

Modeling and Design of a MEMS Piezoelectric Vibration Energy Harvester

by

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Abstract

The modeling and design of MEMS-scale piezoelectric-based vibration energy harvesters (MPVEH) are presented. The work is motivated by the need for pervasive and limitless power for wireless sensor nodes that have application in structural health monitoring, homeland security, and infrastructure monitoring. A review of prior milli- to micro-scale harvesters is provided. Common ambient low-level vibration sources are characterized experimentally. Coupled with a dissipative system model and a mechanical damping investigation, a new scale-dependent operating frequency selection scheme is presented. Coupled electromechanical structural models are developed, based on the linear piezoelectric constitutive description, to predict uni-morph and bi-morph cantilever beam harvester performance. Piezoelectric coupling non-intuitively cancels from the power prediction under power-optimal operating conditions, although the voltage and current are still dependent on this property. Piezoelectric material selection and mode of operation ($\{3-1\}$ vs. $\{3-3\}$) therefore have little effect on the maximum power extracted. The model is verified for resonance and off-resonance operation by comparison to new experimental results for a macro-scale harvester. Excellent correlation is obtained away from resonances in the small-strain linear piezoelectric regime. The model consistently underpredicts the response at resonances due to the known non-linear piezoelectric constitutive response (higher strain regime). Applying the model, an optimized single prototype bi-morph MPVEH is designed concurrently with a microfabrication scheme. A low-level (2.5 m/s^2), low-frequency (150 Hz) vibration source is targeted for anti-resonance operation, and a power density of $313 \text{ }\mu\text{W/cm}^3$ and peak-to-peak voltage of 0.38 V are predicted per harvester. Methodologies for the scalar analysis and optimization of uni-morph and bi-morph harvesters are developed, as well as a scheme for chip-level assembly of harvester clusters to meet different node power requirements.

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List of Symbols

Symbol	Description	Units
$\mathbf{B}_f$	modal forcing matrix with elements $B_{f,ij}$	$[kg]$
B_f	scalar modal forcing coefficient	$[kg]$
A	area	$[m^2]$
A_{ij}	modal matrix equation constant	-
a	interdigitated electrode width	$[m]$
b	width of structure	$[m]$
$\mathbf{C}$	damping matrix with elements C_{ij}	$[N \cdot s/m]$
C	scalar damping coefficient	$[N \cdot s/m]$
$\mathbf{c}$	piezoelectric material elastic stiffness matrix with elements c_{ij}	$[Pa]$
c	scalar elastic stiffness	$[Pa]$
$\mathbf{C}_p$	capacitive coefficient matrix with elements $C_{p,ij}$	$[F]$
C_p	scalar capacitive coefficient;	$[F]$
	measured capacitance	$[F]$
$\mathbf{D}$	electric displacement vector with elements D_i	$[C/m^2]$
$\mathbf{d}$	piezoelectric constant matrix with elements d_{ij}	$[m/V]$
$\mathbf{E}$	electric field vector with elements E_i	$[V/m]$
$\mathbf{e}$	piezoelectric constant matrix with elements e_{ij}	$[C/m^2]$
e_i	electrode numbering	-
f	frequency	$[Hz]$
$\mathbf{f}$	discretely applied external force vector with components f_i	$[N]$
i	current	$[A]$
I	second moment of area of structure	$[m^4]$
J_{yy}	proof mass moment of inertia about the center of gravity	$[kg \cdot m^2]$
J_0	proof mass moment of inertia about loading point	$[kg \cdot m^2]$
$\mathbf{K}$	modal stiffness matrix with elements K_{ij}	$[N/m]$
K	stiffness of structure	$[N/m]$
k_{33}	electromechanical material coupling	-
k_{sys}	electromechanical system coupling	-
k_e	electromechanical material coupling	-
L	length	$[m]$
$\mathbf{M}$	modal mass matrix with elements M_{ij}	$[kg]$

M	mass of structure	$[kg]$
m	mass per length;	$[kg/m]$
	mass per area	$[kg/m^2]$
N_{cyc}	number of cycles	-
nf	number of discrete external forces applied	-
nq	number of electrode pairs (electrical modes)	-
nr	number of bending modes (mechanical modes)	-
O	location of proof mass loading on cantilevered structure	-
o_x	horizontal distance from O to proof mass center of gravity	$[m]$
o_z	vertical distance from O to proof mass center of gravity	$[m]$
$\mathbf{P}$	piezoelectric poling vector	$[C/m^2]$
P_{out}	electrical power generated or extracted	$[W]$
p	interdigitated electrode pitch	$[m]$
$\mathbf{q}$	charge vector with elements q_i	$[C]$
q	charge	$[C]$
Q	quality factor	-
R	electrical resistance	$[\Omega]$
$\mathbf{r}$	generalized mechanical coordinate vector with elements r_i	$[m]$
$\mathbf{S}$	strain vector (contracted notation) with elements S_i	$[m/m]$
S_0	moment of proof mass about O	$[N \cdot m]$
$\mathbf{T}$	stress vector (contraction notation) with elements T_i	$[N/m^2]$
t	thickness;	$[m]$
	time	$[s]$
T_d	damped cycle period	$[s]$
T_k	kinetic energy	$[J]$
U	potential energy	$[J]$
$\mathbf{u}$	mechanical relative displacement vector with elements u_i	$[m]$
$\mathbf{v}$	voltage vector of elements v_i	$[V]$
v	voltage across electrode pair or electrical load	$[V]$
V	volume	$[m^3]$
W	external work	$[J]$
W_e	electrical energy or work	$[J]$
w	absolute displacement	$[m]$
w_B	absolute base displacement	$[m]$
x_1, x_2, x_3	cartesian coordinate directions	-
x_a	general beam structure axial coordinate	-
x_t	general beam structure thickness coordinate	-
z	relative displacement	$[m]$

α	dimensionless time constant	-
δ	first variation of a parameter (Calculus of Variation); logarithmic decrement	-
δ_v	alternative (velocity) logarithmic decrement	-
η	structural loss factor	-
ε	permittivity matrix with elements ε_{ij}	$[F/m]$
χ	transfer function phase angle	$[rad]$
κ	electromechanical structure/system coupling	-
λ_N	convenient modal analysis constant	-
μ	viscosity of air	$[N \cdot s/m^2]$
∇	gradient of variable	$[m^{-1}]$
ν	Poisson's ratio	-
Ω	frequency normalized to resonance (short-circuit) frequency	-
ω	driving or operating frequency	$[rad/s]$
ω_d	damped natural frequency	$[rad/s]$
ω_N	natural frequency	$[rad/s]$
∂	partial derivative of variable	-
φ	scalar electrical potential	-
ψ_r	mechanical mode shape vector of elements $\psi_{r,i}$	-
ψ_v	electrical mode shape vector of elements $\psi_{v,j}$	-
ρ	density	$[kg/m^3]$
Θ	coupling coefficient matrix with elements θ_{ij}	-
θ	scalar coupling coefficient	-
ζ	mechanical damping ratio	-
Subscripts		
0	proof mass property or variable	-
1	first bending mode index (subscript $N = 1$)	-
1, 2	piezoelectric element numbers	-
a, b	half-power frequency indices	-
ar	variable evaluated at the anti-resonance frequency	-
B	variable at the base of the structure/beam	-
c	parameter associated with a cluster of devices	-
e	electrical domain parameter;	-
	effective parameter;	-
	parameter associated with the electrode	-
l	electrical load;	-
m	mechanical domain parameter	-
N	mode number during modal analysis	-
opt	power-optimized variable	-
p	piezoelectric material or element property	-

pl	plate stiffness	-
pt	platinum material/layer property	-
r	variable evaluated at the resonance frequency	-
s	structural layer or section property	-
t	variable at the tip of the structure/beam	-
T	total	-
ti	titanium material/layer property	-
Superscripts		
t	transpose of matrix or vector	-
E	variable at constant electric field	-
D	variable at constant electric displacement	-
T	variable at constant stress	-
S	variable at constant strain	-
$< \dot{a} >$	time derivative of illustrative variable a	-
$< a' >$	spatial derivative of illustrative variable a	-
$< \bar{a} >$	normalized variable of illustrative variable a	-
$< a^* >$	reduced piezoelectric material properties of illustrative variable a ;	-
	local coordinate system	-

Chapter 1

Introduction

In recent years, the development of distributed wireless sensor networks has been a major focus of many research groups. Research projects include SmartDust at UC Berkeley [1], μ -AMPS at MIT [2], and i-Bean wireless transmitters from Millennial Net, Inc. [3]. Distributed wireless micro-sensor networks have been described as a system of ubiquitous, low-cost, self-organizing agents (or nodes) that work in a collaborative manner to solve complex problems [4]. A node has been defined as "... a single physical device consisting of a sensor, a transceiver, and supporting electronics, and which is connected to a larger wireless network" [5]. Applications envisioned for these node-networks include building climate control, warehouse inventory and supply chain control, identification and personalization (RFID tags), and the smart home [6]. Other applications include structural health monitoring (aerospace and automotive), agricultural automation, and homeland security applications [7]. A major concern for these node networks remain the power supply to each node [8].

Advances in low power DSP's (Digital Signal Processors) and trends in VLSI (Very Large Scale Integration) system-design have reduced power requirements for the individual nodes [9]. Power consumption of tens to hundreds of μW is predicted [6, 10, 11, 12] and a current milli-scale commercial node has an average power consumption of $6 - 300 \mu W$, depending on the application and/or mode of operation [13]. This lowered power requirement has made self-powered sensor nodes a possibility. Power solutions envisioned for these self-powered nodes will convert ambient energy into

usable electric energy, resulting in self-sustaining nodes.

Many ambient power sources (*e.g.*, thermal gradients, vibration, fluid flow, solar, *etc.*) have been investigated and it is clear that ambient energy harvesters are well suited for long-term implementation of sensor nodes networks. In this and the following chapter, it will be shown that harvesting mechanical vibrations is a viable source of power, well matched to the needs of wireless sensor nodes. The conversion of ambient mechanical vibrations to electrical energy will be the focus of the current research.

1.1 Wireless Sensor Nodes

In order to evaluate or develop a power solution for a wireless sensor node, it is imperative to understand the sub-components and power drains of the node. The general architecture and applications of the nodes will be considered first. Next, the power drains of each component will be discussed, and certain design considerations and restrictions will be noted.

1.1.1 Architecture and Power Drains

In order to assess the power consumption of nodes, it is important to understand the architecture of wireless sensor nodes. A typical node description is adapted from [14] and is illustrated in Figure 1-1. The node can be divided into four subsystems:

- Computing/processing unit (logic)
- Communication unit
- Sensing unit (with Analog-Digital-converter)
- Power supply (with voltage up-converter, if necessary)

The specific application of a wireless sensor node network affects the power consumed by the individual nodes and will have an effect on the power solution(s) chosen

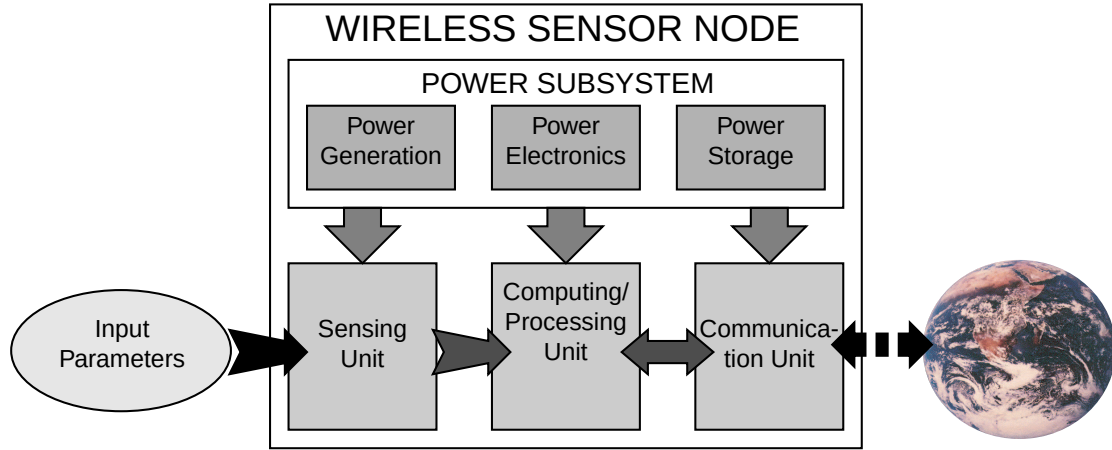


Figure 1-1: Wireless sensor node architecture.

for the nodes in the network. For example, high data transfer rates necessitates larger power sources. Each of the node subsystems will be discussed briefly, with the emphasis on power consumption.

The computing unit includes the memory and a microprocessor or micro-controller (MCU). The power consumption of the MCU will vary greatly, depending on the processor used and its operational mode (active, sleep, or idle mode). The processor will in turn depend on the application. For example, two commonly used processors are the Intel StrongARM and Atmel's AVR [15]. The StrongARM has an active power consumption of 400 mW , whereas the AVR consumes only 16.5 mW in active mode, but has much less processing capability. The consumptions of the Intel processor in its other operational modes are: 50 mW in idle mode and $160\text{ }\mu\text{W}$ in sleep mode. This puts in context power consumptions of $10 - 100\text{ }\mu\text{W}$ that have been predicted [6, 10, 11, 12].

For the communication unit, power consumption will be influenced by the modulation type, data transmission rate, transmission power, and operational modes. These operational modes include transmit, receive, idle and sleep. Switching between the modes also consumes power.

The sensing unit power consumption is difficult to assess since there are numerous sensors available and compatible with these nodes. In addition, some nodes include

both digital and analog inputs and support multiple sensors [13]. Common power drains include: signal sampling, signal conditioning, and analog-to-digital conversion (if necessary). According to Raghunathan *et al.*, the power consumed by passive sensors (*e.g.*, accelerometers, thermometers, pressure sensors, strain sensors, etc.) is negligible when compared to the other subsystems [14]. This is certainly the case in the example that follows.

The power source has to energize the entire sensor node. To ensure a constant voltage supply, a DC-DC voltage up-converter may be necessary. In the case of a battery power source, the voltage decreases as the rate of the chemical reaction decreases. For energy harvesting technologies, the source can be discontinuous or at levels below the peak power requirement of the node. In this case, a storage device (battery) will be necessary to satisfy the temporary high power demand and the average power generated should be greater than the average power consumed by the node. The power consumed by Rockwell's WINS nodes (maximum power consumption $\sim 1\text{ W}$, but average power consumed will be strongly application dependent) is summarized below, adapted from [15]:

1. Processor - 30 – 50% of total consumed power.
 - Active = 360 mW
 - Sleep = 41 mW
 - Off = 900 μW
2. Sensor = 23 mW
3. Transceiver = 50 – 70% of total consumed power.
 - Transmit vs. processor (active) power = (1 to 2) : 1 (range dependent)
 - Receive vs. processor (active) power = 1 : 1
 - Transmit vs. receive power = 4 : 3 at maximum range, 1 : 1 at shorter range

As the example illustrates, advances in reduced-power processing and transceiving will drive the realization of effective, self-powered sensor nodes.

1.1.2 Applications and Node Design Considerations

Node-networks have distinct advantages when compared to macro (traditional) sensor networks. On the one hand, traditional sensors are very accurate, but they tend to be bulky and expensive. Due to the high cost, macro sensors are normally implemented in isolation with the effect that the system is not fault tolerant. If one device fails, the system performance is greatly influenced. On the other hand, wireless sensor nodes are relatively low cost. As a consequence, the node's sensors are inherently less accurate, but a region can be flooded with multiple nodes. The individual node-measurements can be conditioned with advanced DSP's (Digital Signal Processors) to obtain accurate measurements. Furthermore, a network of nodes is inherently fault tolerant as the contribution of each individual node to the total system will be small and a few faulty sensors will not adversely affect the overall system performance [4]. Lastly, a self-powered sensor network has become plausible, as the power requirements of the nodes have decreased.

Nodes powered from scavenging ambient energy can be deployed to inaccessible and/or environmentally sensitive regions, as there will be no battery that has to be maintained or replaced. The excellent fatigue characteristics of MEMS devices make the long-term deployment of these networks viable, if a sustaining power source can be developed. Automation and the ease with which these networks can be deployed or extended also make these networks very attractive.

Considerations for the design and power source analysis of a wireless sensor are discussed along four dimensions: node network lifetime, size of node, cost of node, and node placement and the wireless requirement. In certain applications, such as building structural monitoring or environmental control, the node-network will have very long lifetimes. These lifetimes can typically vary from one to 30 years (the typical life of infrastructure) [4]. It is desirable to have a node lifetime spanning the application lifetime. Secondly, the size of the complete node needs to be small enough to ensure

unobtrusive distribution of the node-network. Potentially, thousands of nodes will be distributed. “Small enough” is application dependent, but most applications desire $\mu m - mm$ sized devices. Military applications will benefit from a small form factor for these nodes. The i-Bean wireless node (excluding sensor) of dimensions $25 \times 15 \times 5 \text{ mm}^3$ (volume = 1875 mm^3), which is powered with a Panasonic *CR2032* Lithium battery of dimensions; diameter 20 mm , thickness 3.2 mm (volume = 1005 mm^3) has been reported [13]. For this example, the battery constitutes 35% of the total node volume and weighs 3.1 grams .

In terms of cost, the i-Bean wireless sensor node (from Millennial Net) is available for approximately \$25, with the goal of a sub-\$10 device [4, 7]. If the cost of an individual node is too high, it will not be economically feasible to deploy large networks of these nodes. For the i-Bean nodes, the current power source is a Panasonic *CR2032* Lithium battery. The battery cost is around \$2.99 – \$4.99 [16], so that the battery cost makes up 12 – 20% of the total node price. If the power source can be incorporated into the node, the total cost will decrease. Furthermore, assembly may be eliminated, as there will be no need to install a battery.

Some applications may inherently limit access to the node once it has been deployed. Limited, or no, node access negates the use of power sources with fixed energy densities since operational support (repair and maintenance) for these nodes is impossible. Thus, an infinite power source is desirable. Furthermore, due to the placement and cost constraints on the nodes, it is oftentimes not viable to have physical (wired) connections with the nodes, either for communication or power supply. Wiring cost constitutes up to 90% of the total sensor cost in building environment control [6].

1.2 Competing Power Solutions

The power source selected for a node will be determined by the specific application. As discussed in the last section, general considerations when selecting a power source for a node include: node network lifetime, cost and size of nodes, node placement and resulting ambient energy availability, and communication requirements. Power or

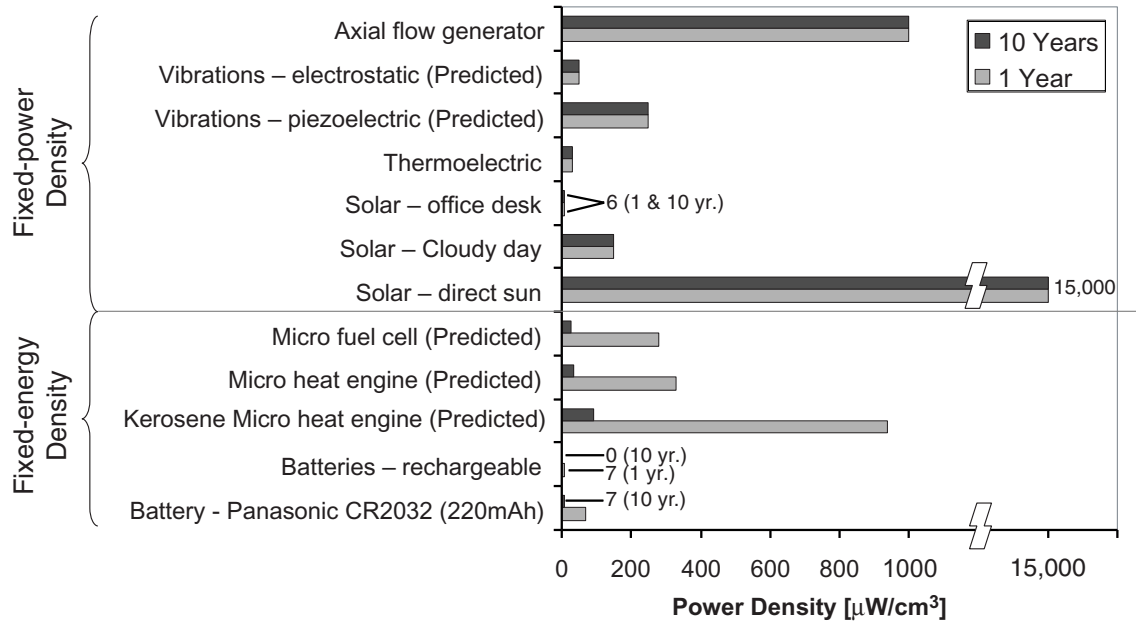


Figure 1-2: Comparison of power sources for wireless sensor nodes. Adapted from [1,5,17,18,19,20,21].

energy sources for nodes can be divided into two groups: sources with a fixed energy density (*e.g.*, batteries) and sources with a fixed power density (such as ambient energy harvesters). These source types for sensor applications are compared in Figure 1-2. Fixed energy density sources have limited life - the source either needs to be replaced or the fuel replenished. Many applications are envisioned where maintenance and repair is not desired or even possible (*e.g.*, an embedded sensor for structural health monitoring). Thus, it is desirable to utilize a power source with a life that matches that of the application. Fixed power density sources, such as ambient energy harvesters, are better suited for long-term implementation than fixed energy density sources.

1.2.1 Fixed Energy Density Power Sources

Fixed energy density sources have a limited amount of energy available, such as batteries or engines. After the energy available has been exhausted, the source needs to be replaced or fuel replenished. As a consequence, these sources have a fuel-imposed

limited life. Five fixed energy density sources will be discussed here: non-rechargeable batteries, the micro gas turbine engine, micro fuel cells, nuclear powered harvesters, and hybrid power sources.

Non-rechargeable battery technologies (such as Lithium batteries) can have high energy densities ($> 120 \text{ Wh/kg}$ [17, 18]) and their lifetimes have been extended, but even these extended lifetimes are not sufficient for the long-term deployment of wireless sensor node networks. The ten year shelf life of Lithium battery technologies has been questioned [19]. “Battery technology...could be the most limiting factor in the design (of mobile devices) with respect to size, weight and cost”, and “Owing to the physics involved, batteries will fall behind other mobile technology trends” [20]. Further reasons why batteries are unattractive for wireless node applications include: Batteries contain chemicals that can be hazardous and are normally large compared to a micro-scale device. State-of-the-art batteries are also considered expensive, as discussed earlier (Panasonic *CR2032* Lithium battery: \$2.99 – \$4.99, depending on the connection type [16]). Due to sensor placement, battery maintenance can be limited or impossible, and costly. Some environments, such as high- or low-temperature applications, significantly degrade normal battery performance. Different battery supplies have been compared by others and it was concluded that a battery is viable only for short lifespan applications (~ 5 years at $10 \mu\text{W}$ of power) [5].

The micro gas turbine engine is a technology under development at MIT. A micro gas turbine engine is theoretically capable of generating $10 - 50 \text{ W}$ while consuming 7 grams of fuel per hour [21, 22]. This translates to very high power densities, of the order of $6 - 30 \text{ W/cm}^3$ for the device (excluding the fuel). However, the device will require $10,683 \text{ cm}^3$ of fuel (assuming Kerosene at room temperature as fuel) for each year of operation, which results in a total device volume of around $\sim 11,000 \text{ cm}^3$, excluding the fuel storage structure. Thus, the total energy density decreases to $900 - 4,700 \mu\text{W/cm}^3$. Alternatively, a fuel supply line will be necessary, which negates the use of the source for a wireless sensor node. Other drawbacks for powering wireless nodes with this technology are that once the process is initiated it

is hard to stop and the output voltages are too high [5].

The micro fuel cell is a technology currently under development at MIT, among others. The fuel cell converts chemical energy to electrical energy through an electrolytic reaction. These devices are generally based on either a solid oxide fuel cell (SOFC), or a proton-exchange membrane (PEM) electrolyte [23]. Due to the low ionic conductivity for SOFC electrolytes, high temperature operation ~ 1000 °C is generally required. This temperature can be reduced by using thinner electrolytes, which is enabled through miniaturization. Stable operation for a miniaturized SOFC has been obtained for temperatures ranging from 480 – 570 °C, with a measured output of ~ 110 mW/cm^2 at 570 °C [24]. Each fuel cell has a reported thickness of ~ 20 μm (~ 300 μm including the silicon substrate). Multiple fuel cells can be stacked to form a power supply, resulting in a power density of around 10 W/cm^3 . However, the volume upon which this power density is based does not include the fuel required for operation (the fuel consumption is not reported). Other issues arise during miniaturization, including: fuel storage and supply, system start-up, peak power requirements, cell stacking, and thermal management [23].

A recent development in the area of replacement technologies for batteries in electronic devices is the nuclear micro-battery. Research teams at Cornell University and the University of Wisconsin - Madison have been investigating the conversion of radiation energy (from a radio-active isotope, such as Polonium-210, Nickel-63, or Tritium) to electricity [25]. Radioisotopes have a much higher energy density ($\sim 57,000$ $mW \cdot hours/milligram$ for Polonium-210 at 8% efficiency) than conventional energy sources (~ 0.3 $mW \cdot hours/milligram$ for Lithium-ion batteries). This power source has a life equal to the half-life of the isotope (ranging from a few seconds to 100's of years for different materials) [26]. The small amount of radiation from the radioisotope can be contained with minimal packaging. The nuclear energy is first converted to mechanical (vibration) energy and then converted to electrical energy through the piezoelectric effect. A cantilevered structure (with a bottom electrode) is placed over the radioisotope source, separated by an air gap. Electrons fly from the source, cross the air gap, and hit the electrode. The impinging electrons

charge the electrode negatively, which is attracted by the positively charged source (electrostatic force). The cantilevered structure bends down until the electrode and source touch and discharges. The now neutrally charged cantilever is released (the electrostatic force is zeroed) and oscillates at its resonance frequency (free vibration). A piezoelectric element is incorporated into the cantilevered device, which converts the mechanical (strain) energy into electrical energy. Once the vibrations die out, the charge cycle repeats. A peak power of $2.5 \mu W$ ($1.5 V$ with a $1 M\Omega$ electrical load) is reported from a prototype device. The device has a footprint of $5 cm \times 5 mm$ and an efficiency of 7.2%, giving a power density (per footprint area) of $1.0 \mu W/cm^2$ [26].

