ME 115(b): Problem Set #6
(Due May 26, 2016)

Problem #1 (18 points): This problem has you consider the contact equations for the case of one planar ellipse rolling/sliding on another planar objects. Recall that the boundary of the ellipse can be parametrized as:

$$\begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \begin{bmatrix} a\cos(u) \\ b\sin(u) \end{bmatrix}$$

Note that when $a = b$, the ellipse is a circle.

Part (a) (15 point): Derive the contact equations for the condition shown in Figure 1, involving one ellipse roll/sliding over another ellipse. Are there any conditions under which the relative curvature become ill defined?

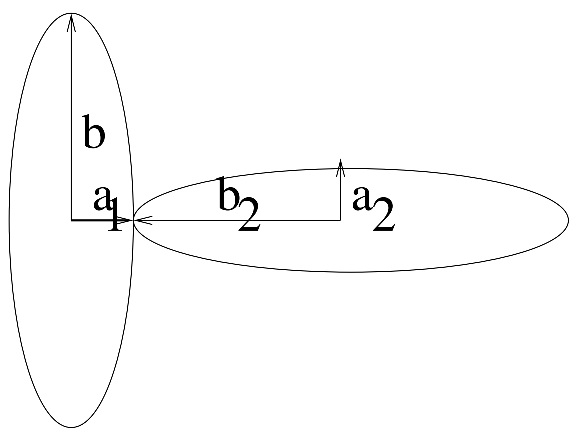

Figure 1: Rolling Ellipses

Part(b) (3 points) Derive the contact equations when the ellipse is roll-sliding on a flat plane.

Problem #2 (25 Points): Recall the involute gear tooth profile which was studied in homework #5. A parametrized formula for the involute curve of a circle is:

$$x(t) = R(\cos(t) + t\sin(t)) \tag{1}$$
$$x(t) = R(\sin(t) - t\cos(t)) \tag{2}$$

where R is the radius of the circle which defines the involute. Note that this curve is the profile of most gear teeth.

Part(a) (10 points): Derive the contact equations for the evolute of a circle (the gear tooth profile).

Part (b) (15 points): For simplicty, assume that the two mating gears have exactly the same diameter base and pitch circles, so that the gear ratio is one. Assume that the two gears are properly meshed so that the gear teech profiles are always in contact over their respective length of interaction. Are there any points of contact between the two teeth where there is NO slipping between the two teeth?

Part (c): (extra credit, 5 points) What are the signs of the slipping velocities on either side of this point.

Problem #2 (15 Points): Let's consider the kinematic equations of motion governing the planar two-fingered grasp of Problem 10(b) in Chapter 5 (page 261) of the MLS text. The key issue explored by this problem is the fact that the fingertips are now round disks, instead of point fingers. Show the structure of the hand Jacobian, grasp map, and contact equations for this example. note, you need not derive the rolling equations for a circle on a plane (which was done in Problem 1).