ME 115(b): Homework #3

(*Revised Version of 4/30/2012*)

(Due Friday, May 4, 2012)

Problem 1: (50 points) Using the methodology discussed in class, design a 4-bar mechanism for 4-point rigid body guidance. The body passes through the following four positions, where the x, y values are the coordinates of the origin of the moving body frame and the angles denote the rotation of the body-fixed frame relative to a fixed reference frame.

1. **position 1:** $\{x, y, \theta\} = (0.0, 0.0, 0^o)$
2. **position 2:** $\{x, y, \theta\} = (1.8, 2.9, 36^o)$
3. **position 3:** $\{x, y, \theta\} = (3.7, 3.3, 48^o)$
4. **position 4:** $\{x, y, \theta\} = (6.0, 3.0, 60^o)$

You need design only one of the two dyads. Designing both dyads will earn a small amount of extra credit (5 points). This problem may be best solved using Mathematica or an equivalent symbolic/numeric package which can handle complex numbers.

This problem can be broken down into the following parts:

1. Using the given information, compute the problem parameters δ_2, δ_3, δ_4, α_2, α_3, α_4.
2. Construct the compatibility equation from the problem parameters:
$$e^{i\beta_2}\Delta_2 + e^{i\beta_3}\Delta_3 + e^{i\beta_4}\Delta_4 + \Delta_1 = 0$$
where:
$$\Delta_2 = \det\begin{bmatrix}(e^{i\alpha_3}-1) & \delta_3 \\ (e^{i\alpha_4}-1) & \delta_4\end{bmatrix} \qquad \Delta_3 = -\det\begin{bmatrix}(e^{i\alpha_2}-1) & \delta_2 \\ (e^{i\alpha_4}-1) & \delta_4\end{bmatrix}$$
$$\Delta_4 = \det\begin{bmatrix}(e^{i\alpha_2}-1) & \delta_2 \\ (e^{i\alpha_3}-1) & \delta_3\end{bmatrix} \qquad \Delta_1 = -(\Delta_2 + \Delta_3 + \Delta_4)\ .$$
3. Choose a value of β_2, and then solve the compatibility equation for the given β_2. You can use either an algebraic approach, or the geometric approach studied in class. To make your life easier, pick $\beta_2 = 30^o$ if you wish.
4. Solve the standard dyad equations for w, z. That is, the compatiblity equation was derived from the set of equations
$$\begin{bmatrix}(e^{i\beta_2}-1) & (e^{i\alpha_2}-1) \\ (e^{i\beta_3}-1) & (e^{i\alpha_3}-1) \\ (e^{i\beta_4}-1) & (e^{i\alpha_4}-1)\end{bmatrix}\begin{bmatrix}w \\ z\end{bmatrix} = \begin{bmatrix}\delta_2 \\ \delta_3 \\ \delta_4\end{bmatrix}\ . \tag{1}$$
With the compatibility equation solved, you can choose any two of the three equations in (1) to solve for w and z.

5. Plot the dyad in the four positions to verify your solution.

If you solve this program using Mathematica (or equivalent), please turn in your program listing.

Extra Credit: (20 points) For the situation described in Problem #1, plot the circle point and center point curves for this problem. Since there are two solutions to the compatibility equations there will be 2 circle point and center point curves. You need only plot one curve.