The combination of rechargeable batteries with an alternative power source to supplement and charge the battery is a growing field of interest (alternative power sources are discussed in Section 1.2.2). This approach has the following drawbacks: The cost and spatial penalty for using batteries are not eliminated, only reduced. Furthermore, there is a lack of miniature secondary batteries [27], and rechargeable batteries have lower energy densities than their non-rechargeable counterparts. Lastly, the performance of rechargeable batteries typically deteriorate after a number of cycles. The first microscopic secondary battery, capable of powering a node for around two days, has been reported [27]. This battery has a footprint of $0.3 cm^2$ ($0.15 cm^2$ per cell) and is capable of 1000 charging cycles, which translates into 5.5 years of operation. This miniature rechargeable battery is compatible with MEMS fabrication processes. Further development in hybrid power supply technology includes the combination of solar cells and rechargeable batteries [4]. Also, an optimized circuit for the transfer of power from the piezoelectric harvesting element to an electrochemical battery has been investigated [28, 29].

1.2.2 Fixed Power Generation Sources

Ambient energy can be defined as energy that is not stored explicitly, but is available in the device surroundings. Various sources of ambient energy have been explored, and are discussed in the following section. These sources have the advantages that they are essentially free, their conversion mechanisms are clean (there is no pollution

associated with the conversion process), and the source has a potentially infinite lifespan. A brief summary of research to date is presented below. Source types and harvesting technologies include solar, thermoelectric, acoustic energy harvesting, the axial-flow micro-turbine generator, and mechanical vibration energy harvesting.

Solar energy harvesting is the most common mechanism of energy harvesting. Solar panels consist of photovoltaic cells and can generate up to $15,000 \mu W/cm^2$ in direct sunlight [5, 30]. However, their performance rapidly reduces to $150 \mu W/cm^2$ on a cloudy day and a mere $6 \mu W/cm^2$ at a desk in an office [6].

Thermoelectric energy harvesting devices generate electricity when placed in a temperature gradient. This is the same principle (the Seebeck effect) upon which a thermocouple works. Some published results include: $2.2 \mu W/cm^2$ is generated for $\Delta T = 5 K$ and $8.6 \mu W/cm^2$ for $\Delta T = 10 K$ [31]. Lim and Snyder *et al.* report a device that generates $40 \mu W/cm^2$ of power for $\Delta T = 1.25 K$ temperature gradient [32, 33]. Strasser *et al.* report a device capable of $1 \mu W/cm^2$ for a $\Delta T = 5 K$ [34].

Acoustic energy harvesters convert energy from acoustic waves to electrical energy. This approach has the major drawback that large areas ($\sim 10 cm^2$) and high-level (decibel) sound fields are required to make scavenging viable. The power densities for these devices are of the order of $1 \mu W/cm^2$ at $100 dB$ [5, 6].

The axial-flow micro-turbine generator is a development which is applicable where fluid motion is available in the ambient [35]. The rotor is fabricated from SU-8 with UV lithography and laser micro machining. The device rotates at $30,000 rpm$ and generates $1000 \mu W$ in a $q = 35 \ell/min$ airflow, with a differential pressure of $\Delta p = 8 mbar$. The device volume is reported to be $0.5 cm^3$, which translates into a power density of $2000 \mu W/cm^3$. The electrical output (voltage and current) is not reported.

The final mechanism of energy harvesting to be discussed, and the focus of the current research, is mechanical vibration energy harvesting. Low-level mechanical vibrations occur pervasively in the environment (refer to Section 2.4 and Appendix A) and high levels occur on machinery and vehicles (*e.g.*, an automobile or aircraft). These devices can be divided into two groups [36]: non-resonant and resonant energy

harvesters (*i.e.*, device resonance frequency is matched to vibration input frequency). These devices are most effective in different vibration regimes and are thus not competing, but rather complimentary configurations. The non-resonant energy harvester is more efficient where the input contains very low frequency ($< 10\text{ Hz}$), irregular vibrations with amplitudes larger than the device critical dimensions. Irregular vibrations are defined as inconsistent or discontinuous motions (such as the movements of a body). This configuration finds application in human movement energy harvesters (for example with wearable computing applications). Several teams have been pursuing this line of research [36, 37, 38]. On the other hand, the resonant energy harvester finds application where the input vibrations are regular, frequencies are higher ($> 100\text{ Hz}$), and the input vibration amplitude is smaller than the device critical dimensions. Regular vibrations are continuous with stable and well defined vibration spectra, such as vibrations generated by an unbalanced machine. Resonant energy harvesters are the focus of the current research.

There are three methods of conversion from mechanical vibration energy to electrical energy: a variable capacitor (electrostatic), an inductor (electromagnetic), and by utilizing the piezoelectric effect. These methods will be reviewed in greater detail in Chapter 2 based on previous work.

1.3 Overview of Thesis

The current research is aimed at realizing a MEMS-scale piezoelectric vibration energy harvester (MPVEH), eventually applied to powering a micro-scale sensor node. This specific project involves the development and validation/verification of coupled electromechanical models, and the investigation of a micro-fabrication process for the eventual development of a micro-scale device.

A cantilever beam configuration was chosen for its simplicity of analysis, compatibility with MEMS manufacturing processes, and low structural stiffness. A low resonant frequency is desired since the ambient vibration measurements (see Section 2.4 and Appendix A) indicate that the majority of ambient sources have significant

vibration components at frequencies below 300 Hz .

Application of the developed and validated/verified model will be the design of a MEMS-scale Piezoelectric Vibration Energy Harvester (MPVEH). This project will focus on the power generation sub-system design necessary to generate usable levels of power from ambient vibrations, rather than optimizing specific components of the sub-system. The project objectives include:

- Experimentally investigate ambient vibration sources and develop a general model to assist in the interpretation of vibration source data for harvesting.
- Develop electromechanically coupled models to predict the power generation for piezoelectric harvesters and analyze the effect of device geometry on performance.
- Validate the models by comparison to available literature data and analyses.
- Develop an experimental setup and test a macro-scale device to verify the developed models through direct comparison of the results.
- Apply the model as a design tool to obtain an optimal MPVEH prototype design targeting a low-level, low frequency application. Evaluate the performance of the single prototype device.
- Investigate a previous fabrication process for a high-level, high-frequency prototype [39] and apply the obtained experience to develop a viable microfabrication sequence for the MPVEH prototype.
- Investigate the chip-level integration of individual harvesters to form a harvester component of the power sub-system of the wireless sensor node.

The following aspects fall outside the scope of the current project:

- Scaling of the sub-system mechanical design (optimizing the mechanical design) to achieve the smallest form factor for a given natural frequency.

- Investigating the electronics associated with a harvester circuitry, for example when the harvester is connected to a capacitor or rechargeable battery, or a rectifier and control electronics. The current work will assume a purely resistive electrical load.
- Development of an efficient fabrication process. An existing fabrication process will be investigated [39].

Chapter 2

Vibration Energy Harvesting

Vibrational energy can be converted to electric energy utilizing one of three mechanisms: the electrostatic, electromagnetic, or piezoelectric effects. These mechanisms will be discussed briefly, before focussing on piezoelectric energy harvesting.

Some experimental and predicted results on vibration harvesting have been published, and are summarized in Table 2.1. From the published results it is clear that the power generated varies greatly, according to device size, mechanism of conversion, and input vibration parameters. The device sizes vary from the micro-scale ($\sim 0.01 \text{ cm}^3$) to the macro-scale ($\sim 75 \text{ cm}^3$). Ideally, a normalization scheme can be used to compare the performance and efficiency of the devices relatively. One method is to report the power density (W/cm^3 or W/kg). However, it was found that generally the volume is not very well documented in the literature. When the device volume is documented, it is typically not specified whether the complete power subsystem is included, or only the power generation unit. The same is true for the device mass. The ideal comparison is through an efficiency parameter. Input vibration amplitude and frequency must be documented as the power output depends on the power input.

Table 2.1: Previous work in vibration energy harvesting.

Mech [†]	Power [μW]	Volume [cm^3]	Freq. [Hz]	Ampl. [μm]	Acc. [m/s^2]	Sim. or Exp.	Ref.	Comment
ES	58	0.5	120	4	2.25	Sim	[30]	-
ES	5.6	-	2520	-	-	Sim	[9]	Note I
EM	200	75	28	6.462	0.2	Exp	[40]	-
EM	3600	75	28	32.31	1	Exp	[40]	-
EM	1	0.025	70	30.00	5.8	Sim	[41]	Note II
EM	100	0.025	330	30.00	12.0	Sim	[41]	Note II
EM	530	0.24	322	25.00	102	Exp	[42]	
EM	830	1	110	150.0	71.7	Exp	[43]	-
EM	1000	-	102	8	3.29	Exp	[44]	Note III
P	93	0.5	120	3.96	2.25	Sim	[5]	PVDF
P	130	0.5	120	3.96	2.25	Sim	[5]	PVDF
P	121	0.5	120	3.96	2.25	Sim	[5]	PZT
P	138.5	0.5	120	3.96	2.25	Sim	[5]	PZT
P	60	1	100	5.70	2.25	Sim	[5]	Note IV
P	2	0.9	80	-	-	Exp	[45]	-

[†] Mechanism of conversion used.

Note I: The power density is not reported [9], but is calculated by Roundy *et.al.* [30].

Note II: The volume reported is extremely small and likely does not include the transformer that will be necessary to convert the output voltage to usable levels.

Note III: No device size is given, but appears to be on a mm^3 scale (from figure).

Note IV: The dimensions of the prototype device are not clearly documented.

2.1 Previous Work

Small-scale vibrational energy harvesting work, utilizing the three common mechanisms of conversion, are reviewed in this section. The three mechanisms are the electrostatic, electromagnetic, and piezoelectric energy conversions. Each mechanism will be discussed next, focusing on applications developed at the small scale.

2.1.1 Electrostatic Vibration Energy Harvesting

The first mechanism of conversion is based on the variable capacitor concept. A variable capacitor consists of two conductors separated by a dielectric material. When the conductors are placed in an electric field and the conductors are moved relative to each other, current is generated. An overview of the properties for the different MEMS

electrostatic harvesting configurations is given by Meninger *et al.* [9]. The major drawback with this method is that a separate voltage source is required to create an electric field [5]. On the other hand, the capacitor configuration is easily integrable into micro-systems using standard MEMS manufacturing processes. Roundy *et al.* report considerable research on vibration energy harvesting, initially concentrating on electrostatic energy harvesters [5, 30]. Different configurations are compared and an optimal configuration is identified. The simulation results are given in Table 2.1 [30]. Meninger *et al.* investigated a low-power integrated circuit controller (*e.g.*, the controller electronics) for an electrostatic energy harvester [9]. With the proposed control scheme, a usable power output of $5.6 \mu W$ was predicted from the generated $8.6 \mu W$ total load power. The emphasis of the research was on the circuit design, and not harvester device optimization. Sterken *et al.* report the use of an electret (permanently polarized dielectric) for a constant charge supply, thus eliminating the need for a separate voltage supply [46]. However, the life of the electret is limited as the charge degrades over time.

2.1.2 Electromagnetic Vibration Energy Harvesting

When a coil within a magnetic field is moved, current flows according to Faraday's Law. This mechanism is utilized to convert motion (vibration) into electrical energy and has the following properties, [5]:

- No separate voltage source is required.
- A permanent magnetic field is required. Permanent magnets are normally bulky and scale poorly to a MEMS size [47].
- The output voltage is normally around $0.1 - 0.2 V$, so it is necessary to transform the voltage to usable levels for nodes.

Please refer to Table 2.1 for a summary of the results from previous research efforts. Lee *et al.* reported an electromagnetic device with a total size of $4 cm^3$ (including transformer circuitry) and a harvester size of around $1 cm^3$ [43]. It is important to

note the portion of space that the required circuitry (to transform the voltage to usable levels) and the support electronics occupy ($\sim 3 \text{ cm}^3$). A slight variation, but very interesting configuration, using frequency up-conversion, has been proposed by Kulah *et al.*, [48]. Low frequency vibration is magnetically up-converted to higher frequency oscillations using a large magnet as a seismic mass of a low-frequency oscillator. The low resonance frequency of the large mass-structure is matched to the vibration frequency. A set of smaller cantilever beams, with inductive coils for energy conversion, is placed near the magnetic field created by the seismic mass. As the large mass moves towards the array of smaller beams, the beams are “caught” in the magnetic field and deflect toward the seismic mass. When the seismic mass moves away, the small beams are “released” and vibrate at their elevated resonance frequencies. This motion can be used for energy harvesting. A power generation of $2.5 \mu W$ per cantilever beam is predicted from the low frequency (10 Hz) vibrations. The total device size is not reported.

2.1.3 Piezoelectric Vibration Energy Harvesting

Interest in the application of piezoelectric energy harvesters for converting mechanical energy into electrical energy has increased dramatically in recent years, though the idea is not new. An overview of research in this field is given by Sodano *et al.*, [49]. In early work, the predicted power output of a poly-vinylidene fluoride (PVDF) bi-morph energy harvester was so small that it was not a feasible power source at the time [50]. This result caused a lapse in the application of piezoelectricity to energy harvesting. However, the application of piezoelectric elements to vibration damping (both active and passive) has received much attention [51]. Some authors proposed using the energy extracted from the system to power sensors or electronics instead of dissipating the energy through resistive heaters or other dissipative elements [52, 53, 54]. Conversely, when an energy harvester is applied to a system, structural damping can be achieved if the harvester size is on the same order as the structure [55, 56, 57].

With the decrease in power requirements for sensor nodes, the application of

piezoelectricity to energy harvesting has become feasible. Piezoelectric elements in several geometries have been applied for this purpose. The most common is the cantilever beam configuration [5, 26, 44, 45, 48, 55, 56, 58, 59, 60], and is the focus of this work. This configuration has the advantage that it is compatible with MEMS fabrication processes, analysis is relatively straightforward, and the structure is very compliant (allowing for low resonance frequencies). A power density for this type of harvester was predicted to be the highest of the three conversion mechanisms and is the focus of a recent text on vibration energy harvesting for wireless nodes [8]. Other harvesting configurations using piezoelectric elements include: membrane structures to harvest energy from pulsing pressure sources [61, 62, 63] and converting energy from walking [64, 65]. Furthermore, research focusing on the power electronics to optimize the transfer of energy from the piezoelectric element to a storage device has been undertaken [9, 11, 28, 29]. Using the piezoelectric effect to convert mechanical vibration energy into electrical energy has inherent advantages and disadvantages [5]. The advantages are: it is the most efficient conversion mechanism (based on simulations), no separate voltage source is required (as for electrostatic harvesters), and the output voltages are of the correct order ($3-8\text{ V}$). The disadvantage is that it is more complex to integrate piezoelectric material configurations into micro-systems since the material needs to be poled in a strong electric field ($> 1\frac{\text{kV}}{\text{mm}}$ [66]) and at an elevated temperature (PZT-5A and PZT-5H have Curie temperatures of $\sim 365\text{ }^{\circ}\text{C}$ and $\sim 190\text{ }^{\circ}\text{C}$, respectively [67]). Please refer to Section 3.1 for a description of the poling process. The integration can be simplified by using a thin film piezoelectric configuration since a lower voltage is required for poling ($\sim 200\text{ V}$ for a $127\text{ }\mu\text{m}$ thick film). The temperature necessary can also be decreased.

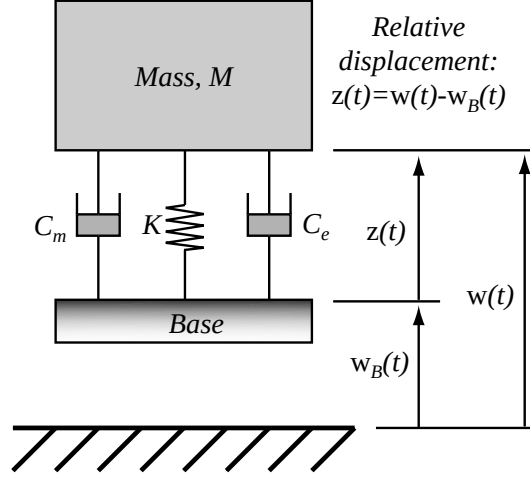


Figure 2-1: 1-D dissipative vibration energy harvester model.

2.2 Dissipative Model to Analyze Available Power Spectra

A basic 1-D dissipative model, illustrated in Figure 2-1, is used to analyze the power generated from a generic vibration energy harvester to understand conversion [41, 68]. This model is strictly valid only for harvesters where the damping term is linear and proportional to the velocity (*e.g.*, certain electromagnetic converters [5]), but is useful in understanding the relative importance of system (structural and electrical) and input parameters on the power extracted. The electrical energy is extracted from the mechanical system, which is excited by a mechanical input. This extraction is not necessarily linear, or proportional to velocity, however, it is a dissipative process and can generally be viewed as damping, as in the 1-D model presented here.

The dynamics of the system are described through eq. (2.1). ω_N is the natural frequency and ζ_m , ζ_e , and ζ_T are the mechanical, electrical, and total damping ratios, respectively. The natural frequency is defined as $\omega_N = \sqrt{\frac{K}{M}}$, where M is the mass and K is the spring stiffness. The damping ratios, ζ_m or ζ_e , are related to their corresponding damping coefficients, C_m or C_e , through $C = 2M\omega_N\zeta$. The base excitation is $w_B(t) = w_B e^{i\omega t}$, where w_B is the magnitude of the base displacement,

and ω is the base input frequency.

$$\ddot{z} + 2(\zeta_m + \zeta_e)\omega_N\dot{z} + \omega_N^2 z = -\ddot{w}_B \quad (2.1)$$

$\ddot{w}_B$ is the magnitude of the base acceleration and is related to the base displacement magnitude through $\ddot{w}_B = \omega^2 w_B$ for harmonic inputs, as is assumed here. The overhead dot indicates the time derivative of the variable. The magnitude of the displacement of the system in response to a base input of frequency ω , can be written in terms of the standard amplification factor, $|G(i\omega)|$.

$$|z| = \frac{M\ddot{w}_B}{K} \frac{1}{\sqrt{[1 - \Omega^2]^2 + [2(\zeta_m + \zeta_e)\Omega]^2}} = \frac{M\ddot{w}_B}{K} |G(i\omega)| \quad (2.2)$$

Use was also made of the definition of the frequency ratio, $\Omega = \frac{\omega}{\omega_N}$. The electrical power extracted through the electrical damping can be determined, $|P_{out}| = \frac{1}{2}C_e\dot{z}^2$, and is written in terms of the input vibration parameters:

$$|P_{out}| = \frac{M\zeta_e\Omega^2\omega^4 w_B^2}{\omega_N [(1 - \Omega^2)^2 + (2\zeta_T\Omega)^2]} = \frac{M\zeta_e\Omega^2\ddot{w}_B^2}{\omega_N [(1 - \Omega^2)^2 + (2\zeta_T\Omega)^2]} \quad (2.3)$$

The power generated can be maximized by determining the optimal operating frequency ratio.

$$\left(\frac{\omega_{opt}}{\omega_N}\right)^2 = \Omega_{opt}^2 = 2(1 - 2\zeta_T^2) \pm \sqrt{4(2\zeta_T^2 - 1)^2 - 3} \quad (2.4)$$

When the damping ratio is small, eq. (2.4) suggests $\omega_{opt} \approx \omega_N$. A second optimum around $\omega_{opt} \approx \sqrt{3}\omega_N$ is suggested by eq. (2.4), but is a local minimum. Since the mechanical damping will typically be small, it is sufficient to let $\omega_{opt} = \omega_N$, or $\Omega = 1$, for optimal power extraction in this dissipative model.

Next, the power generated can be maximized with respect to the electrical damp-

ing ratio. The optimum is calculated as:

$$\zeta_{e,opt} = \sqrt{\zeta_m^2 + \frac{(\Omega^2 - 1)^2}{4\Omega^2}} \quad (2.5)$$

When $\Omega = 1$, the optimal electrical damping of $\zeta_{e,opt} = \zeta_m$ is obtained. These results can be substituted into eq. (2.3) to obtain:

$$|P_{out}|_{opt} = \frac{M\ddot{w}_B^2}{16\zeta_m\omega_N} \quad (2.6)$$

From the analysis we can see that the electrical power harvested is a function of both the geometry of the device, and the vibration input parameters. The result can be re-written, noting that the natural frequency is a function of the mass, M , and the spring stiffness, K .

$$|P_{out}|_{opt} = \frac{\sqrt{M^3}\ddot{w}_B^2}{16\zeta_m\sqrt{K}} \quad (2.7)$$

In order to maximize the power extracted from the system, it is necessary to maximize the mass of the device, as well as the input vibration magnitude. At the same time, it is beneficial to minimize the stiffness of the device. The last term in the optimized power equation is the mechanical damping (the electrical damping is matched to the mechanical damping at resonance). The mechanical damping term is analyzed in detail in the following section. For the time being it is sufficient to state that mechanical damping should be minimized to maximize the power extracted from the vibration source.

2.3 Damping Analysis and Frequency Selection

In order to interpret the optimal power result in eqs. (2.6) and (2.7), the damping ratio needs to be investigated more closely. Since $\zeta_{e,opt} = \zeta_m$ for optimal power generation, the ratio will be determined by the mechanical damping ratio, ζ_m . This damping ratio has four dominant components for a MEMS-scale cantilever beam structure [48, 69, 70, 71]: drag force (airflow force), squeeze force, support losses, and

structural damping. These four components can be modeled as adding linearly, as in eq. (2.8), and are defined in eqs. (2.9) – (2.12).

$$\zeta_m = \zeta_{m,drag} + \zeta_{m,squeeze} + \zeta_{m,sup} + \zeta_{m,struct} \quad (2.8)$$

$$\zeta_{m,drag} = \frac{3\pi\mu b + \frac{3}{4}\pi b^2 \sqrt{2\rho_{air}\mu\omega}}{2\rho_{beam}btL\omega} \quad (2.9)$$

$$\zeta_{m,squeeze} = \frac{\mu b^2}{2\rho_{beam}g_0^3 t\omega} \quad (2.10)$$

$$\zeta_{m,struct} = \frac{\eta}{2} \quad (2.11)$$

$$\zeta_{m,sup} = \frac{0.23h^3}{L^3} \quad (2.12)$$

Here μ is the viscosity of air, ρ_{air} and ρ_{beam} are the densities of air and the beam structure respectively, g_0 is the gap between the bottom surface of the beam and a fixed floor, t is the thickness of the beam, b is the beam width, and L is the length. The structural loss (or damping) factor, η , is related to the equivalent damping coefficient through $C_m = \frac{\eta K}{\omega}$ [71]. It is assumed that the operating frequency coincides with the natural frequency, $\omega = \omega_N$.

For a MEMS-scale device, the drag force damping term is dominant when the device is operated in free space (*e.g.*, away from a wall) and under atmospheric conditions (in air). When the device is operated near a wall, the squeeze force damping becomes dominant. Alternatively, when the device is operated in vacuum, and away from a wall, the structural damping term becomes dominant. The structural damping factor is determined empirically, so for the purpose of the analysis that follows, $\eta = 5 \times 10^{-6}$ was used (from [70]).

For the proposed micro-scale design (a cantilever beam operating away from a wall, under atmospheric conditions), the only significant source of damping is the damping due to the drag force. This component has two terms, $\zeta_{m,drag} \propto \frac{\gamma}{\omega_N} + \frac{\chi}{\sqrt{\omega_N}}$, where γ and χ are proportionality constants. Thus, for the purposes of the analysis, the damping-frequency relation is approximated as $\zeta_m \propto \frac{1}{\omega_N}$. This relation also holds for squeeze film damping. Substituting this result into eqs. (2.6) or (2.7) it is concluded

that the input vibration parameter that most influences the generated power for the current conditions is the input acceleration, $\ddot{w}_B$:

$$|P_{out}|_{opt} \propto M\ddot{w}_B^2 \quad (2.13)$$

Thus, when comparing different sources of vibration for a micro-scale device under atmospheric conditions, it is important to maximize the acceleration. When the micro-scale device is operated in vacuum, the dominant damping components are independent of frequency, and the evaluation of vibration sources will be different. The vibration input for vacuum operation will be related to the power generated through:

$$|P_{out}|_{opt} \propto \frac{M\ddot{w}_B^2}{\omega_N} \quad (2.14)$$

In summary, the optimal operating point for power harvesting is a function of the harvesting device/system and is strongly influenced by the dominant damping mechanism in the system. The damping mechanism for a MEMS-scale device is influenced by whether the device is operated in vacuum or atmospheric conditions. For MEMS-scale devices the volume scales down as length cubed, and the surface area scales down as length squared. As a result, the surface-fluid interactions become dominant over inertial effects for microscopic devices. These fluidic damping mechanisms are generally dependent on frequency, which must be accounted for when analyzing the generated power. On the other hand, surface-fluid interactions are negligible for macro-scale systems, and the dominant damping mechanisms (structural damping and support losses) are generally independent of frequency [71].

