}_i; \text{ and} \\ \theta_i &= \text{the angle between } \hat{X}_{i-1} \text{ and } \hat{X}_i \text{ measured about } \hat{Z}_i. \end{aligned}$$

We usually choose $a_i > 0$ since it corresponds to a distance; however, α_i , d_i , and θ_i are signed quantities.

A final note on uniqueness is warranted. The convention outlined above does not result in a unique attachment of frames to links. First of all, when we first align the \hat{Z}_i axis with joint axis i , there are two choices of direction in which to point \hat{Z}_i . Furthermore, in the case of intersecting joint axes (i.e., $a_i = 0$), there are two choices for the direction of \hat{X}_i , corresponding to the choice of signs for the normal to the plane containing \hat{Z}_i and \hat{Z}_{i+1} . When axes i and $i+1$ are parallel, the choice of origin location for $\{i\}$ is arbitrary (though generally chosen in order to cause d_i to be zero). Also when prismatic joints are present there is quite a bit of freedom in frame assignment. (See also Example 3.5.)

Summary of link frame attachment procedure

The following is a summary of the procedure to follow when faced with a new mechanism in order to properly attach the link frames:

1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the i th axis, assign the link frame origin.
3. Assign the \hat{Z}_i axis pointing along the i th joint axis.
4. Assign the \hat{X}_i axis pointing along the common perpendicular, or if the axes intersect, assign \hat{X}_i to be normal to the plane containing the two axes.
5. Assign the \hat{Y}_i axis to complete a right-hand coordinate system.
6. Assign $\{0\}$ to match $\{1\}$ when the first joint variable is zero. For $\{N\}$ choose an origin location and \hat{X}_N direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

EXAMPLE 3.3

Figure 3.6a shows a three-link planar arm. Because all three joints are revolute, this manipulator is sometimes called an "RRR (or 3R) mechanism." Fig. 3.6b is a schematic representation of the same manipulator. Note the double hash marks indicated on each of the three axes which indicate that these axes are parallel. Assign link frames to the mechanism and give the Denavit-Hartenberg parameters.

We start by defining the reference frame, frame $\{0\}$. It is fixed to the base and aligns with frame $\{1\}$ when the first joint variable (θ_1) is zero. Therefore we position frame $\{0\}$ as shown in Fig. 3.7 with \hat{Z}_0 aligned with the joint 1 axis. For this arm, all joint axes are oriented perpendicular to the plane of the arm. Since the arm lies in a plane with all \hat{Z} axes parallel, there are no link offsets (all d_i are zero). Since all joints are rotational, when they are at zero degrees, all \hat{X} axes must align.

With these comments in mind it is easy to find the frame assignments shown in Fig. 3.7. The corresponding link parameters are shown in Fig. 3.8.

Note that since the joint axes are all parallel and all the \hat{Z} axes are taken to point out of the paper, all α_i are zero. This is obviously a very simple mechanism. Note that our kinematic analysis always ends