ME 115(a): Notes on the Elliptical Trammel

The Elliptical Trammel (also known as the *Trammel of Archimedes*) is a simple mechanism which can trace an exact elliptical path. Figure 1 shows the geometry of this mechanism, which consists of two *prismatic* (or *sliding*) joints and two *revolute* (or *rotational*) joints. These joints guide the movement of a central rigid body.

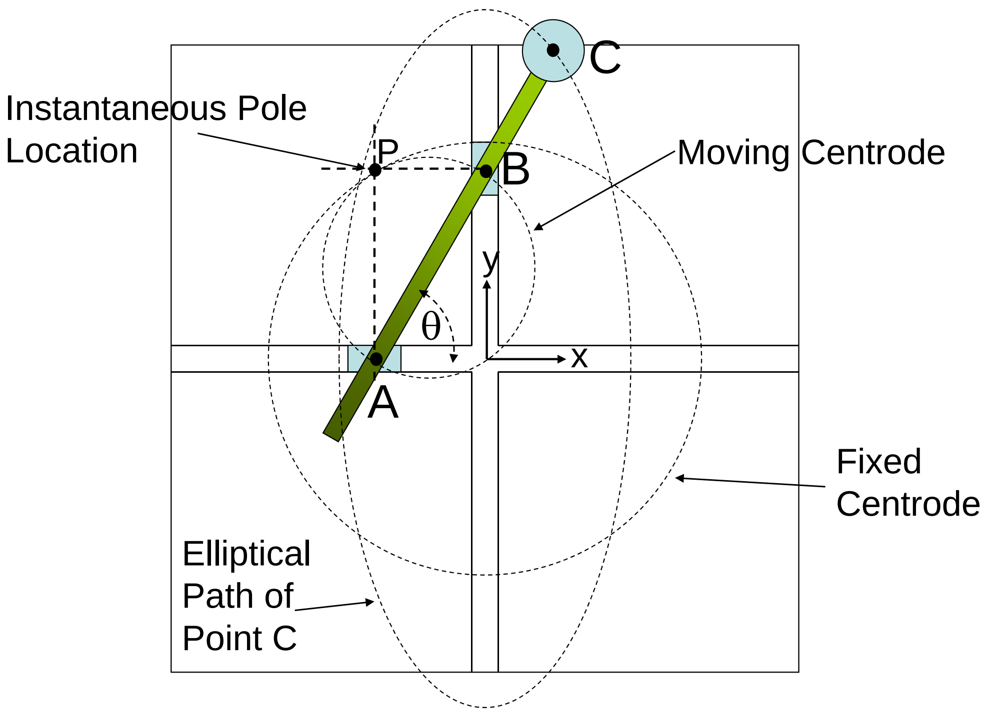

Figure 1: Diagram of the Elliptical Trammel, showing the geometry of the elliptical path, as well as the fixed and moving centrodes (dashed circles).

Let $\mathbf{A}$ and $\mathbf{B}$ denote the points on the moving rigid body that coincide with the revolute joint axes. Let $\mathbf{C}$ denote that point on the moving body that traces a path. Define the following distances:

$$a = |\mathbf{AC}| \qquad b = |\mathbf{BC}| \qquad c = |\mathbf{AB}| \ . \tag{1}$$

For convenience, assign a reference frame with origin at the intersection of the two prismatic joint axes, and with basis vectors colinear with the joint axes (see Figure 1). Let θ denote the angle between the line $\vec{\mathbf{AC}}$ and the x-axis. In this coordinate system, the x and y-coordinates of the point $\mathbf{C}$ are given by:

$$x = b\cos\theta; \qquad y = a\sin\theta \ . \tag{2}$$

Consequently

$$\left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2 = 1 \tag{3}$$

which is the equation for an ellipse with major and minor axes having dimensions a and b.

At each instant, the central body moves as if it was rotating about a pole (also known as instantaneous center of rotation, or ICR). At each location of the mechanism, each sliding block in the prismatic joints moves with a velocity that is equivalent to a rotation about any point on the line which passes through the center of the block and which is perpendicular to the axis of sliding. Both of these lines intersect at a unique point (denoted **P** in Figure 1). Hence, at each instant, the central body moves as if it were rotating about the ICR located at the center of these two lines.

Recall that the *fixed centrode* consists of the set of instantaneous pole locations (or ICRs) in the fixed reference frame, while the *moving centrode* consists of the set of pole locations as described in the moving frame of the central body. The mechanism moves as if the moving centrode rolls without slipping on the fixed centrode. The geometry of the fixed centrode can be found as follows. For each feasible orientation of the central body (denoted by θ), the x and y-coordinates of the ICR P is:

$$x_{\mathbf{P}} = -c\cos\theta \qquad y_{\mathbf{P}} = c\sin\theta \ . \tag{4}$$

Consequently, the moving centrode traces out the circle

$$x_{\mathbf{P}}^2 + y_{\mathbf{P}}^2 = c^2 \ . \tag{5}$$

You will show in Homework #2 that the moving centrode is also a circle centered about the midpoint of the line $\vec{\mathbf{AB}}$, and having radius $\frac{c}{2}$. The central body thus moves as if the moving centrode circle rolls around the inside of the fixed centrode circle.

In general the term *Cardan motion* (or *Cardanic motion*, named after Gerolamo Cardano, a 16^{th} century physician and mathematician) refers to the motion generated when a circle of radius R rolls (without slipping) on the inside of another circle with radius $2R$. Points on the moving circle trace out elliptical paths. Thus, the Elliptic Trammel implements Cardanic motion.