When selecting the vibration peak (in terms of acceleration and frequency) to design the MPVEH for maximum power generation, the maximum value of the input acceleration squared ($\ddot{w}_B^2$) must be considered as the device is to be operated in atmospheric conditions. Equivalently, the input parameters can be written in terms of the input frequency and the displacement, $\omega^4 w_B^2$. To facilitate operating point selection from a given vibration source spectrum, lines of constant “reference power”, RP, can be added to any measured acceleration-frequency plots. Reference power

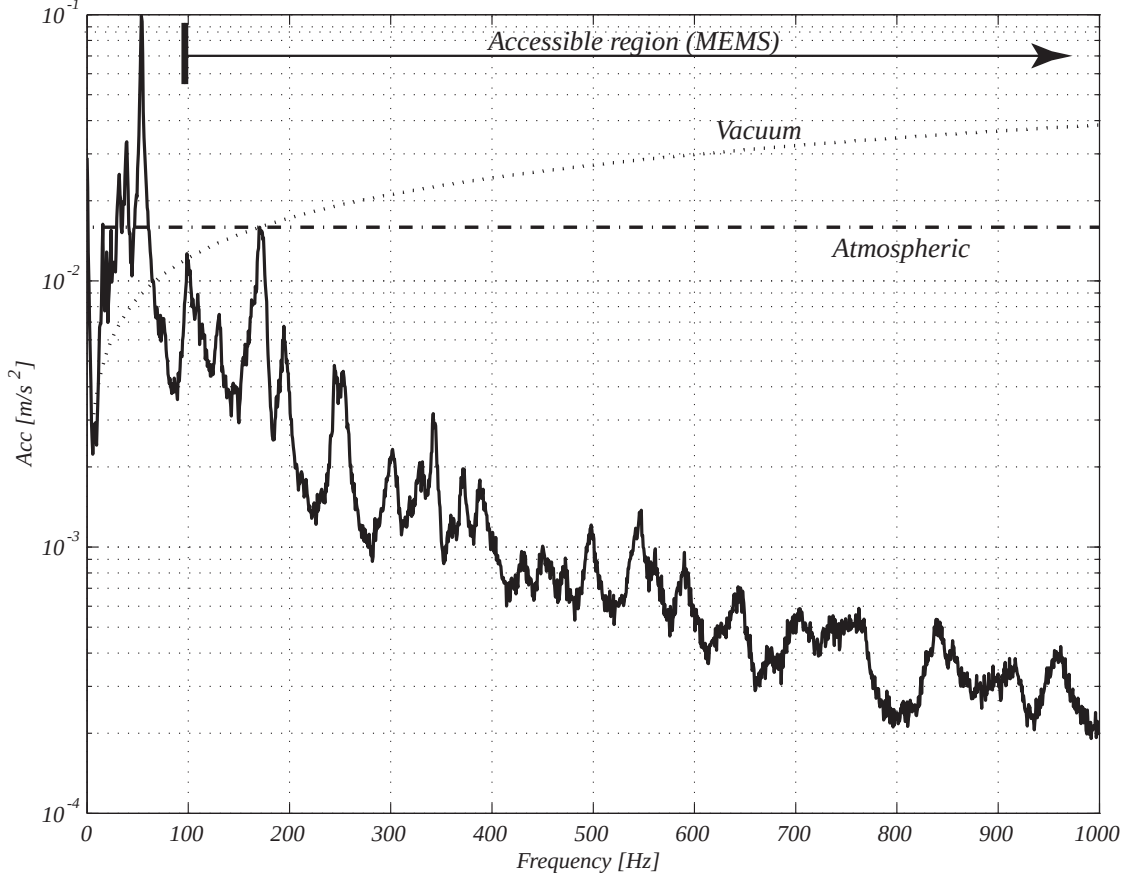


Figure 2-2: Interpreting ambient acceleration-frequency spectra to determine target acceleration peak. The example spectrum is for an AC Duct (measurement 2 in Table 2.2).

is defined as $|P_{out}|_{opt}$ in eqs. (2.6) or (2.7), at the acceleration and frequency of the highest acceleration peak above 100 Hz (the 100 Hz limit is explained below). Please refer to Figure 2-2 for an illustration of these lines. These constant power lines indicate the maximum contribution to the power generated from the input spectrum, assuming damping ratio-optimized resonant harvesters of equal mass. When the MEMS-scale device is operated in vacuum, the damping ratio is independent of the frequency, and the ratio of acceleration squared to the frequency should be used to interpret the measurement data (Figure 2-2). In some cases, the optimal operating frequency can have a lower acceleration than other peaks.

Consistent with our conclusions, others have found it difficult to design a MEMS

device with a resonant frequency below 100 Hz (as size scales down, resonant frequency increases) [5]. The lower limit for practical vibration peaks has been set to 100 Hz for the current investigation, thus defining the “accessible region” above 100 Hz, as in Figure 2-2.

Two other schemes for the interpretation of the measurement data have been put forward in the literature. In the first, the peak power generated is written as in eq. (2.3) (with $\Omega = 1$), but the frequency dependence of the damping term is not considered [30, 5]. In the second scheme, the power generated is written in terms of both the input and output parameters, which can be simplified to obtain eq. (2.3) (with $\Omega = 1$), and similarly does not account for the damping-frequency dependence for a MEMS device [72].

2.4 Power Available from Ambient Vibrations (Summary of Experimental Results)

In order to better understand the characteristics of low-level ambient sources, vibrations measurements for a variety of everyday objects were taken. The purpose of the measurements was to get an indication of the frequency range and amplitude of vibrations from these sources. Eight separate sources under different conditions were analyzed (14 cases in total), over the frequency range 10 – 1,000 *Hz*. Details of the sources/conditions and the experimental procedures are given in Appendix A.

The interpretation schemes for input vibration frequency selection from Section 2.3 is used. It was found that macro- and micro-systems require separate schemes since the dominating damping mechanisms for these systems vary. This is in contrast to previous findings where the mechanisms of the damping were ignored and/or assumed independent of frequency. For micro devices, the operating environment will further influence the selection of a vibration frequency to target. Lastly, ambient vibration sources generally exhibit multiple peaks of significant power. This observation motivated an investigation into the effect of higher device resonance modes on

the power generation and static failure of the structure in Section 2.5.1.

The interpretation scheme for a MPVEH device operated under *atmospheric* conditions, $|P_{out}|_{opt} \propto \ddot{w}_B^2$, discussed in Section 2.3, is used to identify three acceleration peaks for each source, and are listed in Table 2.2. The first peak has the maximum power content (*e.g.*, the highest acceleration squared) and is referred to as the “Highest Peak”, or HP. The “Reference Peak”, RP, is the highest acceleration peak in the accessible region (*i.e.*, above 100 Hz). The “Alternate Peak”, AP, is a secondary peak in the accessible region. For some of the sources, the HP and the RP coincide. It can be seen that the ranges and levels of ambient vibrations vary greatly: for HP, the levels varied from 10^{-3} m/s^2 to around 4 m/s^2 . However, not all these peaks are accessible (*e.g.*, above 100 Hz), and RP values range from 10^{-4} m/s^2 to 4 m/s^2 . At this time, the efficiency of power conversion is not known for the various energy harvesters and a minimum vibration level required for positive power generation has not been established. The measured spectra show good agreement with published ambient vibration data [5].

Upon comparing RP and AP values, two important observations are made. Firstly, in 7 of the 14 cases investigated, an AP was identified at a lower frequency than the RP. The significance of this becomes clear when the device is operated in vacuum. As is illustrated in Figure 2-2, the constant power lines for a device operated in vacuum tends to zero as the frequency decreases. In other words, the acceleration amplitude required at low frequencies for a constant power tends to zero since the power is inversely proportional to the frequency. From the example it is clear that the AP will have the same power content as the RP (refer to Figure 2-2). If the operating environment is vacuum, the optimal harvesting point for a MEMS-scale beam harvester might not correspond to the highest vibration level peak (RP as defined here), but will be at a lower acceleration amplitude and frequency. A reference vibration of $w_B = 2.5 \text{ m/s}^2$ at 150 Hz (approximately vibrations measured on a microwave oven side panel) is used for the preliminary design of an MPVEH in Section 6.4.

Table 2.2: Summary of measured ambient vibration sources: quantitative comparison for harvester operated in atmospheric conditions.

#	Source [†]	HPP		RP		AP	
		Acc. [m/s^2]	Freq. [Hz]	Acc. [m/s^2]	Freq. [Hz]	Acc. [m/s^2]	Freq. [Hz]
1	A/C duct: center, low	0.0328	15.7	0.0254	171.9	0.0130	113.8
2	A/C duct: side, high	0.0990	53.8	0.0159	170.6	0.0127	98.8
3	A/C duct: center, high	0.0398	55.0	0.0366	173.1	0.0186	108.3
4	Computer side panel	0.0402	276.3	0.0402	276.3	0.0360	120.0
5	Microwave oven: top	1.11	120.0	1.11	120.0	0.570	240.0
6	Microwave oven: side	4.21	148.1	4.21	148.1	1.276	120.0
7	Office desk	0.0879	120.0	0.0879	120.0	0.00516	546.3
8	Harvard bridge railing	0.0215	171.3	0.0215	171.3	0.0193	136.3
9	Parking meter: Perp.	0.0327	13.8	0.00172	120.0	0.000697	148.1
10	Parking meter: Par.	0.0355	13.8	0.00207	923.8	0.000509	977.5
11	Car hood: 750 <i>rpm</i>	0.0744	35.6	0.0143	148.8	0.0103	510.6
12	Car hood: 3000 <i>rpm</i>	0.257	147.5	0.257	147.5	0.102	880.6
13	Medium tree	0.000985	16.3	0.000229	115.3	0.000226	240.0
14	Small tree	0.003	30.0	0.000465	293.1	0.000425	99.4

[†] See Appendix A for a detailed description of sources and experiments.

2.5 Other Design Considerations

Since the vibration source signals do not in general contain only one frequency component, it is possible to excite some of the higher order vibration modes of the structure. The effect of the the higher order modes are investigated next. Also, high quality factors are obtainable with MEMS-devices. The effects of the quality factor on the system response is analyzed, with important design and system implications.

2.5.1 Higher-Order Modes

Some sources exhibit peaks with comparable power content at much higher frequencies. See, for example, source 12 (Car hood at 3000 *rpm*) in Table 2.2. The higher

frequency peak can excite a second or third resonance mode of the structure and strain cancellation (and therefore power reduction) in the harvester device is possible. The example has a reference peak of amplitude 0.257 m/s^2 at 148 Hz , and an alternate peak at 881 Hz with an acceleration of 0.102 m/s^2 . This finding prompted an investigation into the effect of higher frequency modes of the beam structure when aligned to an alternate peak of the source vibration.

As an example, a simple cantilever beam was modeled with the first resonance frequency at 140 Hz and the second resonance frequency at 875 Hz . Thus, with the variability of vibration sources, it is possible for the source alternate peak and the second resonance peak of the beam to align. To investigate strain cancellation for this simple cantilever, a modal analysis of the device with two input vibration components was conducted. The first component was aligned with the first resonance of the device (as would occur per design), and the second component coincided with the second resonance of the beam. The vibration level of the second frequency component was varied to analyze the effect on the developed strain as an indication of power converted. Please refer to Figure 2-3 for strain vs. beam length under these assumptions. The power of the second (or alternate) input peak is zero, half, and equal to the power of the reference peak, respectively. The strain developed over the first region of the beam (near the base) increases with the additional excitation (second mode), whereas the strain is reduced towards the tip. Assuming that the power is proportional to strain, it is necessary to look at the total area under the strain curve. Since the second mode causes the total area to decrease, the total power will decrease as well (assuming that the electrodes cover the whole surface). It is also possible to identify an optimal electrode length in the region where the strain is increased, but that is not affected by the cancellation of strain, for this simple case. Furthermore, the second mode contributes by increasing the maximum developed strain at the base of the beam, affecting the static failure design of such devices. For the purpose of the analysis in this section, it is assumed that the mode deflections due to the two inputs are in phase. Should this not be the case, a different final strain distribution will be obtained, but the important conclusion is that strain cancellation is possible

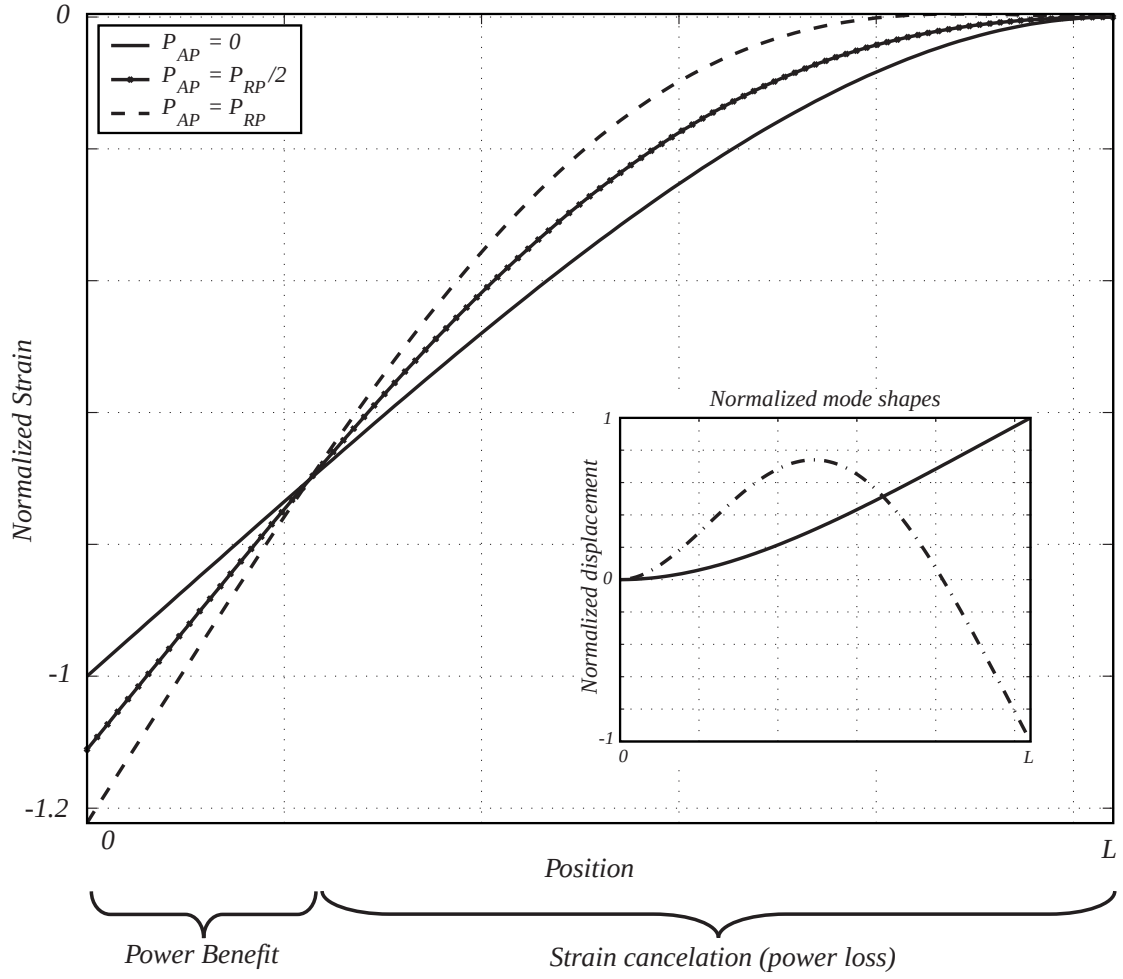


Figure 2-3: Strain distribution for a beam excited by multiple input vibration components, illustrating net power loss.

when higher vibration modes are excited.

2.5.2 High Quality Factor Devices

Four dominating damping components were identified in Section 2.3 for a MEMS-scale device. These were the drag force (airflow force), squeeze force, support losses, and structural damping (defined in eqs. (2.9) – (2.12)). It was shown that for MEMS devices, the dominant damping component depends on the operating environment (*e.g.*, vacuum or atmospheric conditions). When these devices are packaged in vacuum, ex-

tremely high quality factors are obtainable. Quality factors of tens of thousands have been reported for such MEMS resonators [73, 74, 75, 76]. The quality factor is significantly reduced when the device is operated in air. Pourkamali *et al.* reports a resonator with a quality factor of 300 in air, compared to 1800 for the same resonator operated in vacuum [77]. Under vacuum conditions, the dominant damping terms are the support losses and the structural damping.

The quality factor determines the response of the device to the input around the resonance frequency. At resonance ($\Omega = 1$), and with no electrical damping ($\zeta_e = 0$), the displacement is related to the quality factor (and the mechanical damping) through eq. (2.15).

$$|z| = \frac{M\ddot{w}_B}{K} \frac{1}{2\zeta_m} = \frac{M\ddot{w}_B}{K} Q \quad (2.15)$$

$\frac{M\ddot{w}_B}{K}$ is the static deflection. The amplification factor, defined in eq. (2.2), reduces to the quality factor, ($Q = \frac{1}{2\zeta_m}$), at resonance. Refer to Figure 2-4 for an illustration of the effect of the quality factor on the system response. The quality factor determines the amplification of the input (static deflection) by the device at resonance. The quality factor can also be defined in terms of the peak width at the half-power points: $\zeta_m = \frac{\Omega_a - \Omega_b}{2}$ [78]. The half-power points are defined at the frequencies where the maximum amplification is reduced by a factor of $\frac{1}{\sqrt{2}}$, or about 70.7 %. These points have been included in Figure 2-4 for the quality factors of 100 and 500 respectively. A high quality factor results in a very large amplification of the input and a very narrow peak. The response decreases rapidly away from the peak frequency. These effects on the response of the system have different design implications, which will be discussed next.

The first implication is that the quality factor determines the displacement for the structure around resonance. For a beam in bending, the strain is directly related to this displacement and the strain determines the power generated by a piezoelectric energy harvester. Thus, the higher the quality factor, the higher the power generated for a given input. The effect of the quality factor on the power can also be seen in eq. (2.6). Conversely, given a minimum power requirement, it is possible to determine

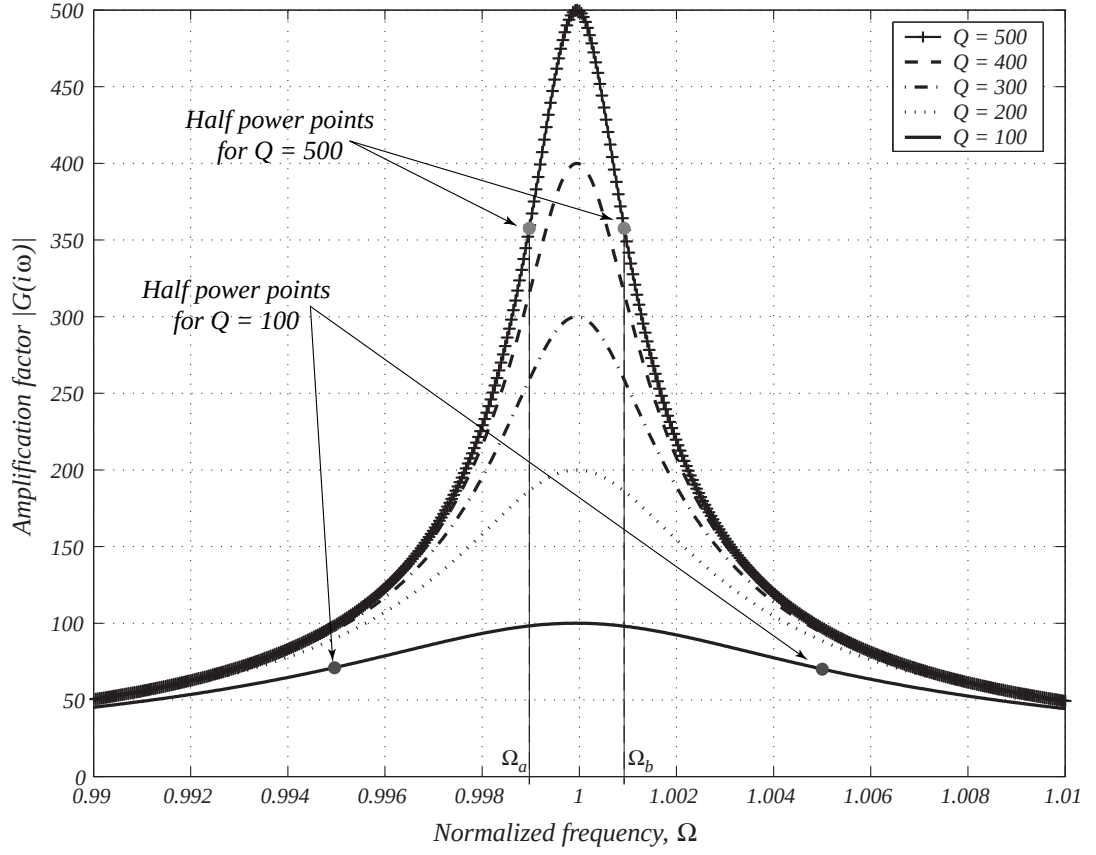


Figure 2-4: Effect of quality factor on system response: displacement magnitude vs. normalized frequency.

the minimum input necessary to generate that power with a device with a known quality factor, using eq. (2.6).

Secondly, the quality factor will determine the width of the response peak. For resonating harvesters, the objective is to align the resonance frequency of the device with the optimal source vibration peak (which was identified in Section 2.3). When the quality factor is very high, the response peak is very narrow. Thus, if the input peak and resonance frequency of the device are perfectly aligned, the power generated will be high. However, when the alignment of these peaks deviate even slightly, the power extracted from the system will decrease dramatically. As a result, incorporating a tuning mechanism into the device may be desirable, or even necessary. A possibility to accomplish resonant frequency tuning is with a variable capacitor [79], though it is not in the scope of the this project to investigate the mechanism

further. For the variable capacitor, the lateral deflection of the resonator will change the bending stiffness of the structure, thus changing the resonance frequency. The disadvantage is that this mechanism consumes power. A trade-off study will be necessary to determine the range of tunability achievable and the electrical cost vs. the increased power generation. A second motivation for investigating the tuning mechanism is the variability of current MEMS manufacturing processes. MEMS fabrication processes can have high variability (up to 10% variation in dimension transfer from the mask to the wafer for some photolithography processes). As a result, the resonant frequency cannot be controlled as accurately as desired. This variability may make a tunable device a necessity.

2.6 Design Implications for a MPVEH

Previous research has mostly focused on macro-scale harvesting devices. Three mechanisms of vibration to electrical energy have been investigated, and utilizing the piezoelectric effect appears to be superior. Ambient sources were measured, and harvestable levels of vibration were identified below 300 Hz . From the dissipative 1-D model, it was shown that the mass of the resonating structure should be maximized, while the stiffness should be minimized (eq. (2.7)). This can be accomplished through material selection and changing the device geometry. For example, the cantilevered beam structure (used in this project) is very compliant and yet simple to analyze and compatible with MEMS fabrication processes. Modal beam models are needed to investigate the effect of higher resonance modes on the power generation and the failure of the device. Furthermore, the damping in the system should be minimized (the quality factor, Q , should be maximized). MEMS devices have the advantage that extremely high Q 's are achievable. A high Q implies a narrow response peak and high input amplification around the resonance. The narrow peak may require the implementation of a tuning mechanism so that the resonance frequency of the device is controllable. This is due to the rapid reduction in the power generated when the resonance frequency of the device and the input vibration peak are not

perfectly aligned. MEMS configurations also impose a lower limit on the resonance frequency (around 100 Hz). This is because the resonant frequency scales up as size scales down. Thus, the target frequency range has been set to $100 - 300\text{ Hz}$. MEMS devices (with high Q 's) will enable ambient vibration energy harvesting since the high Q 's will reduce the minimum vibration input amplitude required to generate a useable level of power. Lastly, one of the advantages of MEMS fabrication is the ability to add additional harvesting structures to a device (as the footprint area allows) at negligible additional cost. By interconnecting the individual harvesters in series or parallel (or combinations thereof), the electrical output (voltage and current) from the complete device can be controlled. This will allow maximization of power per unit area and the tailoring of the electrical output for a specific application.

Chapter 3

Piezoelectric Energy Harvester Models

Coupled electromechanical models are developed for design and performance prediction of a micro-scale piezoelectric energy harvester and for validation to macro-scale harvester experiments. A basic 1-D accelerometer-type closed-form model is shown to characterize the behavior of beam configuration harvesters adequately by comparison to more rigorous models based on a modal analysis. Uni-morph and bi-morph beam configurations, as well as {3-1} and {3-3} actuation modes (the latter using interdigitated electrodes) are considered. The modal analysis for the less common beam geometry including a proof mass at the free end is also included.

3.1 The Piezoelectric Effect

In 1880, Jacques and Pierre Curie discovered the piezoelectric effect in certain crystalline materials: when these materials are subjected to a mechanical force, their crystals become electrically polarized. The polarities for tensile and compressive forces are opposite and the polarity is proportional to the applied force. The converse relationship is also true: when the crystalline material is subjected to an electric field it lengthens or shortens according to the polarity of the electric field. The latter is known as the inverse piezoelectric effect. Please note that this section has been

adapted from [80, 81] and focusses on perovskite-type piezoceramics such as PZT (lead zirconium titanate). In general, there is a distinction between piezoelectric and ferroelectric materials. Materials with crystals that have a dipole are termed piezoelectric materials. A subset of these materials, termed ferroelectric materials, can be poled (*i.e.*, the dipole direction can be changed). Poled ferroelectric materials oftentimes exhibit stronger piezoelectric coupling than their un-poled counterparts and are used in many actuation and sensing applications. Thus, to be consistent with convention, reference to piezoelectric materials in this work will include poled ferroelectric materials.

Bulk piezoelectric ceramics consist of multiple individual perovskite crystals. The crystals are typically randomly oriented in the polycrystalline ceramic and are arranged in grains with multiple domains in a grain. Each crystal consists of a small tetravalent metal ion (titanium, zirconium, etc.) in a lattice of divalent metal ions (lead, barium, etc.) and O_2 - ions. Above a critical temperature (the Curie point), the crystals have a simple cubic symmetry with no dipole moment and no piezoelectric effect. However, below this temperature, the metal ions are displaced and the crystals lose their cubic structure. The crystals obtain a tetragonal symmetric structure and each crystal has a dipole moment (also referred to simply as a dipole, or remnant polarization). The cubic (non-piezoelectric) and tetragonal (piezoelectric) single crystal structures of perovskite ceramics are shown in Figure 3-1.

Adjoining dipoles form regions of local alignment and are called domains. These domains, like the crystals that comprise them, possess net dipole moments, resulting in a net polarization. The directions of polarization among neighboring domains and grains in the bulk are random and the ceramic has no overall polarization. The piezoelectric element as a whole is exposed to a strong electric field, ($\sim 1 \frac{kV}{mm}$ [66]), usually at a temperature slightly below the Curie point. This is referred to as the “poling” of the ferroelectric to create a ceramic with strong piezoelectric coupling. The randomly-oriented domain dipoles become somewhat aligned with the strong electric field and the element expands in the direction of the field. This expansion is an artifact of the crystals (with the tetragonal crystal structures) that are not in

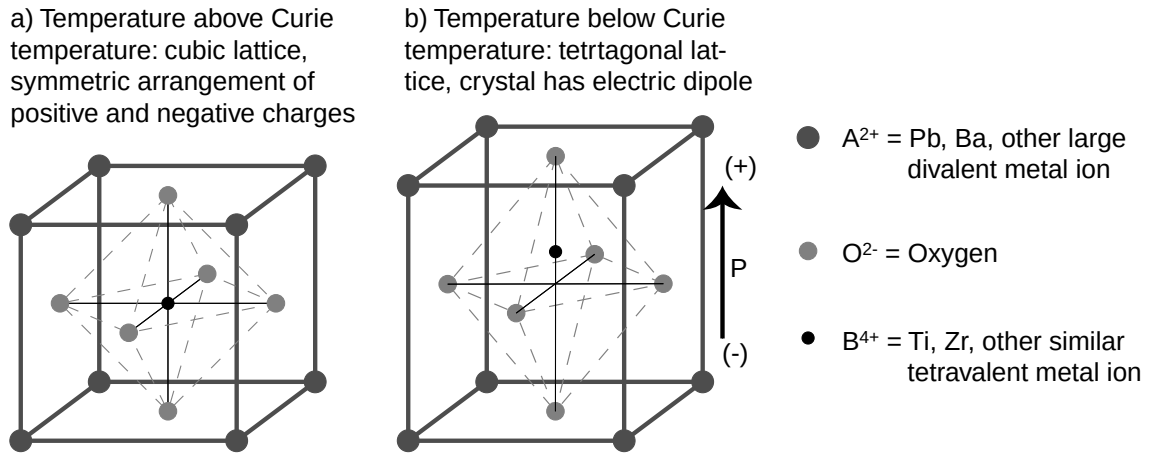


Figure 3-1: Cubic and tetragonal crystal structure of a traditional perovskite (ABO_3) ceramic, such as PZT.

random orientations anymore, but are better aligned with the electric field. When the electric field is removed, many of the dipoles are locked into the configuration of near alignment, primarily due to domain wall motion which occurs during poling. The ceramic now has a permanent polarization, the bulk remnant polarization, and is permanently elongated. Examples of ferroelectric materials with high piezoelectric coupling are PZT (lead zirconate titanate) and $BaTiO_3$ (barium titanate), as compared to ZnO (zinc oxide), which is a piezoelectric material with relatively little piezoelectric coupling.

