ME 115(b): Solution to Problem Set #4

Problem 1 (15 points): Problem 5(a,b) in Chapt. 5 of MLS.

Part (a): An equilateral triangle cannot be grasped in force closure under the condition of two point contact with friction, and a friction cone angle of 30^o. As long as the two fingers are not placed at the same point along the triangle boundary, the Grasp Map is full rank. However, there is no position of the two fingers around the boundary where there exists a line intersecting both contact such that the line lies in the *interior* of the two friction cones. Hence, force closure is not possible

Part (a): Similarly, in 3-dimensional Euclidean space, a tetrahedron whose faces are equilateral triangles cannot be grasped in force closure under the stated conditions of three point contacts with friction, and a friction cone angle of 15^o. As long as the the fingers are not placed along a common line on the surface of the tetrahedron, the Grasp Map is full rank.

The placement of the three fingers naturally defines a plane which passes through all of the point contacts. If the plane is parallel to one of the faces, then we note that the points lying inside of the three friction cones lie on one side of the plane. Hence, there is a supporting hyper-plane for the possible net forces, and so the grasp cannot be force closure. Otherwise, the plane will lie at some skew angle relative to each of the faces. For planes which are not far from being parallel to a face, the same situation holds: there is a separating hyperplane. However, for more skew orientations of that plane, one of the friction cones will lie entirely on one side of the plane, while the other two will either straddle the plane, or lie on the opposite side of the plane.In this situation, one can show that there does not exist a force in the interior of all the friction cones.

Problem 2 (15 points): Problem 7 in Chapt. 5 of MLS.

First let's prove the **if** part. That is, if the grasp is force closure, then there exists a line connecting the two contact points lying inside both friction cones.

- If the grasp is force closure, then the Grasp map must be full rank.
- For two planar point contacts with friction, if the Grasp Map is full rank, then the two contacts are distinct.
- If the grasp is force closure, then there must exist a squeeze force in the interior of the friction cone.
- For planar contact involving two distinct points, a squeeze force consists of equal magnitude contact forces applied along a common line line passing through the two contacts, but pointing in opposite directions along the line.
- Hence, if the grasp is force closure, both of contact forces involved in the squeeze forces must lie in the interior of their respective friction cones.

Hence, there must be a line connecting both contact points which lies in the interior of the two point contact friction cones.

Now let's prove the **only if** part. That is, if there exists a line connecting two contacts on the boundary of a planar object, and the line lies in the interior of the friction cones, then the grasp must be force closure.

- if such a line exists, then the two point contacts are not coincident. Hence, the grasp map is full rank.
- if such a line exists, then there is a squeeze force for the grasps that consists of equal magnitude forces applied along opposite directions of the line.
- Since the line lies in the interior of the friction cones, each of the contact forces lies in the interior of their respective friction cones. Hence, there exists a squeeze force in the interior of the friction cones.

Any grasp with a full rank grasp map an an squeeze force in the interior of the friction cones is a force closure grasp.

Problem 3 (15 points) Problem 8(a) in Chapt. 5 of MLS.

The symmetrical trapezoid has a boundary of length $2 + 1 + 1 + 1 = 5$. Hence, the contact configuration space can be represented by a square whose sides are length 5. In Figure **??** the dashed lines bound the cells whose interiors represent finger locations on pairs of edges. For the answer shown in Fig. **??**, distance along the boundary is measured counter-clockwise starting from the upper right vertex. For every planar two-point contact, the force-closure regions are symmetric about the diagonal line.

Problem 4 Problem 9 in Chapt. 5 of MLS. The contact space of a circle is quite simple. It consists of a diagonal band, whose width depends upon the friction coeffficient. The maximally independent contact region occurs when the two contacts are antipodal on the circle.

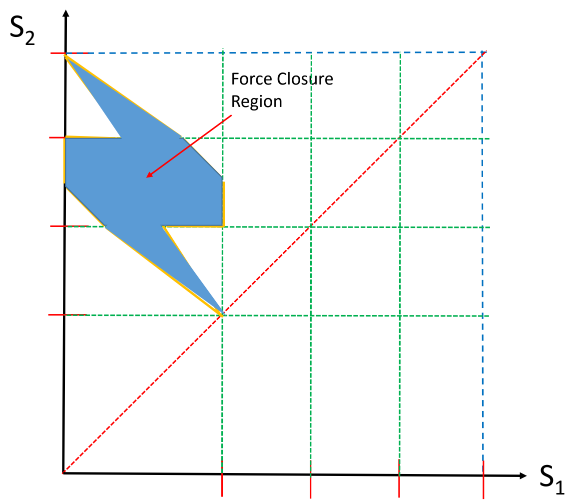

Figure 1: Contact Configuration space for two point contact (with friction) grasp of a planar trapezoid. The force closure regions are symmetric about the diagonal (i.e., one hafl of the force closure regions is not showing).

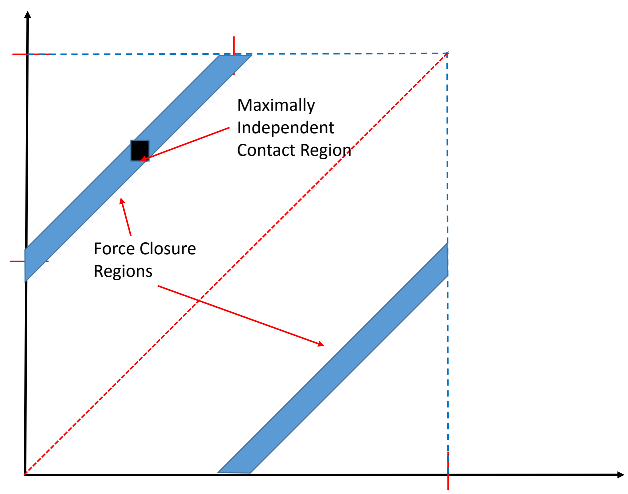

Figure 2: Contact Configuration space for two point contact (with friction) grasp of a planar circular object.