ME 115(b): Solution to Problem Set #3

Problem #1: Problem 1(a,b) in MLS Chapter 4.

Part (a): The lines of force underlying the three frictionless contacts all intersect at a common point. If one places a reference frame origin at that point, with x-axis parallel to the base of the triangle, then the grasp map is:

$$G = \begin{bmatrix} 0 & \cos(30^o) & -\cos(30^0) \\ 1 & -\sin(30^0) & -\sin(30^0) \\ 0 & 0 & 0 \end{bmatrix} .$$

The grasp is **not** force closure since the three contact forces cannot counterbalance a torque about the normal to the plane.

Part (b): Position a reference frame in the same location as Part (a). In this frame, the grasp map is:

$$G = \begin{bmatrix} 0 & \cos(30^o) & -\cos(30^0) & -\cos(30^o) \\ 1 & -\sin(30^0) & -\sin(30^0) & -\sin(30^0) \\ 0 & 0 & \tau & -\tau \end{bmatrix} .$$

where the magnitude of τ depends upon the separation of the two contacts on the right side of the triangle. This gras is force closure.

Problem #2: Problem 2(a,c) in MLS text.

Part (a): The grasp map is a 3×6 matrix. Assuming the same reference frame as used in the solution to Part(a) in the previous problem,

$$G = \begin{bmatrix} 0 & 1 & \cos(30^o) & -\sin(30^0) & -\cos(30^0) & -\sin(30^o) \\ 1 & 0 & -\sin(30^0) & -\cos(30^0) & -\sin(30^0) & \cos(30^0) \\ 0 & \tau & 0 & \tau & 0 & \tau \end{bmatrix} .$$

where τ will depend upon the size of the triangle. The grasp is force closure. One way to show this is to realize that there is an internal force, and the grasp map is full rank.

Part (c): The grasp map is a 6×9 matrix. In a reference frame whose origin lies at the center of the sphere,

$$G = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & -R & 0 & -R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & 0 \\ R & 0 & 0 & R & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

The grasp is force closure

Problem #3: Problem 3(b) in MLS text.

The grasp map for this problem was done in class. The grasp is force closure.

Problem #4: Problem 4(a) in MLS text.

Depending upon your assumptions, there may be more than one answer to this problem. A reasonable answer follows. Choose a contact frame whose z-axis is normal to the planar contact surface, and whose x-axis is parallel to the contact edge. Clearly, normal forces and tangential frictional forces can be supported by this contact. It seems reasonable to assume that no torque can be supported about a line collinear with the contact edge. It also seems reasonable to assume that torque can be supported about the contact normal. The only point of debate centers on the possibility to support a torque about the y-axis of the contact frame. We will assume that it can be supported, but opposing arguments can also be made. In this frame, the wrench basis is thus:

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The friction cone is defined by

$$\begin{aligned} f_n &\geq 0 \\ \sqrt{f_x^2 + f_y^2} &\leq \mu f_n \\ |\tau_n| &\leq \gamma_n f_n \\ |\tau_y| &\leq \gamma_y f_n \end{aligned}$$

where f_n is the normal force, f_x and f_y are the tangential frictional forces, τ_n is the torque about the contact normal, and τ_y is the torque about the y-axis.