Individual crystals of a piezoelectric (such as those described above) have large piezoelectric coupling coefficients relative to the bulk, randomly oriented poly-crystalline form of piezoelectrics typically encountered (such as PZT). The reduction in coupling in the bulk form is due to misalignment of domains relative to the poling axis and incomplete lock-in of poled crystals. Calculations can be made of the degree of incomplete lock-in based on measured piezoelectric coupling in the bulk ceramic vs. the single crystal form [82]. Thin films of piezoelectrics, including PZT, oftentimes are formed with a preferred texture (tetragonal axis is aligned with out-of-plane film axis) along the normal to the film depending on the processing employed. In-plane texture in the thin-film plane, however, is random. Regardless of the texture of the film, the piezoelectric and elastic constants can be formulated and measured based

Using the 1-D configuration, we can simplify the 3-D linear elastic (small-signal) constitutive relations [83, 84] in eq. (3.1) to eqs. (3.2) and (3.3):

$$\begin{Bmatrix} \mathbf{T} \\ \mathbf{D} \end{Bmatrix} = \begin{bmatrix} \mathbf{c}^E & -\mathbf{e}^t \\ \mathbf{e} & \boldsymbol{\epsilon}^S \end{bmatrix} \begin{Bmatrix} \mathbf{S} \\ \mathbf{E} \end{Bmatrix} \quad (3.1)$$

$$T_3 = c_{33}^E S_3 - e_{33} E_3 \quad (3.2)$$

$$D_3 = e_{33} S_3 + \epsilon_{33}^S E_3 \quad (3.3)$$

The $\mathbf{D}$, $\mathbf{E}$, $\mathbf{S}$, and $\mathbf{T}$ matrices are defined as developed electric displacement, applied electric field, applied strain, and developed stress, respectively. The reader is referred to Appendix B for the sign convention of the electrical parameters used in this work. $\boldsymbol{\epsilon}$ is the permittivity of the piezoelectric element and $\mathbf{e}$ is the piezoelectric constant relating charge density and strain and is defined as positive in the direction indicated (the element local coordinates coincide with the global coordinates). The poling direction of the piezoelectric element defines the element's local coordinate system, and the piezoelectric constant needs adjustment (sign correction) when the element local coordinates does not coincide with the global coordinates. The treatment of poling direction and the piezoelectric constant is discussed in Appendix B, and [85, 86]. $\mathbf{e}$ is also referred to as the piezoelectrically induced stress tensor as it relates the stress to electric field. $\mathbf{c}^E$ is the stiffness matrix. The superscripts E and S indicate a parameter at constant (typically zero) electric field and strain respectively, while superscript t indicates the transpose of the matrix. For the analysis in later sections, the superscripts D and T indicate a parameter at constant (typically zero) electric displacement and stress, respectively. Note that c_{33}^E in the 1-D model would be measured under short circuit conditions, *i.e.*, by connecting the electrodes on the piezoelectric block.

From a force equilibrium analysis, the governing equations can be found in terms of the constitutive relations (eqs. (3.2) and (3.2)) and device parameters, defined in Figure 3-2. The piezoelectric element is poled in the x_3 direction. The strain is related to the device parameters through $S_3 = \frac{z}{t_p}$ and the electric field is defined as $E_3 = -\frac{v}{t_p}$.

z is the relative displacement, as in Figure 3-2. The approximate total mass¹ of the system is $M_T = M + \frac{1}{3}M_p$ and $m_e = \frac{M_T}{A_p}$ is the mass per cross-sectional area. A_p is the area of the electrodes (or the piezoelectric element). The stress is the force per area, or $T_3 = -\frac{M_T(\ddot{z} + \ddot{w}_B)}{A_p} = -m_e(\ddot{z} + \ddot{w}_B)$. $\ddot{w}_B$ is the base acceleration and q is the charge developed on the electrodes. The overhead dot indicates the time derivative of the variable. The electric displacement, D_3 , is the charge on the electrodes per unit area. This all gives:

$$m_e\ddot{z} + c_{33}^E \frac{z}{t_p} + e_{33} \frac{v}{t_p} = -m_e\ddot{w}_B \quad (3.4)$$

$$D_3 = e_{33} \frac{z}{t_p} - \varepsilon_{33}^S \frac{v}{t_p} = \frac{q}{A_p} \quad (3.5)$$

Both equations are multiplied by A_p and a convenient electromechanical coupling term, $\theta = -\frac{e_{33}A_p}{t_p}$, is defined. Furthermore, $K = \frac{c_{33}^E A_p}{t_p}$ is the effective stiffness. The capacitance is defined in terms of the constrained permittivity: $C_p = \frac{\varepsilon_{33}^S A_p}{t_p}$. The overhead dot indicates the time derivative. The second governing equation (sensing equation), eq. (3.5), can be written in terms of the current, $i = \frac{dq}{dt}$, by taking the time derivative. For the purely resistive electrical load (as has been assumed), the current can be related to the voltage developed through $v = iR_{par}$. The equivalent resistance, R_{par} , is the parallel resistance of the load and the leakage resistances, R_l and R_p respectively. In general, the leakage resistance is much higher than the load resistance [87], so that $R_{par} \approx R_l$. With these definitions and substitutions, eqs. (3.6) and (3.7) are obtained from eqs. (3.4) and (3.5).

$$M_T\ddot{z} + Kz - \theta v = -M_T\ddot{w}_B \quad (3.6)$$

$$\theta\dot{z} + C_p\dot{v} + \frac{1}{R_l}v = 0 \quad (3.7)$$

Note that eq. (3.6), without the θv coupling term, is the familiar dynamic equation of motion for a 1 degree-of-freedom spring-mass system. Eq. (3.6) can be further reduced by dividing through by the approximate total mass, M_T , and using the defini-

¹For longitudinal vibrations of a rod, the resonance frequency is often estimated by lumping one third of the mass of the rod at the tip. Together with the proof mass, this defines an approximate effective mass for the system.

tion for the resonance frequency, $\omega_N^2 = \frac{K}{M_T} = \frac{c_{33}^E}{m_e t_p}$. Note that ω_N , due to the fact that the structure is a piezoelectric block, is related to the speed of sound (or a pressure wave) through the piezoelectric material at short circuit ($E = 0$) conditions. Viscous damping (proportional to velocity, $\dot{z}$) is added through the mechanical damping ratio, ζ_m , so that eq. 3.6 becomes:

$$\ddot{z} + 2\zeta_m \omega_N \dot{z} + \omega_N^2 z - \frac{\theta}{M_T} v = -\ddot{w}_B \quad (3.8)$$

Using Laplace transforms, the governing equations can be evaluated and the magnitudes of the relative displacement, voltage, and electrical power extracted can be determined. These are given in eqs. (3.9) – (3.11), normalized by the input force ($M_T \ddot{w}_B$). The dimensionless parameter $\Omega = \frac{\omega}{\omega_N}$ and electromechanical coupling coefficient, $k_e^2 = \frac{k_{33}^2}{1 - k_{33}^2} = \frac{e_{33}^2}{c_{33}^E \epsilon_{33}^S}$, can be defined [81, 82]. $k_{33}^2 = \frac{d_{33}^2}{s_{33}^E \epsilon_{33}^T}$ is the electromechanical coupling coefficient relating converted electrical energy to input mechanical energy, or vice versa, for the 1-D system. Lastly, the dimensionless time constant of the system, $\alpha = \omega_N R_l C_p$, is defined. $R_l C_p$ is the time constant of the electrical circuit, while $\frac{1}{\omega_N}$ is related to the period of the mechanical resonator.

$$\left| \frac{z}{M_T \ddot{w}_B} \right| = \frac{1}{K} \frac{\sqrt{1 + (\alpha \Omega)^2}}{\sqrt{[1 - (1 + 2\zeta_m \alpha) \Omega^2]^2 + [(2\zeta_m + (1 + k_e^2) \alpha) \Omega - \alpha \Omega^3]^2}} \quad (3.9)$$

$$\left| \frac{v}{M_T \ddot{w}_B} \right| = \frac{1}{|\theta|} \frac{\alpha k_e^2 \Omega}{\sqrt{[1 - (1 + 2\zeta_m \alpha) \Omega^2]^2 + [(2\zeta_m + (1 + k_e^2) \alpha) \Omega - \alpha \Omega^3]^2}} \quad (3.10)$$

$$\left| \frac{P_{out}}{(M_T \ddot{w}_B)^2} \right| = \frac{\omega_N}{K} \frac{\alpha k_e^2 \Omega^2}{[1 - (1 + 2\zeta_m \alpha) \Omega^2]^2 + [(2\zeta_m + (1 + k_e^2) \alpha) \Omega - \alpha \Omega^3]^2} \quad (3.11)$$

3.2.2 Power Optimization

In all known prior energy harvester work, eqs. (3.9) – (3.11) are simplified by setting $\Omega = 1$, which will be shown to miss important features of the coupled response (particularly anti-resonance). It is of interest to optimize the power extracted from the source. Again, taking the leakage resistance as much larger than the load resistance

($R_{par} \approx R_l$), the power can be optimized with respect to the load resistance, R_l , or equivalently with respect to the dimensionless parameter, α . In performing this optimization, it is convenient to first expand the denominator in powers of α . The result gives:

$$\alpha_{opt}^2 = \frac{\Omega^4 + (4\zeta_m^2 - 2)\Omega^2 + 1}{\Omega^6 + (4\zeta_m^2 - 2[1 + k_e^2])\Omega^4 + [1 + k_e^2]^2\Omega^2} = \frac{1}{\Omega^2} \frac{(1 - \Omega^2)^2 + (2\zeta_m\Omega)^2}{([1 + k_e^2] - \Omega^2)^2 + (2\zeta_m\Omega)^2} \quad (3.12)$$

This α_{opt} gives the load resistance, R_l , to extract maximum power at any frequency ratio, Ω . The corresponding power generated is found by substituting the α_{opt} back into the power expression, eq. (3.11). Since a piezoelectric element exhibits both open- and short-circuit stiffness, there will be two important optimal-power operating frequencies to consider. First, set $\zeta_m = 0$ and let $R_l \rightarrow 0$ in eq. (3.9) to obtain the short-circuit natural frequency:

$$\Omega_r = 1 \quad \text{or,} \quad \omega_r = \sqrt{\frac{c_{33}^E A_p}{M_T t_p}} \quad (3.13)$$

This has previously been defined as the resonance frequency, ω_N . This frequency is defined at the minimum impedance of the circuit [82, 84]. For the open-circuit analysis, set $\zeta_m = 0$ and let $R_l \rightarrow \infty$ in eq. (3.9) to obtain the open-circuit natural frequency:

$$\Omega_{ar} = \sqrt{1 + k_e^2} = \frac{1}{\sqrt{1 - k_{33}^2}} \quad \text{or,} \quad \omega_{ar} = \sqrt{\frac{c_{33}^D A_p}{M_T t_p}} \quad (3.14)$$

This frequency is known as the anti-resonance frequency and is defined at the maximum impedance of the circuit [82, 84]. The ratio of the anti-resonance to resonance frequency (Ω_{ar}) is governed solely by the electromechanical coupling coefficient (k_{33}^2 or k_e^2) for this case where the entire structure is coupled. Finally, a system coupling factor is often defined in terms of the resonance and anti-resonance frequencies as $k_{sys}^2 = \frac{\Omega_{ar}^2 - \Omega_r^2}{\Omega_{ar}^2}$ [57, 84]. For this 1-D case, where the entire structure is made up of a piezoelectric, this system coupling factor is related to the other coupling factors through: $k_{sys}^2 = k_{33}^2 = \frac{k_e^2}{1 + k_e^2}$. Note that $\Omega_{ar} \geq \Omega_r$ since the coupling is always positive

or zero. Using these frequency ratios, it is possible to obtain optimal resistances with eq. (3.12) under the respective operating conditions. First, the optimal electrical loading at resonance (Ω_r) is determined:

$$\alpha_{opt,r}^2 = \frac{4\zeta_m^2}{4\zeta_m^2 + k_e^4} \quad (3.15)$$

ζ_m is generally at least an order of magnitude smaller than k_e^2 , thus this result can be approximated very well as:

$$\alpha_{opt,r} \approx \frac{2\zeta_m}{k_e^2} \quad (3.16)$$

This result, with $\Omega = \Omega_r = 1$, can be substituted into eq. (3.11) to obtain maximum normalized power at resonance:

$$\left| \frac{P_{out}}{(M_T \ddot{w}_B)^2} \right|_{opt,r} = \frac{\omega_N}{K} \frac{2\zeta_m}{16\zeta_m^2 \left(\frac{\zeta_m^2}{k_e^2} + 1 \right)} \quad (3.17)$$

The term in the denominator, $\frac{\zeta_m^2}{k_e^2} \ll 1$, such that this result reduces to eq. (3.18):

$$\left| \frac{P_{out}}{(M_T \ddot{w}_B)^2} \right|_{opt,r} = \frac{\omega_N}{K} \frac{1}{8\zeta_m} \quad \text{or} \quad |P_{out}|_{opt,r} = \frac{(M_T)^{\frac{3}{2}} \ddot{w}_B^2}{\sqrt{\frac{c_{33}^E A_p}{h}} 8\zeta_m} \quad (3.18)$$

A parallel approach is to substitute eq. (3.16) into the voltage developed, eq. (3.10), which then reduces to $|v|_{opt,r} = \frac{M_T \ddot{w}_B}{2|\theta|}$. The power is related to the voltage developed and the electrical load (purely resistive) through $|P_{out}|_{opt,r} = \frac{|v|_{opt,r}^2}{R_{lr}}$. The optimal resistance is $R_{lopt,r} \approx \frac{2\zeta_m}{k_e^2} \frac{1}{\omega_N C_p}$ using eq. (3.16). Substituting these into the optimal power equation defined above, eq. (3.18) is obtained, as expected.

A similar analysis can be completed at the anti-resonance frequency ratio, Ω_{ar} . The anti-resonance frequency is determined by the coupling term, as seen in eq. (3.14). Substituting this result into eq. (3.12), the following is obtained:

$$\alpha_{opt,ar}^2 = \frac{(1 + k_e^2)^2 - 2(1 + k_e^2) + 1 + 4\zeta_m^2(1 + k_e^2)}{4\zeta_m^2(1 + k_e^2)^2} \quad (3.19)$$

The last term in the numerator will be much smaller than the other terms. The result can be simplified to:

$$\alpha_{opt,ar} \approx \frac{k_e^2}{2\zeta_m(1 + k_e^2)} \quad (3.20)$$

Again, this result and $\Omega = \Omega_{ar}$ can be substituted into the power expression, eq. (3.11).

$$\left| \frac{P_{out}}{(M_T \ddot{w}_B)^2} \right|_{opt,ar} = \frac{\omega_N}{K} \frac{\frac{1}{2\zeta_m} k_e^4}{4k_e^4 + 4\zeta_m^2 (1 + k_e^2)} \quad (3.21)$$

The second term in the denominator will be much smaller than the first due to the small damping and eq. (3.21) reduces to eq. (3.18). This is a very surprising result. The piezoelectric coupling, represented by k_e^2 , cancels from the power equations at optimal conditions, both at resonance and anti-resonance (provided that $\zeta_m \ll k_e^2$). First, this would suggest that the power generated at the resonance and anti-resonance frequencies are equal (under optimal conditions). Next, the only material properties that would then influence the power generated are the piezoelectric element short-circuit stiffness, c_{33}^E , and the density (of the mass and the piezoelectric element). As with the resonant frequency voltage optimal, the voltage optimal at anti-resonance is a function of the piezoelectric coupling and $\alpha_{opt,ar}$ through $|v|_{opt,ar} = \frac{M_T \ddot{w}_B}{2|\theta|} \alpha_{opt,ar} = |v|_{opt,r} \alpha_{opt,ar}$. These results will be discussed in greater detail for the coupled beam model in Section 3.3.

3.2.3 1-D Model Results

It should be noted that the above model assumes linear behavior of the piezoelectric element over all frequencies. This is not the case for known piezoelectric materials; particularly the impedance ($R - C$) of the piezoelectric deviates significantly from non-resonant values at material resonance and anti-resonance [81, 82]. The power optimization and solutions here are relevant to understanding power optimums of structural elements (beams) with piezoelectric elements, but not prediction due to unmodeled effect.

Power is plotted against normalized frequency in Figure 3-3 for the parameters

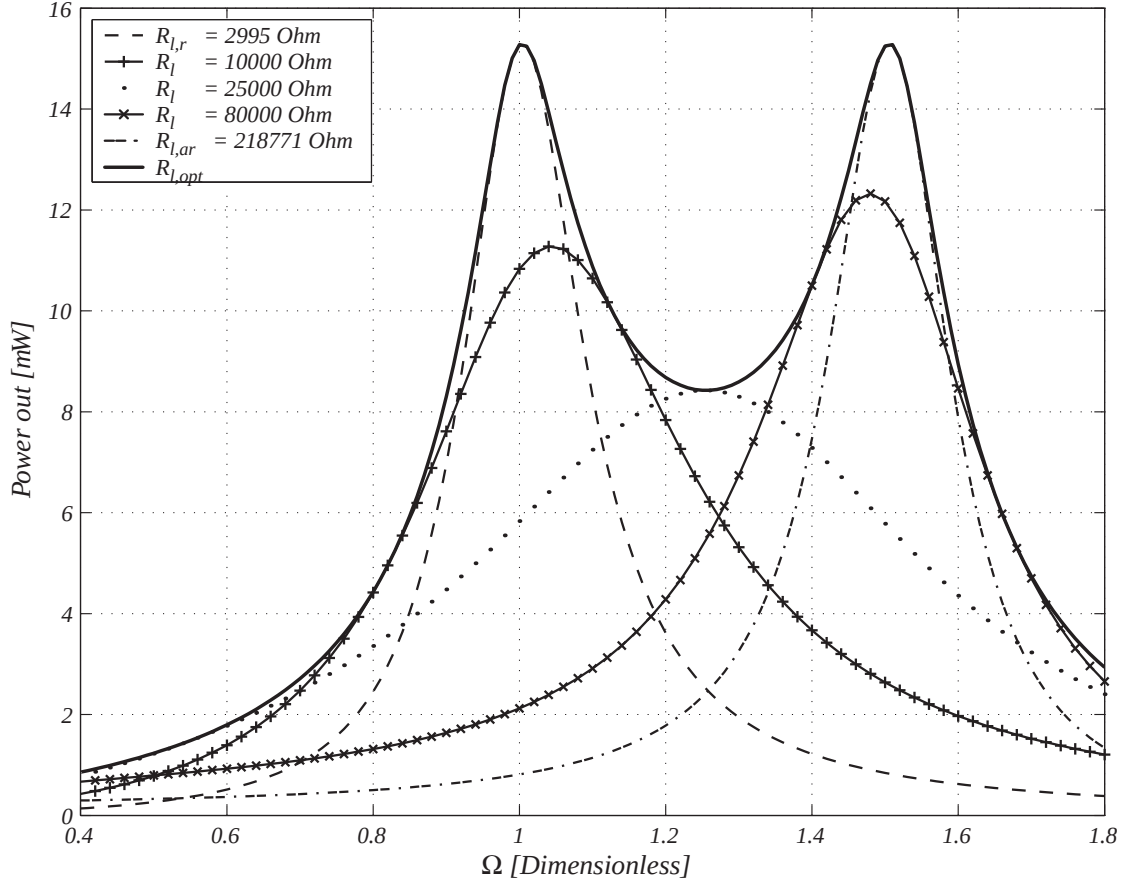


Figure 3-3: Power vs. normalized frequency with varying electrical resistance for 1-D model in Figure 3-2. R_l is the electrical load resistance, $R_{l,r}$ and $R_{l,ar}$ are the power optimized electrical loads at resonance and anti-resonance, respectively. The solid line is the optimized power (optimal electrical load, $R_{l,opt}$, at all frequencies).

given in Figure 3-2 with a base input acceleration of 9.81 m/s^2 , or 1 g . Note that $\Omega_{ar} \approx 1.5$ here. The solid line forms the envelope of maximum power possible if the electrical resistance (R_l) is optimized at each frequency ratio. Switching between the two peaks, corresponding to the resonance and anti-resonance frequencies, is achieved by varying the electrical load. The power increases as the resonance frequency is approached, and reaches a maximum, before decreasing to a local minimum at $\Omega \approx 1.25$. This local minimum corresponds to a local minimum in proof mass displacement. The power then increases to a second maximum, corresponding to the anti-resonance frequency. Note the sharpness of the tuned peaks around the two natural frequencies.

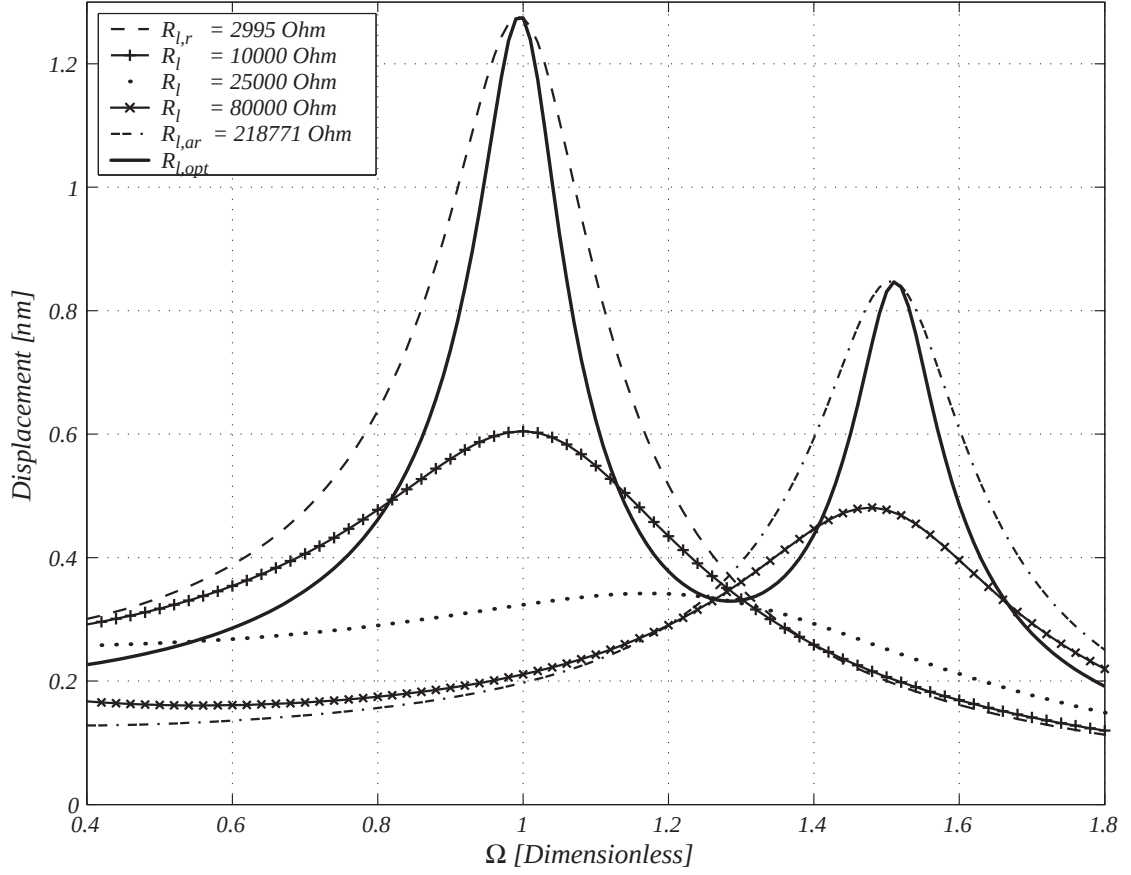


Figure 3-4: Displacement vs. normalized frequency with varying electrical resistance for 1-D model in Figure 3-2. R_l is the electrical load resistance, $R_{l,r}$ and $R_{l,ar}$ are the power optimized electrical loads at resonance and anti-resonance, respectively. The solid line is the optimized power (optimal electrical load, $R_{l,opt}$, at all frequencies).

A higher mechanical damping ratio would broaden, but also lessen, these peaks. While the extracted powers predicted at these peaks are equal, the voltage and current differ significantly.

Next, the relative displacements of the proof mass at the two peaks are compared, as shown in Figure 3-4. The electrical resistance is still optimized for maximum power. Unlike the power, the displacement is higher at resonance than at anti-resonance. Also note that the relative displacement of the proof mass is not locally minimized at either the resonance or the anti-resonance peak (where the power extracted is maximized), but at an intermediate position. Operating at the second peak could

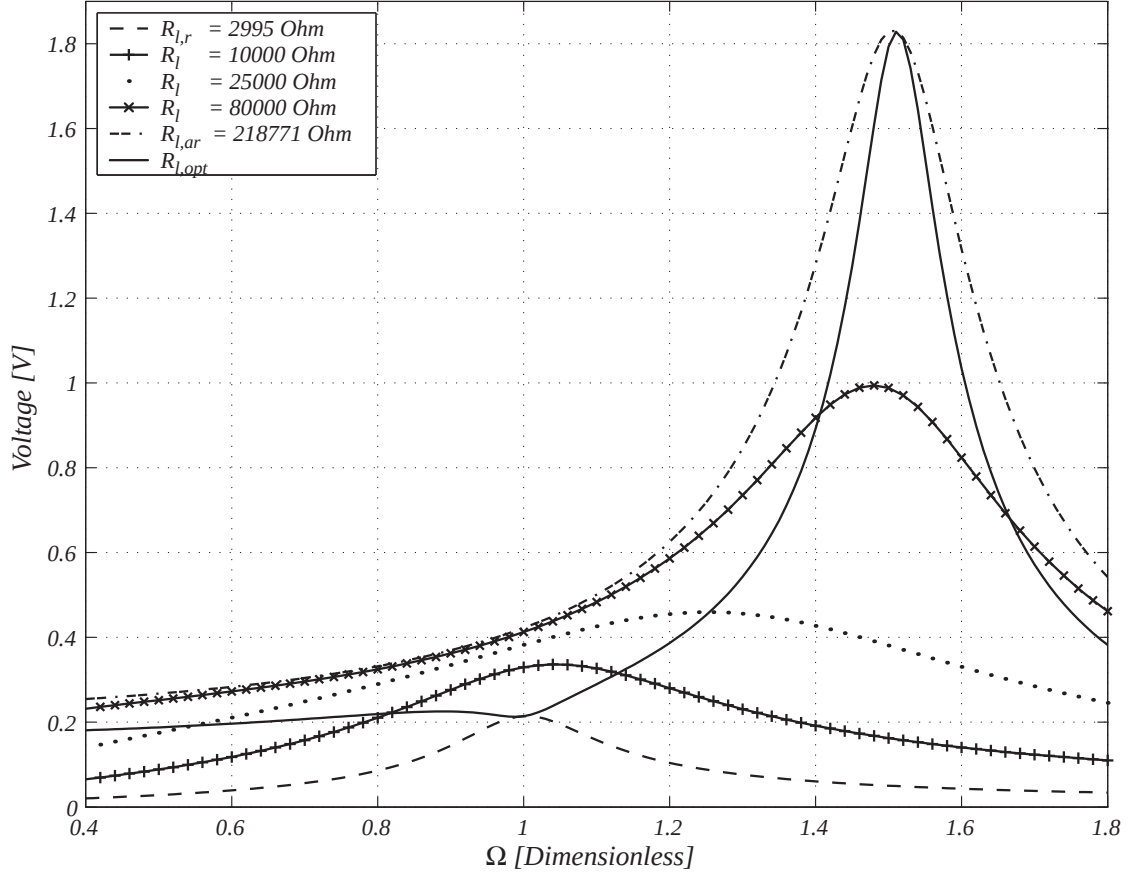


Figure 3-5: Voltage vs. normalized frequency with varying electrical resistance for 1-D model in Figure 3-2. R_l is the electrical load resistance, $R_{l,r}$ and $R_{l,ar}$ are the power optimized electrical loads at resonance and anti-resonance, respectively. The solid line is the optimized power (optimal electrical load, $R_{l,opt}$, at all frequencies).

be advantageous since the proof mass displacement will be smaller, allowing for a smaller device operating volume.

Voltage is plotted against normalized frequency in Figure 3-5. As with the displacement, the voltage generated at the two resonances differ, but in the case of the voltage the difference is around an order of magnitude. Since the power generated at these peaks are the same, and power is related to voltage and current, i , through $P_{out} = vi$ (for a resistive load), the current at the first peak will be an order of magnitude larger than at the second peak (inverse of voltage). The capability of a piezoelectric element to charge a secondary battery has been investigated by Sodano *et al.* [88]. From the study it was concluded that certain piezoelectric elements are

not well suited for battery charging applications, as generated current levels are too low. Operation at the resonance frequency could potentially alleviate the problem. On the other hand, the rectifier circuit has a minimum voltage requirement for operation, and the voltage requirement is also governed by the onset of losses at lower voltages [44]. Operating at the anti-resonance frequency could be advantageous in this case.

The power-optimal load resistance is plotted against frequency ratio in Figure 3-6. The shift from the resonance to anti-resonance frequencies is clearly observable. As mentioned, the magnitude of Ω_{ar} is governed by the piezoelectric coupling, k_e^2 . This coupling is a function of the stiffness contribution of the piezoelectric element to the device. In the 1-D model, the shift is most pronounced since the piezoelectric element constitutes the entire structure (and thus the stiffness). The optimal frequencies corresponding to the optimal electrical loads calculated for the resonance and anti-resonance frequencies do not perfectly align with the resonance and anti-resonance frequencies due to the finite mechanical damping in the system. For low damping, these higher peaks will be negligibly offset from the resonance and anti-resonance frequencies. In fact, for progressively higher damping cases, the maximum power peaks shift towards each other, and the two optimal-power operating points (resonance and anti-resonance for low damping) merge to a single peak at an intermediate frequency. For this analysis, it has been assumed that the damping is relatively low ($\zeta_m = 0.005$).

To summarize, as the electrical load is increased, the piezoelectric element operating condition moves from short-circuit to open-circuit. Since the piezoelectric element constitutes the whole structure, there is a significant shift in the resonance frequency. For beams/plates at the macro-scale, this effect is not as pronounced since the piezoelectric element generally does not contribute significantly to the overall structural stiffness (*e.g.*, for a piezoelectric patch applied to a structure for sensing). Also, macro-scale structures normally have more significant damping. Resonance and anti-resonance are very important at the micro-scale. The power generated at short- and open-circuit conditions are equal (for constant acceleration amplitude and mechanical damping), but the voltage and current developed at the different operating

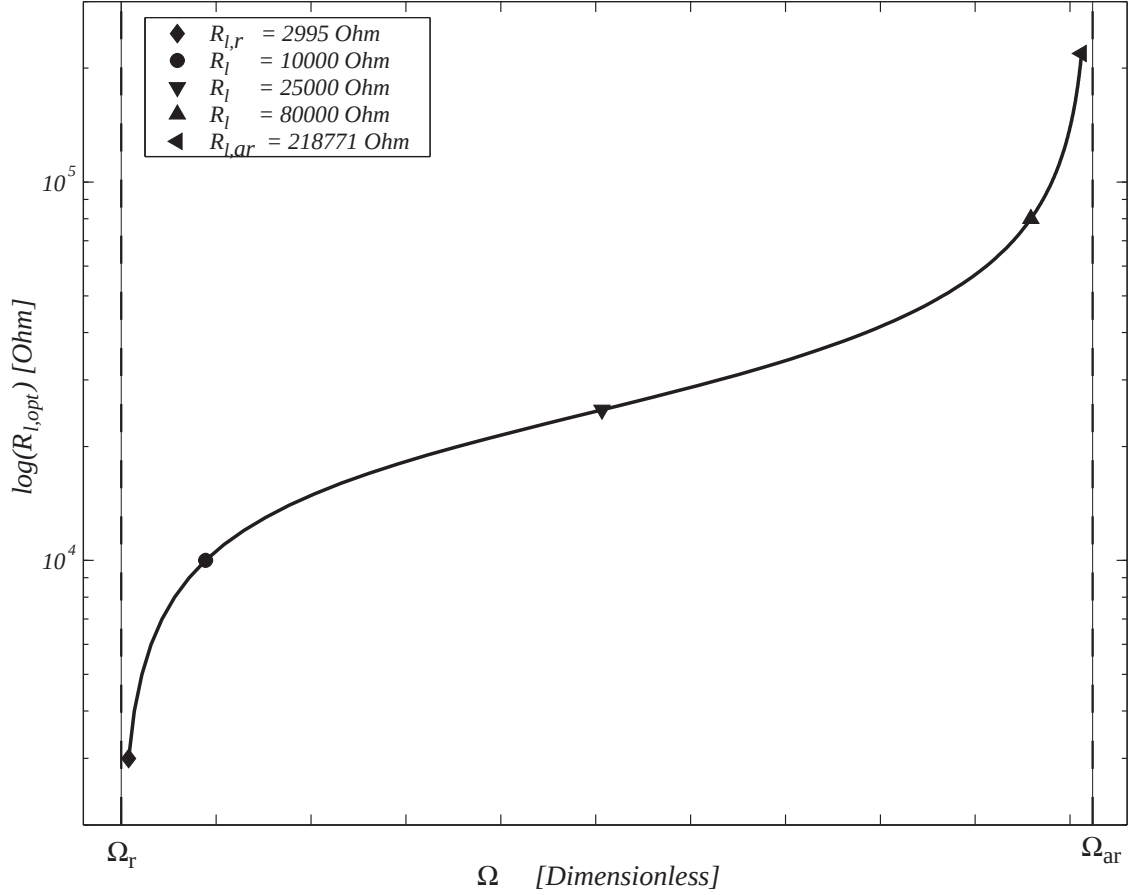


Figure 3-6: Power-optimal resistive load vs. normalized frequency with varying electrical resistance for 1-D model in Figure 3-2.

points differ substantially. The displacement of the proof mass of the device is also different at these operating points, as the system will have more damping at the open-circuit condition (due to higher electrical damping).

3.3 Coupled Beam Models

As stated earlier, the cantilever beam configuration was chosen for its geometric compatibility with the MEMS fabrication processes. It is also a relatively compliant structure, allowing for large strains and thus more power generation. The basic configurations of the uni-morph and bi-morph (for the {3-1} mode of operation) are illustrated in Figure 3-7 and each have the following components: the cantilevered

3.3.1 Modeling Cantilever Beams with Piezoelectric Elements

The model for a cantilever beam with piezoelectric elements can be obtained with an energy method approach. An alternative method, which will not be discussed in this paper, is a force equilibrium analysis [89]. The analysis to follow was adapted from Hagood *et al.* [85]. The generalized form of Hamilton's Principle for an electro-mechanical system, neglecting the magnetic terms and defining the kinetic (T_k), internal potential (U), and electrical (W_e) energies, as well as the external work (W), is given by:

$$\int_{t1}^{t2} [\delta (T_k - U + W_e) + \delta W] dt = 0 \quad (3.22)$$

δ indicates the first variation of the function, as used in Calculus of Variation. It is not in the scope of this thesis to explain Calculus of Variation, as many texts on the topic are available, for example [90]. The individual energy terms are defined as:

$$T_k = \frac{1}{2} \int_{V_s} \rho_s \dot{\mathbf{u}}^t \dot{\mathbf{u}} dV_s + \frac{1}{2} \int_{V_p} \rho_p \dot{\mathbf{u}}^t \dot{\mathbf{u}} dV_p \quad (3.23)$$

$$U = \frac{1}{2} \int_{V_s} \mathbf{S}^t \mathbf{T} dV_s + \frac{1}{2} \int_{V_p} \mathbf{S}^t \mathbf{T} dV_p \quad (3.24)$$

$$W_e = \frac{1}{2} \int_{V_p} \mathbf{E}^t \mathbf{D} dV_p \quad (3.25)$$

The subscripts p and s indicate the piezoelectric element section and the structural (inactive) section of the beam, respectively. Electrodes, each with thickness t_e , if significant to the stiffness of the structure, can be included in the structural section of the beam. V_p and V_s are volumes. The relative displacement is denoted by $\mathbf{u}(\mathbf{x}, t)$ and ρ is the density. The contributions to W_e due to fringing fields in the structure and free space are neglected. nf discretely applied external point forces, $\mathbf{f}_k(t)$ at positions $\mathbf{x}_k$, and nq charges, q_j , extracted at discrete electrodes with positions $\mathbf{x}_j$ are considered. The external work term is defined in terms of the relative displacement,

$\mathbf{u}(\mathbf{x}_k, t)$, and the scalar electrical potential, $\varphi_j = \varphi(\mathbf{x}_j, t)$:

$$\delta W = \sum_{k=1}^{nf} \delta \mathbf{u}_k \mathbf{f}_k(t) + \sum_{j=1}^{nq} \delta \varphi_j q_j \quad (3.26)$$

The above definitions, as well as the small-signal constitutive relations of a piezoelectric material (see eq. (3.1)), are used in conjunction with a Calculus of Variation approach to rewrite eq. (3.22). It should be noted that it has been assumed that the piezoelectric element local coordinates coincide with the global coordinates. The local coordinates are defined by the poling direction, so according to this assumption, the global coordinates are defined to align with the local coordinates and the constitutive law as given in eq. (3.1) holds. Please refer to Appendix B for a scheme to account for the inversion of the local piezoelectric element coordinates (negative poling).

$$\begin{aligned} \int_{t1}^{t2} \left(\int_{V_s} \rho_s \delta \dot{\mathbf{u}}^t \dot{\mathbf{u}} dV_s + \int_{V_p} \rho_p \delta \dot{\mathbf{u}}^t \dot{\mathbf{u}} dV_p - \int_{V_s} \delta \mathbf{S}^t \mathbf{c}_s \mathbf{S} dV_s - \int_{V_p} \delta \mathbf{S}^t \mathbf{c}^E \mathbf{S} dV_p + \int_{V_p} \delta \mathbf{S}^t \mathbf{e}^t \mathbf{E} dV_p + \right. \\ \left. \int_{V_p} \delta \mathbf{E}^t \mathbf{e} \mathbf{S} dV_p + \int_{V_p} \delta \mathbf{E}^t \boldsymbol{\epsilon}^S \mathbf{E} dV_p + \sum_{k=1}^{nf} \delta \mathbf{u}_k \mathbf{f}_k(t) + \sum_{j=1}^{nq} \delta \varphi_j q_j \right) dt = 0 \end{aligned} \quad (3.27)$$

Three basic assumptions are introduced: the Raleigh-Ritz procedure, Euler-Bernoulli beam theory, and constant electrical field across the piezoelectric element. These assumptions are consistent with previous modeling efforts (*e.g.*, [55, 56, 85]). In the Raleigh-Ritz approach, the displacement of a structure is written as the sum of nr individual modes shapes, $\psi_{ri}(\mathbf{x})$, multiplied by a generalized mechanical coordinate, $r_i(t)$ [91]. For a beam in bending, only the transverse displacement (x_t direction) is considered and the mode shape is a function only of the axial position, $\mathbf{u}(\mathbf{x}, t) = \mathbf{z}(x_a, t)$, as in eq. (3.28). Here, $\mathbf{z}(x_a, t)$ is the beam displacement relative to the base of the beam. Furthermore, the base excitation, w_B , is assumed to be in the transverse direction as well. Lastly, the electric potential for each of the nq electrode pairs can be written in terms a potential distribution, ψ_{vj} , and the generalized electrical

coordinate, $v_j(t)$, as in eq. (3.29). It is convenient to let the generalized electrical coordinates coincide with physical voltages, though this is not required [85]. The Euler-Bernoulli beam theory allows the axial strain in the beam to be written in terms of the beam neutral axis displacement and the distance from the neutral axis (x_t), as in eq. (3.30).

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{z}(x_a, t) = \sum_{i=1}^{nr} \psi_{ri}(x_a) r_i(t) = \boldsymbol{\psi}_r(x_a) \mathbf{r}(t) \quad (3.28)$$

$$\varphi(\mathbf{x}, t) = \sum_{j=1}^{nq} \psi_{vj}(\mathbf{x}) v_j(t) = \boldsymbol{\psi}_v(\mathbf{x}) \mathbf{v}(t) \quad (3.29)$$

$$\mathbf{S}(\mathbf{x}, t) = -x_t \frac{\partial^2 \mathbf{z}(x_a, t)}{\partial x_a^2} = -x_t \boldsymbol{\psi}_r'' \mathbf{r}(t) \quad (3.30)$$

Primes indicates the derivative with respect to the axial position, x_a . Next, the above definitions are substituted into eq. (3.27) to yield:

$$\begin{aligned} \int_{t1}^{t2} \left(\int_{V_s} \delta \dot{\mathbf{r}}^t \boldsymbol{\psi}_r^t \rho_s \boldsymbol{\psi}_r \dot{\mathbf{r}} dV_s + \int_{V_p} \delta \dot{\mathbf{r}}^t \boldsymbol{\psi}_r^t \rho_p \boldsymbol{\psi}_r \dot{\mathbf{r}} dV_p - \int_{V_s} \delta \mathbf{r}^t (-x_t \boldsymbol{\psi}_r'')^t \mathbf{c}_s (-x_t \boldsymbol{\psi}_r'') \mathbf{r} dV_s - \right. \\ \left. \int_{V_p} \delta \mathbf{r}^t (-x_t \boldsymbol{\psi}_r'')^t \mathbf{c}^E (-x_t \boldsymbol{\psi}_r'') \mathbf{r} dV_p + \int_{V_p} \delta \mathbf{r}^t (-x_t \boldsymbol{\psi}_r'')^t \mathbf{e}^t (-\nabla \boldsymbol{\psi}_v) \mathbf{v} dV_p + \right. \\ \left. \int_{V_p} \delta \mathbf{v}^t (-\nabla \boldsymbol{\psi}_v)^t \mathbf{e} (-x_t \boldsymbol{\psi}_r'') \mathbf{r} dV_p + \int_{V_p} \delta \mathbf{v}^t (-\nabla \boldsymbol{\psi}_v)^t \boldsymbol{\varepsilon}^S (-\nabla \boldsymbol{\psi}_v)^t \mathbf{v} dV_p + \right. \\ \left. \sum_{i=1}^{nf} \delta \mathbf{r}_k^t \boldsymbol{\psi}_{r,k}^t \mathbf{f}_k + \sum_{j=1}^{nq} \delta \mathbf{v}_j^t \boldsymbol{\psi}_{v,j}^t q_j \right) dt = 0 \end{aligned} \quad (3.31)$$

Using integration by parts, and noting that the variations must disappear at the boundaries, the first two terms can be rewritten as $\int_V \delta \dot{\mathbf{r}}^t \rho \boldsymbol{\psi}_r^t \boldsymbol{\psi}_r \dot{\mathbf{r}} dV = - \int_V \delta \mathbf{r}^t \boldsymbol{\psi}_r^t \rho \boldsymbol{\psi}_r \ddot{\mathbf{r}} dV_s$. Next, the terms are grouped and each variation (*e.g.*, $\delta \mathbf{r}^t$ and $\delta \mathbf{v}^t$), tend to zero sep-

arately. Two governing equations are obtained:

$$\begin{aligned}
& - \int_{V_s} \psi_r^t \rho_s \psi_r \ddot{\mathbf{r}} dV_s - \int_{V_p} \psi_r^t \rho_p \psi_r \ddot{\mathbf{r}} dV_p - \int_{V_s} (-x_t \psi_r'')^t \mathbf{c}_s (-x_t \psi_r'') \mathbf{r} dV_s - \\
& \int_{V_p} (-x_t \psi_r'')^t \mathbf{c}^E (-x_t \psi_r'') \mathbf{r} dV_p + \int_{V_p} (-\nabla \psi_v)^t \mathbf{e} (-x_t \psi_r'') \mathbf{r} dV_p + \sum_{i=1}^{nf} \psi_{r,k}^t \mathbf{f}_k = 0
\end{aligned} \tag{3.32}$$

$$\int_{V_p} (-x_t \psi_r'')^t \mathbf{e}^t (-\nabla \psi_v) \mathbf{v} dV_p + \int_{V_p} (-\nabla \psi_v)^t \boldsymbol{\varepsilon}^S (-\nabla \psi_v) \mathbf{v} dV_p + \sum_{j=1}^{nq} \psi_{v,j} q_j = 0 \tag{3.33}$$

From these equations, the mass ($\mathbf{M}$), stiffness ($\mathbf{K}$), coupling ($\boldsymbol{\Theta}$), and capacitive matrices ($\mathbf{C}_p$) are defined. Please refer to Appendix B for a detailed analysis of the relationship between poling direction, piezoelectric constants, and applied and developed electric fields.

$$\mathbf{M} = \int_{V_s} \psi_r^t \rho_s \psi_r dV_s + \int_{V_p} \psi_r^t \rho_p \psi_r dV_p \tag{3.34}$$

$$\mathbf{K} = \int_{V_s} (-x_t \psi_r'')^t \mathbf{c}_s (-x_t \psi_r'') dV_s + \int_{V_p} (-x_t \psi_r'')^t \mathbf{c}^E (-x_t \psi_r'') dV_p \tag{3.35}$$

$$\boldsymbol{\Theta} = \int_{V_p} (-x_t \psi_r'')^t \mathbf{e}^t (-\nabla \psi_v) dV_p \tag{3.36}$$

$$\mathbf{C}_p = \int_{V_p} (-\nabla \psi_v)^t \boldsymbol{\varepsilon}^S (-\nabla \psi_v) dV_p \tag{3.37}$$

The governing equations, given in eqs. (3.38) and (3.39), are obtained:

$$\mathbf{M} \ddot{\mathbf{r}} + \mathbf{K} \mathbf{r} - \boldsymbol{\Theta} \mathbf{v} = \sum_{k=1}^{nf} \psi_r^t(x_k) \mathbf{f}_k(t) \tag{3.38}$$

$$\boldsymbol{\Theta}^t \mathbf{r} + \mathbf{C}_p \mathbf{v} = - \sum_{j=1}^{nq} \psi_v(x_j) q_j(t) \tag{3.39}$$

The input to the system is a base excitation. To represent the beam's inertial load from this excitation, the structure is discretized into nf elements of length Δx_a and the local inertial load is applied on the k^{th} element, or $f_k = -m_k \Delta x_a \ddot{w}_B$. This results in nf discrete loads. m_k is the element mass per length. The loading is summed for all the elements. In the limit of $\Delta x_a \rightarrow dx_a$, the summation reduces to the integral over the structure length and a mass per length distribution is used, $m(x_a)$. For simplicity, it has been assumed here that the beam is of uniform cross-section in the axial direction so that $m(x_a) = m = \text{constant}$. Substituting the forcing function into the right hand side of eq. (3.38), the “forcing vector”, $\mathbf{B}_f$, is defined.

$$\mathbf{B}_f = \int_0^L m(x_a) \boldsymbol{\psi}_r^t dx_a = m \int_0^L \boldsymbol{\psi}_r^t dx_a \quad (3.40)$$

The right hand side of eq. (3.39) reduces to a column vector, $\mathbf{q}$, of length nq (the number of electrode pairs) with element values q_T ($q_T = \sum_{j=1}^{nq} q_j$). This equation can be differentiated with respect to time to obtain current. The current can be related to the voltage, assuming that the electrical loading is purely resistive, R_l .

Mechanical damping is added through the addition of a viscous damping term, $\mathbf{C}$, to eq. (3.38) to obtain eq. (3.41). When multiple bending modes are investigated, a proportional damping scheme is often used to ensure uncoupling of the equations during the modal analysis [91]. The damping for a physical device is determined by the geometry and the operating environment, as discussed in Section 2.3. Damping is typically measured at the device natural frequency, which is fixed. As a consequence, the damping dependence on frequency need not be considered here, if it can be assumed that the damping ratios at the resonance and anti-resonance frequencies are the same. The following results:

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} - \boldsymbol{\Theta}\mathbf{v} = -\mathbf{B}_f\ddot{w}_B \quad (3.41)$$

$$\boldsymbol{\Theta}^t \mathbf{r} + \mathbf{C}_p \mathbf{v} + \mathbf{q} = 0 \quad (3.42)$$

It is important to note the similarities between the governing equations for the vibrating beam eqs. (3.41) and (3.42) and the basic 1-D device eqs. (3.6) and (3.7). Considering one beam mode and a single electrode pair, eqs. (3.41) and (3.42) reduce to scalar equations that allows for a beam power optimization to be performed, directly analogous to the 1-D device derived in Section 3.2.2. The actuator equation, eq. (3.41), is written in an alternative form by dividing through by M and making use of the definitions for the first resonance frequency, $\omega_1 = \sqrt{\frac{K}{M}}$, and damping ratio $\zeta_m = \frac{C}{2M\omega_1}$. Note that since one vibration mode ($\psi_{r,1}$) is considered, the system will have a single natural frequency. For a piezoelectric structure, the natural frequency will correspond to either the resonance or anti-resonance frequency, depending on the electrical loading. $\omega_N = \omega_1$ corresponds to the resonance frequency. In the sensing equation (eq. (3.42)), the charge can be related to the voltage through $v = \frac{dq}{dt} R_l = i R_l$ to obtain:

$$\ddot{r} + 2\zeta_m\omega_1\dot{r} + \omega_1^2 r - \theta/Mv = -B_f\ddot{w}_B/M \quad (3.43)$$

$$\theta\dot{r} + C_p\dot{v} + \frac{1}{R_l}v = 0 \quad (3.44)$$

The dimensionless factors $\alpha = \omega_1 R_l C_p$, $\kappa^2 = \frac{\theta^2}{K C_p}$, and $\Omega = \frac{\omega}{\omega_1}$ are defined, where ω is the base input frequency, and the system response is calculated. α is again the dimensionless time constant and κ^2 is a structure/system electromechanical coupling coefficient, vs. $k_{33}^2 = \frac{d_{33}^2}{s_{33}^E \epsilon_{33}^T}$ or $k_e^2 = \frac{e_{33}^2}{c_{33}^E \epsilon_{33}^S}$, which are piezoelectric material coupling coefficients (defined in Section 3.2.1). The resulting equations are similar to those derived for the basic 1-D piezoelectric model (see eqs. (3.9) – (3.11)):

$$\left| \frac{r}{B_f\ddot{w}_B} \right| = \frac{1}{K} \frac{\sqrt{1 + (\alpha\Omega)^2}}{\sqrt{[1 - (1 + 2\zeta_m\alpha)\Omega^2]^2 + [(2\zeta_m + \{1 + \kappa^2\}\alpha)\Omega - \alpha\Omega^3]^2}} \quad (3.45)$$

$$\left| \frac{v}{B_f\ddot{w}_B} \right| = \frac{1}{|\theta|} \frac{\alpha\kappa^2\Omega}{\sqrt{[1 - (1 + 2\zeta_m\alpha)\Omega^2]^2 + [(2\zeta_m + \{1 + \kappa^2\}\alpha)\Omega - \alpha\Omega^3]^2}} \quad (3.46)$$

$$\left| \frac{P_{out}}{(B_f\ddot{w}_B)^2} \right| = \frac{\omega_1}{K} \frac{\alpha\kappa^2\Omega^2}{[1 - (1 + 2\zeta_m\alpha)\Omega^2]^2 + [(2\zeta_m + \{1 + \kappa^2\}\alpha)\Omega - \alpha\Omega^3]^2} \quad (3.47)$$

One difference is that eq. (3.45) gives the generalized mechanical displacement, which should be converted to actual displacements by multiplying it with the mode shape (according to eq. (3.28)). The mode shape is defined in Section 3.3.3 as seen in eq. (3.67). It should be noted that the mode shape is normalized to a value of two at the tip.

3.3.2 Power Optimization

Again, the system can be analyzed at short- and open- circuit conditions by letting the mechanical damping tend to zero, with the load resistance tending to zero and infinity, respectively. Two frequency ratios are obtained, which correspond to the resonance and anti-resonance frequencies of the beam structure:

$$\Omega_r = 1 \quad (3.48)$$

$$\Omega_{ar} = \sqrt{1 + \kappa^2} \quad (3.49)$$

It is of interest to note again the system coupling factor, defined by others as $k_{sys}^2 = \frac{\Omega_{ar}^2 - \Omega_r^2}{\Omega_{ar}^2}$, [57, 84]. Substituting eqs. (3.48) and (3.49) into the system coupling, it reduces to $k_{sys}^2 = \frac{\kappa^2}{1 + \kappa^2}$, or $\kappa^2 = \Omega_{ar} - 1 = \frac{\omega_{ar} - \omega_r}{\omega_{ar}}$. In the 1-D case (with the entire structure consisting of piezoelectric), the system coupling, k_{sys} reduced to the piezoelectric material coupling factor, k_{33}^2 (see Section 3.2.1). For the beam/plate structure, this relation is not as simple since the structure is partially inactive (not piezoelectric). Hence, the system coupling depends on both the material coupling and the device geometry.

The power can be optimized with respect to load resistance to obtain an optimal electrical load:

$$\alpha_{opt}^2 = \frac{\Omega^4 + (4\zeta_m^2 - 2)\Omega^2 + 1}{\Omega^6 + (4\zeta_m^2 - 2(1 + \kappa^2))\Omega^4 + (1 + \kappa)^2\Omega^2} = \frac{1}{\Omega^2} \frac{(1 - \Omega^2)^2 + (2\zeta_m\Omega)^2}{([1 + \kappa^2] - \Omega^2)^2 + (2\zeta_m\Omega)^2} \quad (3.50)$$

Comparing these results to Section 3.2, we identify similar behavior for the cantilever configuration as for the basic 1-D configuration. The anti-resonance frequency

is determined by the coupling term $\kappa^2 = \frac{\theta^2}{KC_p}$. Since the structure is made up of both piezoelectric- and non-piezoelectric layers, this term does not correspond to the material coupling coefficient.

Again, it is possible to calculate the optimal electrical loading at both resonance and anti-resonance (see Section 3.2.2). For resonance:

$$\alpha_{opt,r}^2 = \frac{4\zeta_m^2}{4\zeta_m^2 + \kappa^4} \quad (3.51)$$

In general, ζ_m is at least an order of magnitude smaller than κ^2 , thus this result is approximated very well by:

$$\alpha_{opt,r} \approx \frac{2\zeta_m}{\kappa^2} \quad (3.52)$$

This result, with the corresponding frequency ratio, is substituted into eq. (3.47) to obtain:

$$\left| \frac{P_{out}}{(B_f \ddot{w}_B)^2} \right|_{opt,r} = \frac{\omega_1}{K} \frac{2\zeta_m}{16\zeta_m^2 \left(\frac{\zeta_m^2}{\kappa^2} + 1 \right)} \quad (3.53)$$

For the term in the denominator, $\frac{\zeta_m^2}{\kappa^2} \ll 1$, such that the result reduces to eq. (3.54):

$$\left| \frac{P_{out}}{(B_f \ddot{w}_B)^2} \right|_{opt,r} = \frac{\omega_1}{K} \frac{1}{8\zeta_m} \quad \text{or} \quad |P_{out}|_{opt,r} = \frac{B_f^2}{\sqrt{KM}} \frac{\ddot{w}_B^2}{8\zeta_m} \quad (3.54)$$

The analysis can be repeated at the anti-resonance frequency. Substituting eq. (3.49) into eq. (3.50), the following is obtained:

$$\alpha_{opt,ar}^2 = \frac{(1 + \kappa^2)^2 - 2(1 + \kappa^2) + 1 + 4\zeta_m^2(1 + \kappa^2)}{4\zeta_m^2(1 + \kappa^2)^2} \quad (3.55)$$

The last term in the numerator will be much smaller than the other terms and the result simplifies to:

$$\alpha_{opt,ar} \approx \frac{\kappa^2}{2\zeta_m(1 + \kappa^2)} \quad (3.56)$$

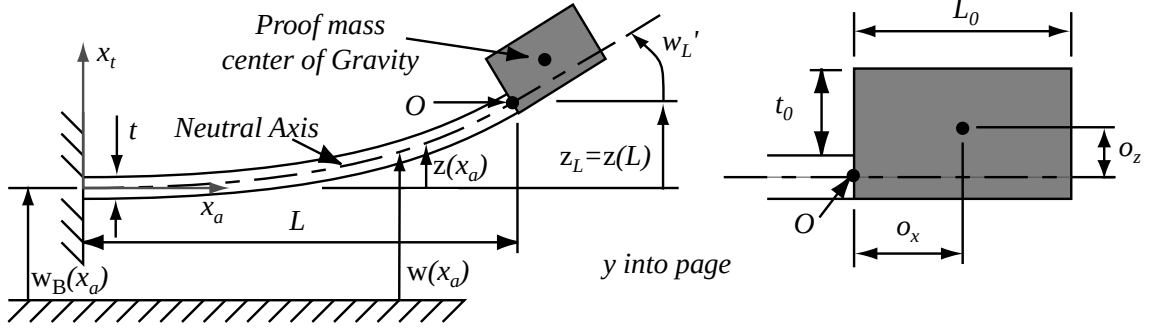


Figure 3-8: Base-excited cantilever beam with tip mass.

This result is substituted into the power equation, eq. (3.47), giving:

$$\left| \frac{P_{out}}{(B_f \ddot{w}_B)^2} \right|_{opt,ar} = \frac{\omega_1}{K} \frac{\frac{1}{2\zeta_m} \kappa^4}{4\kappa^4 + 4\zeta_m^2 (1 + \kappa^2)} \quad (3.57)$$

The second term in the denominator will be much smaller than the first due to the small damping and eq. (3.57) reduces to eq. (3.54). This result is discussed in more detail in Section 3.3.4.

3.3.3 Modal Analysis: Cantilever Beam with a Mass at the Free End

Since the target frequencies of the piezoelectric energy harvester under investigation are very low for MEMS-scale devices, it will be necessary to add a mass at the tip of the cantilever beam. The modal shapes and natural frequencies for a fixed-free cantilever beam are readily available in vibration texts (*e.g.*, [91]), but the analysis with the addition of the mass is not as common, and will be covered briefly. This section is adapted from [92, 93, 94].

Please refer to Figure 3-8 for an illustration of the assumed beam configuration. First, it is not assumed that the center of gravity of the mass coincides with the point of loading on the beam, O . This is the general more case. Euler-Bernoulli beam theory is used to determine the governing equations in terms of the mechanical displacement, ψ_r in eq. (3.58), and can be solved generally for the N^{th} mode, $N \in (1, nr)$ in eq.

(3.59):

$$(cI)_e \psi_{rN}^{IV} - m\omega_N^2 \psi_{rN} = 0 \quad (3.58)$$

$$\psi_{rN} = c \sinh \lambda_N x_a + d \cosh \lambda_N x_a + e \sin \lambda_N x_a + f \cos \lambda_N x_a \quad (3.59)$$

$(cI)_e$ is the effective bending stiffness of the beam. The parameter, $\left(\frac{\lambda_N}{L}\right)^2 = \frac{m\omega^2}{(cI)_e}$ has been defined for convenience. The constants (c , d , e , and f) are solved by enforcing the boundary conditions of the beam with the mass. It is assumed that both the beam and the proof mass are uniform in the axial direction with masses per length of m and m_0 , respectively. Also, it is assumed that the stiffness of the mass is much higher than that of the beam (rigid mass). Using energy methods, it is possible to determine the boundary conditions at the point where the beam and the mass are connected, z_L :

$$(cI)_e z_L'' - \omega_N^2 J_0 z_L' - \omega_N^2 S_0 z_L = 0 \quad (3.60)$$

$$(cI)_e z_L''' + \omega_N^2 M_0 z_L + \omega_N^2 S_0 z_L' = 0 \quad (3.61)$$

where: $M_0 = m_0 L_0$, $S_0 = M_0 o_x$, $J_0 = J_{yy} + M_0(o_x^2 + o_z^2)$. J_{yy} is the mass moment of inertia of the proof mass around its center of gravity, and ω_N is the modal frequency of the beam for the N^{th} mode. By defining $\bar{\lambda}_N = \lambda_N L$, $\bar{M}_0 = \frac{M_0}{mL}$, $\bar{S}_0 = \frac{S_0}{mL^2}$, and $\bar{J}_0 = \frac{J_0}{mL^3}$, the boundary conditions are used to obtain the matrix equation of the constants:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} = 0 \quad (3.62)$$

$$A_{11} = (\sinh \bar{\lambda}_N + \sin \bar{\lambda}_N) + \bar{\lambda}_N^3 \bar{J}_0 (-\cosh \bar{\lambda}_N + \cos \bar{\lambda}_N) + \bar{\lambda}_N^2 \bar{S}_0 (-\sinh \bar{\lambda}_N + \sin \bar{\lambda}_N) \quad (3.63)$$

$$A_{12} = (\cosh \bar{\lambda}_N + \cos \bar{\lambda}_N) + \bar{\lambda}_N^3 \bar{J}_0 (-\sinh \bar{\lambda}_N - \sin \bar{\lambda}_N) + \bar{\lambda}_N^2 \bar{S}_0 (-\cosh \bar{\lambda}_N + \cos \bar{\lambda}_N) \quad (3.64)$$

$$A_{21} = (\cosh \bar{\lambda}_N + \cos \bar{\lambda}_N) + \bar{\lambda}_N \bar{M}_0 (\sinh \bar{\lambda}_N - \sin \bar{\lambda}_N) + \bar{\lambda}_N^2 \bar{S}_0 (\cosh \bar{\lambda}_N - \cos \bar{\lambda}_N) \quad (3.65)$$

$$A_{22} = (\sinh \bar{\lambda}_N - \sin \bar{\lambda}_N) + \bar{\lambda}_N \bar{M}_0 (\cosh \bar{\lambda}_N - \cos \bar{\lambda}_N) + \bar{\lambda}_N^2 \bar{S}_0 (\sinh \bar{\lambda}_N + \sin \bar{\lambda}_N) \quad (3.66)$$

The mode resonance frequencies are obtained by solving for $\bar{\lambda}_N$ such that $\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} =$

0. Successive values of $\bar{\lambda}_N$ correspond to the modes of the beam and the natural frequency of each mode can be determined with: $\omega_N^2 = \bar{\lambda}_N^2 \sqrt{\frac{(cI)_e}{mL^4}}$. The solution, eq. (3.59), can be written in terms of a single arbitrary constant, say f :

$$\psi_{rN} = f \left[(\cosh \lambda_N x_a - \cos \lambda_N x_a) - \frac{A_{12}}{A_{11}} (\sinh \lambda_N x_a - \sin \lambda_N x_a) \right] \quad (3.67)$$

Eq. (3.67) gives the general bending mode shape for a clamped beam, and is normalized to $\psi_{rN}(L) = 2$ at the beam tip. The effective mass of the structure is obtained from the Lagrange equations of motion and is given in eq. (3.68). Note that eq. (3.68) replaces eq. (3.34) when a proof mass is added to the cantilever structure.

$$\begin{aligned} \mathbf{M} = & \int_{V_s} \psi_r^t \rho_s \psi_r dV_s + \int_{V_p} \psi_r^t \rho_p \psi_r dV_p + M_0 (\psi_r(L))^t \psi_r(L) + \\ & 2S_0 (\psi_r(L))^t \psi_r'(L) + I_0 (\psi_r'(L))^t \psi_r'(L) \end{aligned} \quad (3.68)$$

Lastly, the external work term needs to be re-evaluated to include the inertial loading due to the proof mass at the beam tip. In eq. (3.40) the forcing vector, $\mathbf{B}_f$, was defined to account for the inertial loading due to a base excitation. It was previously assumed (for simplicity) that the device is of uniform cross-section in the axial direction. However, the device now consists of two distinct sections, the uniform beam and uniform proof mass. Both sections contribute to the inertial loading of the device. The proof mass displacement is calculated in terms of the displacement and rotation of the tip of the beam. The forcing function definition is extended to account for the proof mass by including two additional terms in the forcing vector (last two integrals in eq. (3.69)):

$$\mathbf{B}_f = m \int_0^L (\psi_r(x_a))^t dx_a + m_0 (\psi_r(L))^t \int_L^{L+L_0} dx_a + m_0 (\psi_r'(L))^t \int_L^{L+L_0} x_a dx_a \quad (3.69)$$

Eq. (3.69) replaces eq. (3.40) when utilizing a proof mass.

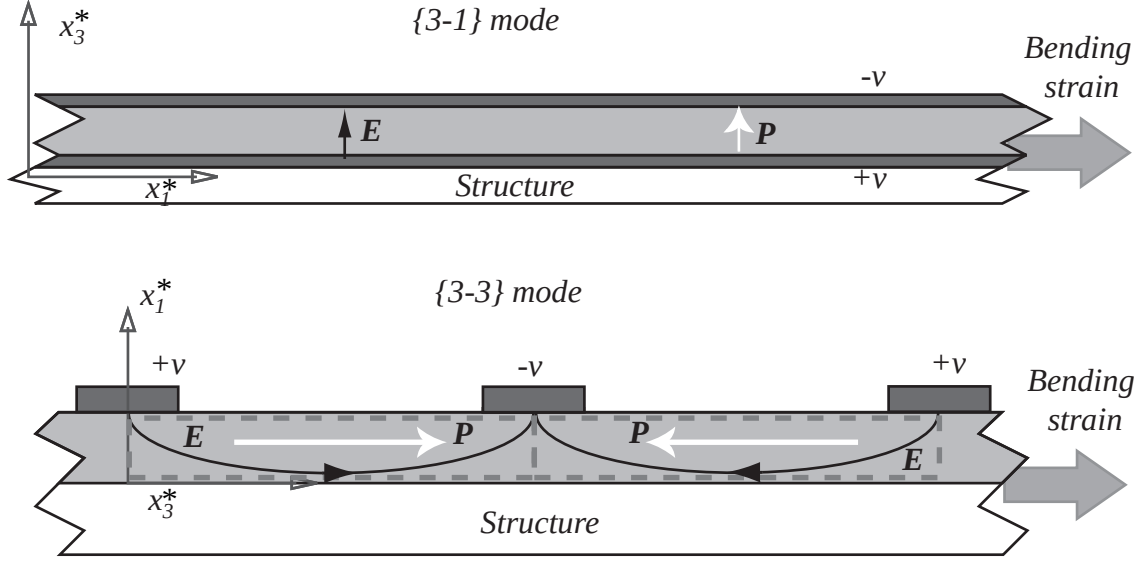


Figure 3-9: {3-1} (*top*) vs. {3-3} (*bottom*) mode of operation. See Appendix B for the definition of the local piezoelectric coordinates (x_1^* , x_3^*).

3.3.4 { 3-1 } vs. {3-3} Operation Modes

For piezoelectric elements, the longitudinal ({3-3}) piezoelectric effect can be much larger than the transverse ({3-1}) effect ($d_{33}/d_{31} \sim 2.4$ for most piezoelectric ceramics [95, 96, 97]). For in-plane actuation, it is desirable to operate the device in the {3-3} mode. The {3-3} mode is used when the electric field and the strain direction coincide. Refer to Figure 3-9 for an illustration of the two configurations. Due to the larger coupling in the longitudinal mode, it has been assumed that this mode will be superior to the transverse mode of operation for vibration energy harvesting [39, 87].

In Section 3.3.2, the power was analyzed under optimal conditions (*e.g.*, at either resonance or anti-resonance and with the corresponding optimal electrical loadings). Eq. (3.54) is obtained (which is repeated below for convenience) at both the resonance and anti-resonance frequencies:

$$\left| \frac{P_{out}}{(B_f \ddot{w}_B)^2} \right|_{opt} = \frac{\omega_1}{K} \frac{1}{8\zeta_m} \quad \text{or} \quad |P_{out}|_{opt} = \frac{B_f^2}{\sqrt{KM}} \frac{\ddot{w}_B^2}{8\zeta_m} \quad (3.70)$$

There are two operating points at which the same maximum power can be ex-

tracted (assuming the base acceleration and mechanical damping are the same). More interestingly, the electromechanical coupling, κ , cancels out of the optimal power equations. This means that the superior piezoelectric coupling factor for the {3-3} mode of operation is not a sufficient justification for the use of this configuration. In fact, the only material properties that affect the maximum power generated are the structural stiffness, and the density. The power is proportional to the density to the power $\frac{3}{2}$, and inversely proportional to the square root of the stiffness, so that the density should be maximized and the stiffness minimized to optimize the power generated. The density is constant, irrespective of the orientation of the element (*e.g.*, relative to the global coordinates). The stiffness for the piezoelectric element will be influenced by the orientation/poling of the material. For the {3-1} configuration, the bending stiffness of a beam structure is linear in c_{11}^E . Conversely, the bending stiffness is given by c_{33}^E for the {3-3} mode configuration. In general, $c_{11}^E > c_{33}^E$, though the difference is relatively small ($\frac{c_{11}^E}{c_{33}^E} = 1.077$ for PZT-5A). This would suggest that the {3-3} mode of operation is marginally better, considering the power only.

The voltage and current developed at the power optimum is not independent of the coupling. In Section 3.2.2 it was shown that, at the resonance frequency, $|v|_{opt,r} = \frac{M_T \ddot{w}_B}{2|\theta|}$ (below eq. (3.18)). Thus, material selection (particularly piezoelectric coupling) will significantly affect the voltage and current developed.

A more pertinent consideration for devices at the micro-scale is the complete device geometry. For a previous high-level, high-frequency prototype device utilizing the {3-3} mode (with interdigitated electrodes), the microfabrication scheme necessitated an asymmetric structure. This is primarily due to the process used to deposit the PZT, as well as the geometry of the electrodes needed. Please refer to Chapter 6 for a detailed discussion of the fabrication process that was considered. The asymmetric configuration necessitates the use of structural layers to ensure that the piezoelectric element is above the neutral axis of the structure (to prevent strain cancellation). In general, these structural layers are silicon based. These layers are less dense and stiffer (higher bending stiffness) than the piezoelectric layer. Both of these characteristics are detrimental to power when the {3-3} mode of operation is used. For the

$\{3-3\}$ mode of operation to be used, the fabrication process needs to be adjusted to maximize the volume of piezoelectric element in the structure (*e.g.*, two elements), or alternative structural layers will have to be considered.

One of the advantages of using the $\{3-3\}$ mode is that the voltage developed can be controlled. For the $\{3-1\}$ mode, the electrode spacing is determined by the thickness of the piezoelectric layer (see Figure 3-9). This electrode spacing determines the voltage developed (due to the relation between the voltage and the electric field). For the $\{3-3\}$ mode configuration, this spacing can be controlled by changing the pitch of the electrodes. Though this seems like a major advantage at first, the final goal is to have a series of energy harvesters on a single die. One of the advantages of MEMS-fabrication is that the cost of the device is not increased when adding additional devices on a chip. These devices will be interconnected. By connecting the devices in series, the voltages that are generated are added and when the devices are connected in parallel, the charges (or currents) developed across the electrodes add (assuming that the device movement is in phase). Thus, by controlling the interconnections of the devices, the electrical output of the system is controllable. Since both configurations are still of interest, the basic geometries are discussed next.

Conventionally, electrodes are placed on the top and bottom surfaces of the piezoelectric element for $\{3-1\}$ operation. The electric field (and poling direction) is through the thickness of the piezoelectric element, while the strain of interest (bending) is in the axial direction, as shown in Figure 3-9. Alternatively, an interdigitated electrode configuration can be used. With this configuration, a large component of the electric field (and thus the poling direction) is in the axial direction, the same direction as the bending strain (see Figure 3-9). As a result, the device is operated largely in the longitudinal, or $\{3-3\}$, mode. Past research on implementing interdigitated electrode configurations include results by Hagood *et al.* and Bernstein *et al.*, among others [95, 96, 97, 98, 99].

The model developed in Section 3.3.1 will be implemented next for both the $\{3-1\}$ mode and the $\{3-3\}$ mode configurations.

3.3.5 Uni-morph Utilizing the {3-1} Mode

The conventional cantilever configuration piezoelectric energy harvester model ({3-1} mode) is presented in this section. Please refer to Figures 3-7 and 3-9 for illustrations of this configuration, as well as the definition of the parameters. Assuming plane stress (refer to [83] for a detailed description of plane stress and strain constitutive reductions), the constitutive relations eq. (3.1) can be simplified by noting that $T_3 = T_4 = T_5 = 0$. Keeping only the primary variables gives:

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_6 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{11}^{E*} & c_{12}^{E*} & 0 & -e_{31}^{*} \\ c_{21}^{E*} & c_{11}^{E*} & 0 & -e_{31}^{*} \\ 0 & 0 & c_{66}^{E*} & 0 \\ e_{31}^{*} & e_{31}^{*} & 0 & \varepsilon_{33}^{S*} \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_6 \\ E_3 \end{Bmatrix} \quad (3.71)$$

For the plate, which only bends in the transverse direction, the constitutive equations can further be reduced by noting that $S_2 = S_6 = 0$:

$$\begin{Bmatrix} T_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{11}^{E*} & -e_{31}^{*} \\ e_{31}^{*} & \varepsilon_{33}^{S*} \end{bmatrix} \begin{Bmatrix} S_1 \\ E_3 \end{Bmatrix} \quad (3.72)$$

It is important to note that due to the plate assumptions, the piezoelectric constants in eqs. (3.71) and (3.72) will not be equal to the fully 3-D constants. These constants are determined from the compliance form of the constitutive relations (*i.e.*, with stress as the independent field variable and in terms of the compliance, $\mathbf{s}^E$), resulting in [83]:

<u>Beam</u>	<u>Plate</u>	
$c_{11}^{E*} = \frac{1}{s_{11}^E}$	$c_{11}^{E*} = \frac{s_{11}^E}{(s_{11}^E)^2 - (s_{12}^E)^2}$	(3.73)

$e_{31}^{*} = \frac{d_{31}}{s_{11}^E}$	$e_{31}^{*} = \frac{d_{31}}{s_{11}^E + s_{12}^E}$	(3.74)
--	---	--------

$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}$	$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E + s_{12}^E}$	(3.75)
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The following electric potential distribution was assumed to give a constant electric field through the thickness of the piezoelectric element. The potential varies from 0 at the top electrode to +1 at the bottom electrode. Assuming a uni-morph structure (see Figure 3-7) with a structural layer of thickness t_s , such that the neutral axis is on the piezoelectric element-structural layer interface, the functions ψ_r and ψ_v in eqs. (3.28) and (3.29) become (considering only the first mode of the beam):

$$\psi_v = \psi_{v,1} = \frac{-x_3^* + t_p}{t_p} \quad (3.76)$$

$$\psi_r = \psi_{r,1} = f \left[(\cosh \lambda_1 x_a - \cos \lambda_1 x_a) - \frac{A_{12}}{A_{11}} (\sinh \lambda_1 x_a - \sin \lambda_1 x_a) \right] \quad (3.77)$$

t_p is the thickness of the piezoelectric element, and $\frac{\lambda_1}{L} = 1.8571$ for the first bending mode (without a proof mass).

3.3.6 Uni-morph Utilizing the {3-3} Mode

An approximate model for the interdigitated electrode configuration has been adopted. It is assumed that the region of the piezoelectric element under the electrode is electrically inactive, whereas the section between the electrodes utilizes the full {3-3} effect. These are first approximations, since the electric field is not completely axial (horizontal in the Figure 3-10) through the thickness of the piezoelectric element, nor is the region of piezoelectric element under the electrode completely inactive. These effects compensate for each other to some extent. Please refer to Figure 3-10 for the geometry of the approximate model.

For a uni-morph configuration and stress/strain assumptions employed as before with the parameters defined in Figure 3-7, the constitutive relations for the active section in Figure 3-10 can be reduced to:

$$\begin{Bmatrix} T_3 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{33}^{E*} & -e_{33}^* \\ e_{33}^* & \varepsilon_{33}^{S*} \end{bmatrix} \begin{Bmatrix} S_3 \\ E_3 \end{Bmatrix} \quad (3.78)$$

The piezoelectric constants will not be equal to the fully 3-D constants, but are

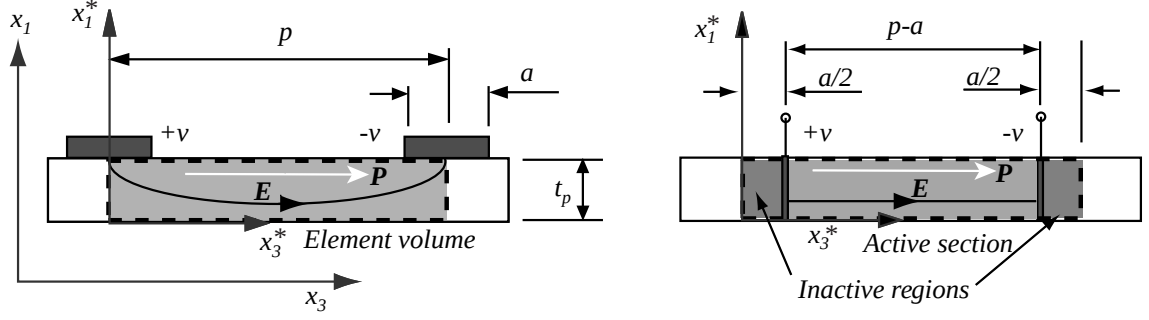


Figure 3-10: {3-3} mode (*left*) and approximate model (*right*) of electric field between interdigitated electrodes. p is the pitch of the electrodes and a is the width of the electrodes. x_1^* and x_3^* are the element local coordinates.

as follows:

<u>Beam</u>	<u>Plate</u>	
$c_{33}^{E*} = \frac{1}{s_{33}^E}$	$c_{33}^{E*} = \frac{s_{11}^E}{s_{11}^E s_{33}^E - (s_{13}^E)^2}$	(3.79)

$e_{31}^{*} = \frac{d_{33}}{s_{33}^E}$	$e_{31}^{*} = e_{33}^{*} = \frac{s_{11}^E d_{33} - s_{13}^E d_{31}}{s_{11}^E s_{33}^E - (s_{13}^E)^2}$	(3.80)
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$\varepsilon_{33}^{S**} = \varepsilon_{33}^T - \frac{d_{33}^2}{s_{33}^E}$	$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{1}{s_{11}^E s_{33}^E - (s_{13}^E)^2} (s_{33}^E d_{31}^2 - 2s_{13}^E d_{31} d_{33} + s_{11}^E d_{33}^2)$	(3.81)
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With the approximation that the region of piezoelectric element beneath the electrode is inactive, while the sections of piezoelectric element between the electrodes experience coupling through the full longitudinal mode, an electrical potential distribution can be specified. To have a constant electric field between the electrodes, the potential distribution needs to vary from +1 at the electrode on the left to 0 at the electrode on the right in Figure 3-10. Note that the electrical potential is defined in the element local coordinates (x_1^* , x_3^*). Again, only the first beam mode is considered.

$$\psi_v = \psi_{v,1} = \frac{-x_3^* + (p - \frac{a}{2})}{p - a} \quad (3.82)$$

$$\psi_r = \psi_{r,1} = f \left[(\cosh \lambda_1 x_a - \cos \lambda_1 x_a) - \frac{A_{12}}{A_{11}} (\sinh \lambda_1 x_a - \sin \lambda_1 x_a) \right] \quad (3.83)$$

It is important to note that even though the device is made up of nq separate piezoelectric control volumes, there is only one electrode pair and the voltage across all the elements will be the same. Since the strain varies along the length of the beam, different amounts of charge will be generated in each element and the charge sums to give the total charge output of the device.

3.4 Conclusions

First, a 1-D electromechanical model was developed to analyze the general response of piezoelectric energy harvesters. Relations for the displacement, voltage developed, and power extracted are obtained. The power is optimized by finding optimal operating frequencies and electrical loading conditions. Two operating points, corresponding to the resonance and anti-resonance frequencies, are identified. The power-optimal electrical loadings at these points are calculated. The results are substituted back into the power extracted, and it is found that the optimal power is independent of the electromechanical coupling. On the other hand, the optimal voltage (and current) and displacement are functions of the coupling. Implications of these results are that the optimal power is determined by the mass and stiffness properties of the structure, and not the piezoelectric coupling as commonly thought. The mass should be maximized and the stiffness should be minimized to optimize the power extracted.

The model is extended for a cantilevered structure and equations of motion are obtained. The model is based on an energy method, and the modal analysis of a beam with a proof mass at the free end is included. From the equations of motion, the displacement, voltage developed, and the power extracted are related to the input vibration frequency and amplitude. In the simplified case (considering a single beam bending mode and having 1 electrode pair), the equations of motion reduce to scalar equations and can be optimized in a similar fashion to the 1-D case.

The piezoelectric harvester can be operated in both the $\{3-1\}$ and the $\{3-3\}$ modes of operation, using conventional and interdigitated electrode configurations, respectively. The developed model is implemented for both these geometries for a uni-morph

cantilever. Bi-morph configurations are considered in detail in Appendices B and C. Modeling and experimental results are compared in Chapter 5 for a macro-scale bi-morph harvester.

Chapter 4

Experimental Methods: Macro-scale Device

Experimental results are required to validate and verify the model presented in Section 3.3. A macro-scale device was used for this purpose. The device, termed Device 2, will be described in detail in Chapter 5, where the results from the experiments are presented and compared to the predicted results. In this chapter, the different tests performed and their procedures are detailed. The tests performed can be divided into three groups:

- Material property measurements
- Damping measurements
- Performance tests

The first and second groups are standard inputs to the dynamic models: the first group include material inputs, focusing on the constitutive relations of the piezo-electric material and the structural layer. All material properties reported are for the Piezo Systems, Inc. T226-A4-503X bi-morph (PZT-5A based), which was used as the validation device. The second test is the device-specific mechanical damping measurements. The third testing type is device tests that will be directly compared to the performance (voltage, power, displacement, etc.) predicted with the model.

4.1 Material Property Measurements

Material properties are necessary inputs into the model for validation. The device consists of both active (piezoelectric) and inactive (structural/shim) layers. For the active layers (refer to Section 3.3 on the coupled beam modeling), the full 3-D constitutive relations for the piezoelectric layers (repeated in eq. (4.1) for convenience) are reduced for the cantilevered beams/plates under investigation. The reduction is due to the simplified stress and strain states. It was shown that, for the {3-1} mode of operation employed in the validation device, eqs. (4.2), (4.3), and (4.4) apply (from Section 3.3.5):

$$\begin{Bmatrix} \mathbf{T} \\ \mathbf{D} \end{Bmatrix} = \begin{bmatrix} \mathbf{c}^E & -\mathbf{e}^t \\ \mathbf{e} & \boldsymbol{\varepsilon}^S \end{bmatrix} \begin{Bmatrix} \mathbf{S} \\ \mathbf{E} \end{Bmatrix} \quad (4.1)$$

<u>Beam</u>	<u>Plate</u>
$c_{11}^{E*} = \frac{1}{s_{11}^E}$	$c_{11}^{E*} = \frac{s_{11}^E}{(s_{11}^E)^2 - (s_{12}^E)^2}$

(4.2)

$e_{31}^{*} = \frac{d_{31}}{s_{11}^E}$	$e_{31}^{*} = \frac{d_{31}}{s_{11}^E + s_{12}^E}$
--	---

(4.3)

$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}$	$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E + s_{12}^E}$
--	--

(4.4)

In this section, the techniques for measuring the material properties required in the model are described. Four properties are of primary interest: the densities of the individual layers, elastic stiffness (at constant electric field for the piezoelectric layers), piezoelectric constant relating the charge density and the strain, and the permittivity of the piezoelectric element at constant strain (clamped). For the inactive (structural) layers, only the density and stiffness are needed. Density is simply calculated from the mass of the device or layers and the corresponding volume. The published piezoelectric element density ($\rho_p = 7800 \text{ kg/m}^3$) was used to infer the density of the shim ($\rho_s = 7165 \text{ kg/m}^3$) from the measured total mass of the as-acquired device (10.564 *grams*) and the corresponding volume ($1.373 \times 10^{-6} \text{ m}^3$). The device mass was measured with a OHAUS Precision Standard TS-4000 (SN:6588) scale, which has

a resolution of 0.001 *grams*. The thicknesses of the nickel electrodes ($\sim 1\mu m$) are negligible.

4.1.1 Elastic Stiffness

The elastic stiffness of both the active and inactive layers of the device are required. From eq. (4.2), it can be seen that the elastic stiffness for the piezoelectric elements at constant electric field, $(c_{11}^E)^*$, is determined from the bulk compliance properties of the piezoelectric material. Specifically, s_{11}^E , or s_{11}^E and s_{12}^E , are required, depending on the device geometry. Given the device configuration under consideration, these properties cannot be measured directly. The bulk piezoelectric material would have to be tested to determine these values independently (refer to [81, 82, 84] for standard procedures to measure these values). The next best option is to rely on published values of the bulk properties, but these are not available either for the device. Instead, the published elastic plate stiffness value is used directly ($c_{11}^{E*} = 66 \text{ GPa}$, from Piezo Systems, Inc. data sheet). This compares to $c_{11}^{E*} = 61 \text{ GPa}$ (beam) for PZT-5A [67].

If the entire structure consisted of piezoelectric material, the stiffness could be verified by measuring the resonance frequency of the structure. The device tested consisted of three distinct layers (two piezoelectric elements separated by a brass shim). No data was available for the brass shim layer. Therefore, the stiffness of the structural layer is determined using the measured resonance frequency. The method to measure the elastic stiffness outlined below can be employed to determine the stiffness of individual elements (if specimens are available) or layers (device-level measurements). The short-circuit resonance frequency was defined in Section 3.3.3 as:

$$\omega_N^2 = \bar{\lambda}_N^2 \sqrt{\frac{(cI)_e}{\rho_c b t_T L^4}} \quad (4.5)$$

ρ_c is the average density of the device, and t_T is the total thickness. For the configurations considered (beam/plate without a proof mass), $\bar{\lambda}_N \approx 1.8571$. The average bending stiffness of the structure, $(cI)_e$, is a function of the piezoelectric material stiffness, $(c_{11}^E)^*$, the structural layer stiffness, c_s , and geometry. Thus, from knowl-

edge of the device geometry (especially cross-sectional thicknesses) and the measured resonance frequency, the bending stiffness of the total device can be calculated. It is assumed that the layers are perfectly bonded, and that the beam/plate obeys the simplified stress-strain assumptions made in Section 3.3.5. The resonance frequency can be measured to within 0.0153 Hz using the laser vibrometer (100 Hz bandwidth and 6400 FFT lines) and is found to be 107 Hz . The device geometry measurements are less precise: the length and width can be measured to within 0.01 mm using a vernier caliper, whereas the thickness can be measured to within $\sim 5\text{ }\mu\text{m}$ using a microscope (at $20\times$ magnification). Thus, the geometric measurements limit the bending stiffness measurements to around two decimal places. Also, the published elastic stiffness is given to 2 digits of accuracy. Using this method, the elastic stiffness for the structural layer of $c_s = 100\text{ GPa}$ is obtained, which is comparable to bulk brass which has an elastic stiffness of $\sim 105\text{ GPa}$ [100].

4.1.2 Piezoelectric Constant

The piezoelectric constant relating the strain and the charge generated, e_{31}^* , is determined from the bulk piezoelectric constants relating the strain to the electric field, d_{31} , and the bulk compliance values (eq. (4.3)). Though d_{31} is normally available from published results, knowledge of the compliance is not as readily available or easily measured. An alternative measurement is obtained by substituting the optimal resistance at the resonance frequency (refer to Section 3.3.2) into the calculated voltage response, which reduces to:

$$|v_r|_{opt} = \frac{B_f \ddot{w}_B}{2\Theta} \quad (4.6)$$

Θ is simply a function of the geometry of the device, and the piezoelectric constant, e_{31}^* (refer to eq. 3.36). B_f is a 'mass' term (function of the density and geometry) and $\ddot{w}_B$ is the base acceleration. Thus, this material constant can be conveniently calculated from knowledge of the voltage developed at the resonance frequency under the optimal electrical loading and the device geometry. In Section 5.4.4 it is found

that the resonance response of the system cannot be accurately predicted using the small signal linear piezoelectric material model as it does not capture the non-linear constitutive response of the piezoelectric element. As a result, the method outlined above is not practical to measure the piezoelectric constant. Instead, the position of the anti-resonance frequency relative to the resonance frequency can be used to calculate this constant.

In Section 3.3.2 it was shown that the resonance frequency does not depend on the piezoelectric coupling in the system. On the other hand, the ratio of the anti-resonance frequency to the resonance frequency is determined by the system coupling term through $\Omega_{ar}^2 = 1 + \kappa^2$. This coupling is defined in terms of the elastic stiffness, the permittivity at constant strain, the piezoelectric constant, and the geometry of the device, or $\kappa^2 = \frac{\theta^2}{K C_p}$. Thus, with the elastic stiffness and the geometry of the structure known, together with the permittivity at constant strain, the piezoelectric constant can be determined from the measured resonance and anti-resonance frequencies ($\Omega_{ar} = 1.0575$ for Device 2). However, the permittivity at constant strain is dependent on the piezoelectric constant (see eq. (4.8)). Thus, an iterative scheme is required to calculate both the piezoelectric constant and the permittivity at constant strain. When this method is applied, the piezoelectric constant of $e_{31}^* = -14 \text{ C/m}^2$ is obtained. This value is compared to the calculated values for both a beam and a plate structure (eq. (4.3)). Published material properties for PZT-5A [67] were used for this calculation, and it was found that the expected value should be in the range of $e_{31}^* = -12 \text{ C/m}^2$ for a beam, to $e_{31}^* = -17.5 \text{ C/m}^2$ for a plate. Thus, a value of -14 C/m^2 is feasible.

4.1.3 Absolute Permittivity

The absolute permittivity of a piezoelectric element can be calculated from the capacitance, which is measured with an impedance analyzer. The absolute permittivity, ε , is related to the relative permittivity (or dielectric constant), ϵ , through $\varepsilon = \epsilon \varepsilon_0$ [101], where $\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ is the permittivity of free space. In the sections hereafter, reference to permittivity will imply absolute permittivity. The permittiv-

ity at constant strain, ε_{33}^S , is calculated from the permittivity at constant stress, ε_{33}^T , which are generally related through eq. (4.7):

$$\boldsymbol{\varepsilon}^T - \boldsymbol{\varepsilon}^S = \mathbf{d}^t \mathbf{e} = \mathbf{e}^t \mathbf{s}^E \mathbf{e} \quad (4.7)$$

From the previous discussion, c_{11}^{E*} and e_{31}^* are known. Also, the reduced permittivity relations (for the simplified stress-strain state of cantilevered structures) were given in eq. (4.4). The permittivity at constant strain for the beam or plate, ε_{33}^{S*} , can be related to the measured permittivity at constant stress, ε_{33}^T , and the measured piezoelectric beam/plate constant, e_{31}^* . It should be noted that the bulk value for the piezoelectric constant, $d_{31} = -190 \times 10^{-12} \text{m/V}$, is required, but is generally available [85]:

<u>Beam</u>	<u>Plate</u>
$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E} = \varepsilon_{33}^T - d_{31} e_{31}^*$	$\varepsilon_{33}^{S*} = \varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E + s_{12}^E} = \varepsilon_{33}^T - 2d_{31} e_{31}^*$

(4.8)

The capacitance measurement method described below is adapted from the ASTM Standard on the testing of insulating materials, D 150 - 98 [102]. The permittivity is inferred through a capacitance measurement, and is related to the capacitance though $C = \varepsilon A / t_p$. A is the area of the electrodes, t_p is the thickness of the piezoelectric element. In general, the permittivity at constant stress can be calculated more accurately from the corresponding capacitance at constant stress measurement since the measurement is performed at a low frequency. Alternatively, the permittivity under constant strain can be measured at frequencies above the resonance frequency, but away from high overtone resonances, and well below any ionic resonances [84]. The latter is more uncertain and leaves more room for interpretation and error and is generally less accurate. For this reason, the capacitance at constant stress is measured and the permittivity at constant stress is calculated.

A practical capacitor can be modeled as either an ideal capacitor in series with an ideal resistor, or an ideal capacitor in parallel with an ideal resistor. The par-

allel configuration is the preferred and accepted representation. The permittivity is calculated from the capacitance measurement and from knowledge of the element geometry. As mentioned above, the capacitance measurement was performed using the ASTM Standard D 150 - 98 as a guideline, though some modifications to the standard method were required due to the geometry of the device. These modifications are described below.

Some of the considerations when measuring the capacitance of a piezoelectric element include: fringing and stray capacitance, guarded electrodes, geometry, and edge, ground, and gap corrections. A guard electrode is an electrode, so connected to divert unwanted conduction from the measurement device [103]. It is assumed that the fringing and stray capacitances are negligible since the plate is wide and long relative to the thickness. Sheet specimens are preferable for capacitance measurements, as is used. No guard electrodes are feasible for the current configurations since the electrodes are pre-deposited. For the device tested, an etch step would have been necessary to remove the electrode. The measurement of the device geometry is critical as the “source of greatest uncertainty in permittivity [measurements] is in the determination of the dimensions of the specimen...” [102]. The measured geometry is also used to calculate the vacuum capacitance for the specimen. Lastly, the frequency at which the permittivity is measured should be well below (~ 0.01) the lowest resonance of the device (in this case the thickness vibration resonance). Lastly, due to loading of the piezoelectric element due to the other device layers, the element is not truly stress free (due to through thickness constraints). As a result, the value measured expected to be is lower than the true value.

An HP 4194A Impedance Analyzer (SN:2830J04586) was used for the constant stress, parallel configuration, capacitance measurements. Open- and short-circuit compensations were necessary since leads were used for the capacitance measurements. Furthermore, the integration time was set to “MEDIUM” and 256 averages were taken to increase the accuracy of the measurement. The voltage applied to the specimen by the analyzer can be specified, and an excitation voltage of 1 V_{rms} was used. The “ $C_p - R_p$ ”-setting of the “Impedance Measurement” function was selected

to obtain the parallel representation of the piezoelectric element. The capacitance was measured at 100 Hz , 1 kHz , and 10 kHz since a first thickness resonance of $\sim 10 MHz$ was calculated. Thus, the measurements are taken well below the resonance frequency.

For the device tested, a capacitance at constant stress of $C_p^T = 52.8 nF$ was measured. For the device dimensions $63.5 mm \times 31.8 mm \times 0.556 mm$, a permittivity of $\varepsilon_{33}^T = 1632\varepsilon_0 F/m$ was calculated. This is lower than the published value of $\varepsilon_{33}^T = 1800\varepsilon_0 F/m$. This lower value is attributed to the loading of the other device layers on the dielectric layer. The permittivity for individual layers were measured (in other work) before they were incorporated into a bi-morph configuration. The ratio of the permittivity of the individual elements to the permittivity of the device (~ 0.913) was approximately equal to the ratio of the published and measured permittivities for the tested device (~ 0.907). It is concluded that the published permittivity for the elements is more accurate and will be used in the analysis. Thus, these measurements served as a verification of the value for the current configuration. Before a device is constructed and the individual layers are accessible, the above procedure can be followed to get an accurate measurement of the permittivity.

4.1.4 Material Properties Results Summary

The material properties of interest are the elastic stiffness, the piezoelectric constant, and the permittivity. The elastic stiffness cannot be measured with ease for the geometries and devices under consideration. The published material properties for the piezoelectric elements in the device are used: $c_{11}^{E*} = 66 GPa$ and $\rho_p = 7800 kg/m^3$. Using these properties, the density of the structural layer was calculated as $\rho_s = 7165 kg/m^3$. This value is lower than expected for a brass shim (published densities range from 8500–9000 kg/m^3 , www.matweb.com), but it should be noted that the model utilizes the average density of the structure such that only the average measured density is important. An elastic stiffness of 100 GPa is calculated using the method outlined in Section 4.1.1. The piezoelectric coupling can be obtained from the voltage measurement at resonance under optimal electrical loading

Table 4.1: Material properties for Device 2 (PZT-5A bi-morph from Piezo Systems, Inc. (T226-A4-503X).

Material property (beam configuration [†])	Used	Published	Ref.
Published properties used			
ρ_p [kg/m^3]	7800	7800	[104]
c_{11}^{E*} [GPa]	66	66	[104]
d_{31} [m/V]	-190×10^{-12}	-190×10^{-12}	[104]
Measured properties			
Device length, L [mm]	63.5	63.5	[104]
Device width, b [mm]	31.8	31.8	[104]
Piezo layer thickness, t_p [μm] [‡]	270	270	[104]
Structure layer thickness, t_s [μm] [‡]	140	130	[104]
Device mass [$grams$]	10.564	-	-
Capacitance (constant stress), C_p^T [nF]	52.8	59.1	-
e_{31}^* [C/m^2]	-14	-	-
Calculated properties			
ρ_s [kg/m^3]	7165	9000	[100]
ε_{33}^T [F/m]	$1800 \times \varepsilon_0$	$1800 \times \varepsilon_0$	[104]
Elastic stiffness, structural layer, c_s [GPa]	100	105	[100]
$\varepsilon_{33}^{S*§}$ [F/m]	$1500 \times \varepsilon_0$	-	-

[†] See beam eqs. (4.2) to (4.4).

[‡] From microscopy measurements.

[§] Utilizing eq. (4.4), where $d_{31} = -190 \times 10^{-12} m/V$ for PZT-5A from [67].

(see Section 4.1.2). However, this method proved impractical due to the non-linear response of the piezoelectric elements to the applied strain. Instead, the constant is calculated from the relative positions of the resonance and anti-resonance frequencies. A piezoelectric constant of $e_{31}^* = -14 C/m^2$ is obtained. Permittivity is inferred though capacitance measurements made with an impedance analyzer. The published bulk value of $1800\varepsilon_0 F/m$ is confirmed through these tests. The permittivity at constant strain is calculated from the published value, using the determined piezoelectric constant, e_{31}^* and the bulk value of the piezoelectric constant relating strain to electric field, $d_{31} = -190 \times 10^{-12} m/V$. The value obtained was $\varepsilon_{33}^S = 1500\varepsilon_0 F/m$. These results are summarized in Table 4.1, and are compared to published results where applicable.

4.2 Damping Tests

All practical systems have damping. The damping affects the response of a system, especially near the resonances (see Section 2.5). A detailed analysis of the different damping terms affecting the system under investigation is given in Section 2.3. This section is concerned with the measurement of the mechanical damping ratio (ζ_m) directly (ζ_m captures all non-electrical damping in the section), first using a slight variation on the standard logarithmic decrement technique [91]. Secondly, a method is used to relate the base and absolute tip displacements at the resonance frequency and under no electrical load (short-circuit). Both methods should agree and are discussed below, starting with the logarithmic decrement.

For the standard logarithmic decrement technique, the free-vibration displacement response of the device is measured in the time domain. The displacement decay of the system is calculated by comparing the relative height of displacement peaks, separated by N_{cyc} cycles. The laser-vibrometer setup (refer to Section 4.3.1) was used to measure absolute displacements for this. However, when using the laser vibrometer, the velocity is measured instead of the displacement. The displacement-time data is required for the standard logarithmic decrement method and the velocity measurements can be converted to displacement data with a numerical integration scheme. An alternative method is to rederive the logarithmic decrement method in terms of the velocity-time data. The following analysis is included to show that the same relations hold when relatively comparing the velocity peaks (this is intuitive since the response is harmonic). The free vibration of a simple 1 degree of freedom system is given by eq. (4.9). It should be noted that the cantilevered structure has infinite degrees of freedom, but since the time constants of the higher overtone modes are very small relative to the first mode, the effects of these higher modes on the response will decay rapidly and can be ignored. Hence, it is assumed that the free-vibration response due to an initial excitation will be represented by the first mode only. In this case, the system can be approximated as a single degree of freedom

system and eq. (4.9) holds [78]:

$$w_t(t) = e^{-\zeta_m \omega_N t} (A \cos \omega_d t + B \sin \omega_d t) \quad (4.9)$$

Here, $w_t(t)$ is the measured absolute tip displacement, ζ_m is the mechanical damping ratio, ω_N is the natural frequency, and $\omega_d = \omega_N \sqrt{1 - \zeta_m^2}$ is the damped natural frequency. A and B are arbitrary constants. The velocity of the system is obtained by taking the time derivative of the displacement:

$$\dot{w}_t(t) = e^{-\zeta_m \omega_N t} [-\zeta_m \omega_N A \cos \omega_d t - \zeta_m \omega_N B \cos \omega_d t - A \omega_d \sin \omega_d t + B \omega_d \cos \omega_d t] \quad (4.10)$$

The logarithmic decrement is generally defined as the ratio of a reference displacement and a displacement N_{cyc} cycles, or $N_{cyc} T_d$ seconds, later. T_d is the period of the damped oscillation. The standard logarithmic decrement is defined as [91]:

$$\delta = \ln \frac{w_t(t)}{w_t(t + N_{cyc} T_d)} = 2\pi N_{cyc} \frac{\zeta_m}{\sqrt{1 - \zeta_m^2}} \quad (4.11)$$

An alternative definition for the logarithmic decrement in terms of the velocity is given as:

$$\delta_v = \ln \frac{\dot{w}_t(t)}{\dot{w}_t(t + N_{cyc} T_d)} \quad (4.12)$$

Substituting the velocity equation for free vibrations (eq. (4.10)) into eq. (4.4), and noting that $N_{cyc} \omega_d T_d = 2\pi$ for all $N_{cyc} \in \mathbb{N}$, the alternate logarithmic decrement reduces to:

$$\delta_v = \zeta_m \omega_N N_{cyc} T_d = 2\pi N_{cyc} \frac{\zeta_m}{\sqrt{1 - \zeta_m^2}} \quad (4.13)$$

This result is the same as for the standard logarithmic decrement (eq. (4.11)). Thus, the logarithmic decrement defined in terms of the velocity can be used to measure the damping of the system. The final result for the damping ratio is:

$$\zeta_m = \frac{\frac{1}{2\pi N_{cyc}} \delta_v}{\sqrt{1 + \left(\frac{1}{2\pi N_{cyc}} \delta_v \right)^2}} \quad (4.14)$$

Thus, the free-vibration absolute velocity-time data can be used to calculate the mechanical damping of the structure. The laser vibrometer is used for the measurement, with the electrical damping experimentally fixed at zero by shorting the piezoelectric elements. Tip location (for measuring velocity) is taken at the center of the device and is known to within $\pm 100 \mu m$, based on the estimated spot size of the laser. For the device tested, a mechanical damping of $\zeta_m = 0.0414$ was first determined. The magnitudes of peak velocities used for the measurement must be of the same order as the velocities obtained during the operation of the device (taken to be $\sim 165 \text{ mm/s}$). Peaks ~ 5 cycles apart were used for the δ_v measurements. The test was repeated about 40 times and an average value for the mechanical damping was obtained. This measurement was taken with the device mounted on the electrostatic shaker. A mechanical damping ratio of 0.0414 seemed high, and the device was mounted on a more rigid structure and the test was repeated. A damping ratio of $\zeta_m = 0.0143$ was measured. It was hypothesized that the shaker added damping to the device and thus influenced the dynamics of the structure. This hypothesis was tested in the following damping measurement.

The effect of the shaker on the device dynamics was a concern as it influences the dynamics of the harvester. A measurement scheme was developed to measure the damping during a dynamic test by comparing the base and absolute tip displacements. At the resonance frequency, and for short-circuit conditions (zero electrical load), absolute tip displacement, w_t , is related to the base displacement through:

$$\left| \frac{w_t}{w_B} \right| = \psi_r(L) \frac{1}{2\zeta_m} \sqrt{\left(\frac{B_f}{M} \right)^2 + 4\zeta_m^2} \quad (4.15)$$

The ratio of M and B_f is simply dependent on the mode shape, ψ_r , and is 0.7830 for the current configuration (a cantilevered structure without a proof mass). For small damping, and noting that the mode shape is normalized to 2 at the tip, this reduces to eq. (4.17).

$$\left| \frac{w_t}{w_B} \right| = \frac{B_f}{M} \frac{1}{\zeta_m} \quad (4.16)$$

Thus, the mechanical damping is conveniently related to the base and absolute

tip displacements through:

$$\zeta_m = 0.7830 \left| \frac{w_B}{w_t} \right| \quad (4.17)$$

Using this result, a mechanical damping ratio of $\zeta_m = 0.0178$ is calculated from base and tip displacements during resonance (short-circuit) device operation. This value is still larger than the more rigid mounting measurement ($\zeta_m = 0.0143$), likely indicating some contribution to the system damping from the shaker. Since $\zeta_m = 0.0178$ is the damping that the system has during operation, this is the value that will be used in the analysis in Chapter 5. The experimental setup is detailed in Section 4.3.1, but the relevant equipment settings for the damping measurements are given below.

For the damping tests, the laser vibrometer sensitivity was set at $25 \frac{mm}{s}/V$. The response was measured over a frequency bandwidth of $5 kHz$ and 6400 FFT lines (resulting in a $1.28 s$ time window, since the time domain response is required) for the logarithmic decrement method. For the method relating the tip and base displacements, a bandwidth of $0.2 Hz$ and 1600 FFT lines were used, resulting in frequency increments of $0.125 Hz$. The device is excited at the resonance frequency, with a base acceleration of $\ddot{w}_B = 2.5 m/s^2$. The frequency domain response is sufficient since only magnitudes of the displacements are required at the resonance frequency.

4.3 Performance Tests

Two performance measures are of interest when concerned with model validation: mechanical and electrical performance. In the mechanical domain, the parameter of primary interest is the lateral tip displacement which is measured with the laser vibrometer. There are two electrical parameters from which the electrical performance of the device can be established; the output voltage and the power generated. The voltage output can be measured directly. The power generation is calculated from the output voltage and knowledge of the electrical loading. In this project, a purely resistive electrical load is used to simplify the calculation and measurement of the

power.

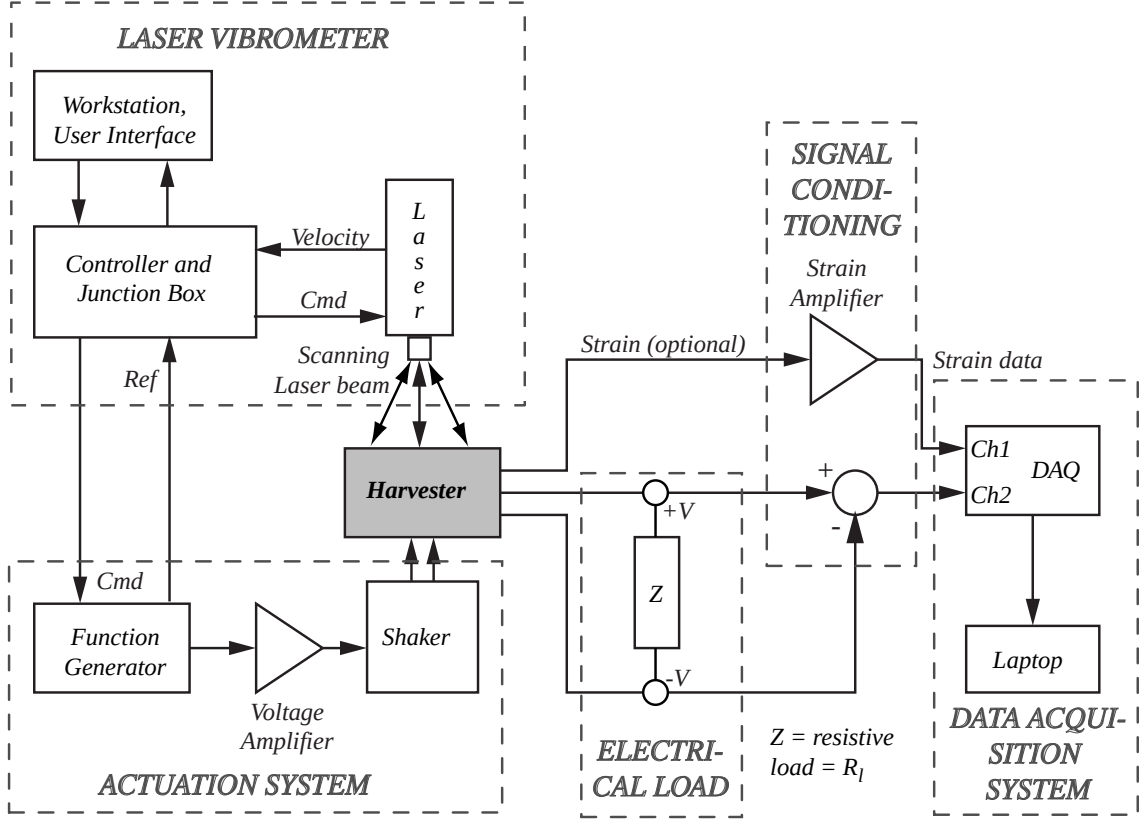


Figure 4-1: Illustration of the experimental setup.

4.3.1 Setup

The experimental setup was jointly developed by the author and Jeffrey Chambers. The system is made up of the following components and sub-components (also refer to Figure 4-1):

1. Laser-vibrometer: Polytec PSV-300H. The laser vibrometer is a doppler-effect interferometer that can perform measurements over a frequency range of $0.1 \text{ Hz} - 250 \text{ kHz}$ and velocity measurement to a maximum of 20 m/s . The vibrometer has a minimum resolution of $0.3 \mu\text{m/s}$, though not all values given here correspond to the same setting. Please refer to Table 4.2 for a complete listing of the ranges and resolutions for the vibrometer for different settings. The conversion

Table 4.2: Laser-vibrometer measurement ranges: PSV-300H, adapted from [106].

Sensitivi- ty [$\frac{mm}{s}/V$]	Max. Velocity [$\frac{mm}{s}$]	Velocity Resolu- tion [$\frac{\mu m}{s}$]	Displ. Resolu- tion at 100 Hz [nm]	Min. Fre- quency [Hz]	Max. Fre- quency [kHz]	Max. Accel- eration [g]
1	20	0.3	0.5	0.1	10	70
5	100	0.3	0.5	0.1	5	200
10	200	0.5	0.8	n/a	200	12,000
25	500	2	3	n/a	250	40,000
125	2,500	5	8	n/a	250	200,000
1,000	20,000	10	16	n/a	250	1,600,000

of this resolution into displacement or acceleration is frequency dependent. An example measurement is illustrated in Figure 4-2. The system also has the capability to measure the deflection of a surface and reconstruct the mode shapes (either as static gradient figures or animations illustrating the mode shapes). The system measures the structure's response at a specified grid of measurement points, and reconstructs the mode shape based on deflection and phase data of each point. Each grid point is known to within $\pm 100 \mu m$, based on the estimated spot size of laser. The phase is captured by monitoring the reference signal from the function generator. An example of two reconstructed mode shapes is given in Figure 4-3. The laser vibrometer consists of the following components:

- (a) Laser
 - i. Scanning-head: OFV-056 (SN: 6 00 1696)
 - ii. Sensor head: OFV-303S (SN: 6 00 1542)
- (b) Controller: OFV-3001 SF6 (SN: 5 00 1296)
- (c) Workstation: PSV-PC-F (SN: 5 00 1319)
- (d) Junction box: PSV-Z-040-F (SN: 5 00 1191 09)

2. Actuation unit:

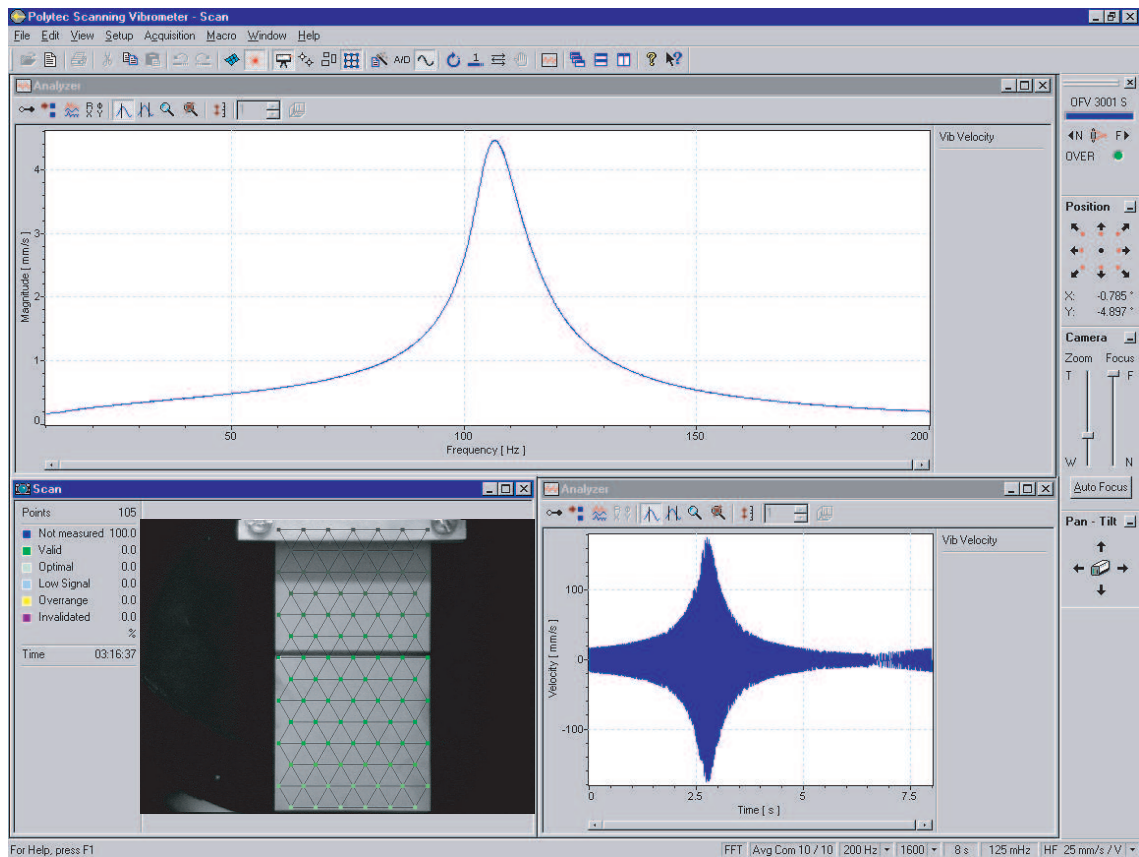


Figure 4-2: Illustration of vibration measurement: the frequency domain (*top*) and time domain (*bottom right*) response of device tip (Device 2) during a frequency test is shown in the two analyzer windows. The planform camera view (*lower left*) of the device with measurement grid points is also shown.

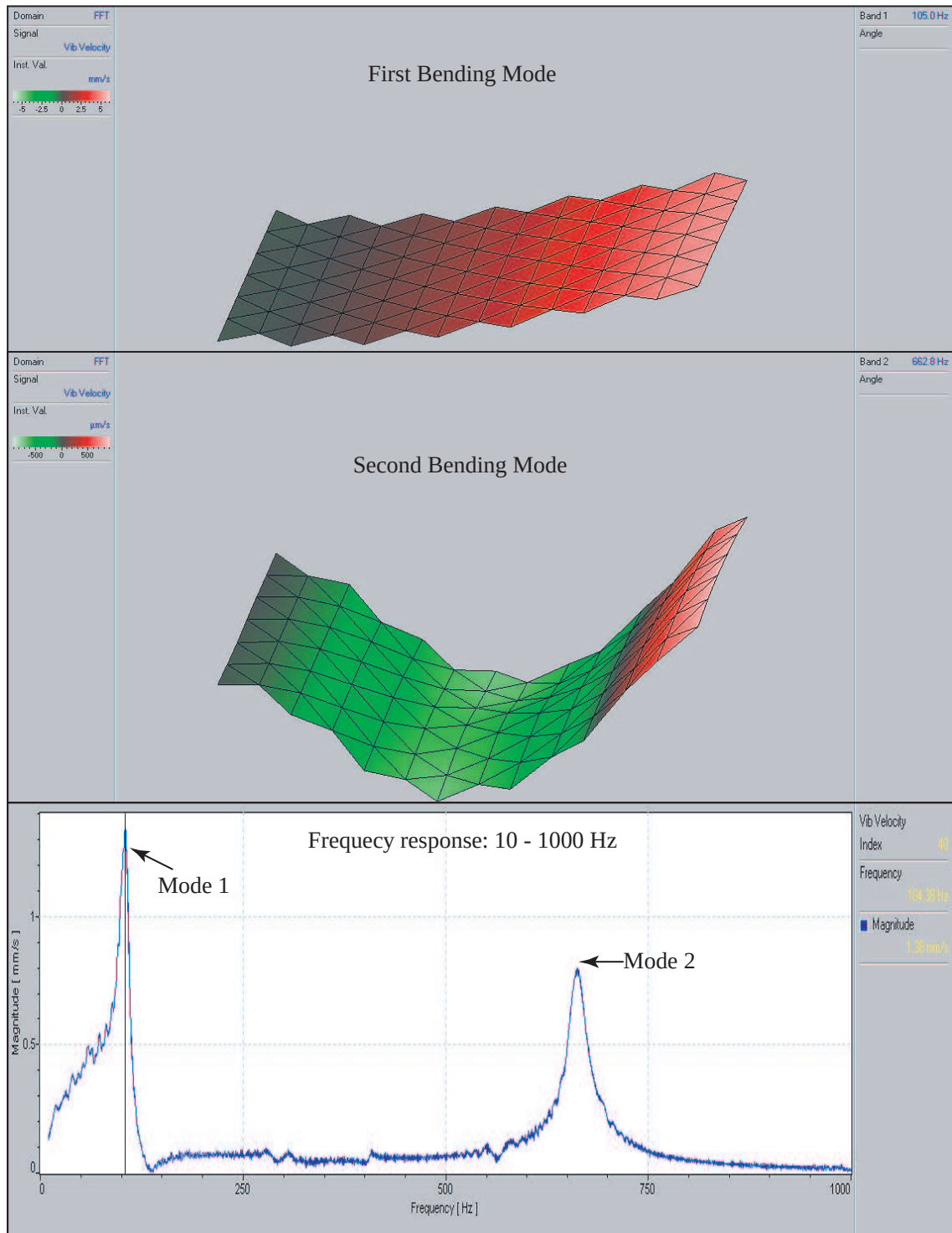


Figure 4-3: Results for a frequency scanning test on Device 2. The scanning grid and the reconstructed short-circuit mode shape (velocity) are shown for (*top*) the first mode and (*middle*) the second mode. Velocity in the frequency domain (*bottom*) shows the location of the first and second modes.

- (a) Function generator: Prema ARB 1000 (SN: 10181). The unit is controlled with the user interface of the laser vibrometer controller. The function generator outputs a signal of maximum $\pm 10\text{ V}$ peak to peak.
 - (b) Power amplifier: Crown DC-300A Series II (SN: 086964). The signal generated by the function generator is normally low power. The voltage is controlled by the function generator, and the current generated is generally not sufficient to drive an electrostatic shaker. Thus, the power amplifier is used to ensure that the signal into the electrostatic shaker has sufficient power.
 - (c) Electrostatic shaker: Brüel & Kjær Type 4809 (SN: 1561261)
3. Piezoelectric resonating harvester (test specimen/device):
- (a) Cantilevered device: the device tested is a Piezo Systems, Inc. bender (T226-A4-503X). The device is pre-poled, as shown in Figure 4-4 (series connection).
 - (b) Electrical connections to piezoelectric elements (illustrated in Figure 4-4).
 - (c) Strain gauges and connectors (optional). Provision is made for the addition of strain gauges on the device, though it was not used in this work.
4. Electrical load: A purely resistive load was used. The load was applied with discrete resistors. The resistance was measured with a multimeter (Omega HHM16, SN: 98000004), which has a measurement accuracy of 0.3%, measurement resolution $0.1\ \Omega$, and maximum measurement $\Omega_{max} = 43\text{ M}\Omega$.
5. Strain Signal Conditioning (optional): Measurements Group, Inc. 2120 Strain Gage Conditioner (SN: 023319) and 2110 Power Supply (SN: 67302). This unit supplies a bias to the strain gauges and converts the charge signal from the strain gauges to a voltage through a calibration factor of $400\ \mu - \text{strain per V}$. Again, strain data was not acquired in this work.
6. Data Acquisition (DAQ) System.

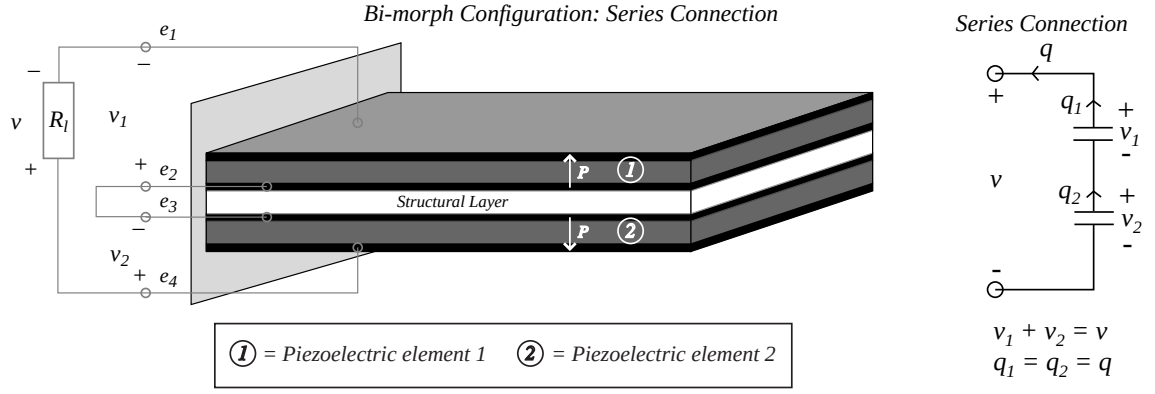


Figure 4-4: Electrical connections of series configuration bi-morph piezoelectric energy harvester tested (*left*) and simplified equivalent circuit (*right*).

- (a) Data acquisition board: National Instruments DAQPad-6070E (MIT0361508) with a configurable connector enclosure (CA-1000). Maximum voltage input is ± 10 V peak-to-peak and a measurement resolution of 0.3 mV.
- (b) Laptop computer with 1394 'fire-wire' connector.

4.3.2 Procedures

As mentioned, there are three performance parameters that are of interest. These include electrical performance (voltage and power), and a mechanical performance measure (lateral tip displacements). Since these measurements form the basis of the performance tests, the procedure for each of these measurements will be described next, starting with the voltage measurements. The results from these tests are presented in Chapter 5.

The voltage across the electrical load is measured. The device tested is a symmetric bi-morph structure (*i.e.*, structures with two active elements) with four electrodes, as illustrated in Figure 4-4. The piezoelectric element numbering is defined in Figure 4-4 (the top element is element 1). There are two electrodes for each element, which can be connected to the electrical circuit in two different configurations: series or parallel. These connections are detailed in Appendix C, as well as the implementation of the configurations in the model developed in Section 3.3. Device 2 is poled to allow

for series connection. Thus, the voltage is measured between e_1 and e_4 . A simplified electrical circuit for the series connections is also shown in Figure 4-4, where the piezoelectric elements are simplistically represented as capacitors.

Since the voltage is measured and the voltages developed across each of the individual elements add, a higher signal-to-noise ratio is obtained with the series connection. It is important to note that the two active elements in the bi-morph are poled in opposite directions (refer to Figure 4-4). The reader is referred to Appendix B for a detailed discussion of the effect of the poling direction on the piezoelectric constants. The opposite poling is required since the bending structure develops opposite strains above and below the neutral axis. The poling direction and applied strain determine the direction of the electric field that is developed (through piezoelectric coupling). When the poling directions are in the same global direction, electric fields of opposite signs are developed for the series connection, which cancels when added. Conversely, when the poling directions are opposite (as in the device tested), the electric fields developed will have the same direction and will add.

The voltage is measured directly using the DAQ, which has a voltage limit of ± 10 V peak-to-peak. Note that the DAQ has an internal resistance of $1\text{ M}\Omega$ when connected across the electric load, which affects the resistance (and power) measurements as described in the section to follow (the $1\text{ M}\Omega$ DAQ resistance is in parallel with the electrical load applied). The voltage measurements were taken on Channel 0 of the DAQ, using a National Instruments LabView graphical user interface (called a “Virtual Instrument”, or *VI*) which was created by Mr. John Kane for the experiment. The adjustable parameters in the *VI* for the measurements were the number of channels sampled (1 channel for current tests), the sampling frequency ($10,000\text{ Hz}$ per channel), and the total number of data-points (or time length) per channel (a time length of 1 second was used, resulting in 10,000 data points). Measurements of resolution 0.3 mV were obtained.

The second electric performance measure is the power extracted from the system. The power is calculated from the known electrical load and voltage measurements. For the current research, a purely resistive load is connected to the harvesting device. The

resistor is connected between the terminals where the voltage, v , is measured (refer to Figure 4-4). The specific terminals depend on the device geometry (bi-morph vs. uni-morph) and the inter-element connection (series or parallel).

Initially, a variable resistor was used to change the electrical loading on the system (for convenience). The variable resistor was constructed from four potentiometers, ranges of: $0 - 100 \Omega$, $0 - 5 k\Omega$, $0 - 10 k\Omega$, and $0 - 1 M\Omega$, respectively. These potentiometers were connected in parallel to give a resistance range of $10 \Omega - 0.8 M\Omega$. The resistance was measured using a standard multi-meter (Omega HHM16, SN: 98000004). This multimeter has an accuracy of 0.3%, a maximum resolution of 0.1Ω , and a maximum resistance measurement of $43 M\Omega$. However, the variable resistor added inductance to the circuit due to the coil geometry of the potentiometers, which affects the dynamics of the circuit. Since the electrical and mechanical domains are coupled through the piezoelectric effect, the additional inductance will invariably influence the mechanical dynamics and performance of the device. For this reason, discrete resistors were used for the electrical loading.

The DAQ has an internal resistance of $1 M\Omega$, which is connected in parallel across the load from the variable resistor for the voltage measurements. When the load resistance is small (below $20 k\Omega$), the effect of the DAQ resistance is negligible. However, for larger resistances ($\sim 100 k\Omega$), the effect becomes more important as can be deduced from simple circuit theory. The effective electrical load, R_l , *as used in the rest of the project* is the parallel resistance of the applied load and the DAQ. The internal resistance of the piezoelectric element ($\sim 31 M\Omega$) is ignored.

For a simple resistive load, the power is related to the voltage and resistance through $P_{out} = \frac{v^2}{R_l}$, where v is the load voltage measured. For more complicated electric circuits, the power calculation becomes more tedious and falls outside the scope of this project. Given the measurement uncertainty in voltage ($0.3 mV$) and resistance ($0.1 k\Omega$ in the $k\Omega$ -range), power is known to $\pm 0.25 \mu W$.

The lateral displacement and acceleration of the structure is measured indirectly using the laser-vibrometer. The system is a laser interferometer that measures absolute velocity, utilizing the Doppler effect. Since the excitation is harmonic, the

time data can be converted to the frequency domain using the Fourier Transform, $\dot{w}(\omega) = \int_{t_1}^{t_2} \dot{w}(t)e^{-i\omega t} dt$. Please refer to mathematics or vibration texts for a detailed description of the transformation, for example [91, 105, 106]. In the frequency domain, the absolute displacement (w) and acceleration ($\ddot{w}$) can be easily calculated, since these are related through frequency (ω):

$$w(\omega) = \frac{1}{\omega} \dot{w}(\omega) \quad (4.18)$$

$$\ddot{w}(\omega) = \omega \dot{w}(\omega) \quad (4.19)$$

Since the system measures velocity, there is a frequency dependence for the minimum displacement resolution that the system is capable of. When setting up the laser vibrometer, there are a number of considerations, including the frequency, displacement, velocity, and acceleration ranges of the measurement. The first is the frequency range of the measurements. The vibrometer operates over a frequency range of $0.1 \text{ Hz} - 250 \text{ kHz}$. It also has a DC capability, though the use and meaning of this is unclear from the hardware manual [107]. The DC setting is not used in this project. The available velocity range settings, with the corresponding frequency ranges, resolutions, and maximum accelerations, are outlined in Table 4.2.

In general, the measurement range of $10 \frac{\text{mm}}{\text{s}}/V$ is sufficient for most applications, but the selection is not solely dependent on the velocity. For the current application, displacements of up to $300 \mu\text{m}$ around 100 Hz are to be measured. This translates into $\sim 380 \frac{\text{mm}}{\text{s}}$ peak to peak. Thus, the required sensitivity is at most $19 \frac{\text{mm}}{\text{s}}/V$. The $25 \frac{\text{mm}}{\text{s}}/V$ setting is used for tests in this work. The acceleration is well below the maximum acceleration allowed ($\sim 24 \text{ gs}$).

The external function generator is controlled from the graphical user interface to create the signal forms that are required. The vibrometer and function generator frequency bandwidth for the tests can be selected on the user interface. Furthermore, the frequency resolution of the measurements can be controlled through the specification of the number of data-point in the FFT analysis. The sampling frequency is automatically adjusted based on the specified bandwidth. The bandwidth

setting is adjusted as needed for the specific tests performed, in general, a bandwidth of 200 Hz was used. Only the initial frequency sweep (refer to Figure 4-3) had a different bandwidth (1 kHz was used).

The vibrometer has the capability to measure and animate the modes of a structure at any measured frequency. This is of specific interest at or near the resonances of the device. A grid of points on the device surface is specified at which the velocity is measured, and from the magnitude and phase data at the different frequencies, the response of the structure is reconstructed. In the scanning mode, the vibrometer takes a set of readings at each grid point, before progressing to the next point. It is important that the measurement window (determined by the bandwidth and the number of points in the FFT analysis) and length of the signal created by the function generator be equal. When this is not the case, inaccurate results are obtained. The number of averages at each of these points on the grid can be specified. Increasing the number of averages increases the accuracy of the measurements. The effect of the number of averages on the obtained response is illustrated in Figure 4-5. The “Complex Averaging” setting was used since excitation was deterministic (refer to the theory manual, [106], for a discussion on the different averaging schemes available). The reference signal need not be measured for the current analysis, but is normally used to determine the frequency response function (transfer function).

Raw data is exported to an output file in ASCII format and a graphing suite such as Excel or MATLAB is used to reduce the data. The data set is selected by selecting the analyzer window of interest. For the time data, two columns of data are obtained; the first column corresponds to the time, and the second column the data (*e.g.*, velocity in mm/s). The unit of the actual data depends on the parameter displayed in the analyzer window. For the time domain data, either voltage or velocity data can be displayed and exported. When the frequency domain data is displayed in the analyzer window, the first column in the output file is the frequency data. The second column is the parameter values that are displayed in the analyzer window at that time. The magnitude of the absolute displacement, velocity, acceleration, and voltage response at the corresponding frequencies are some of the available options.

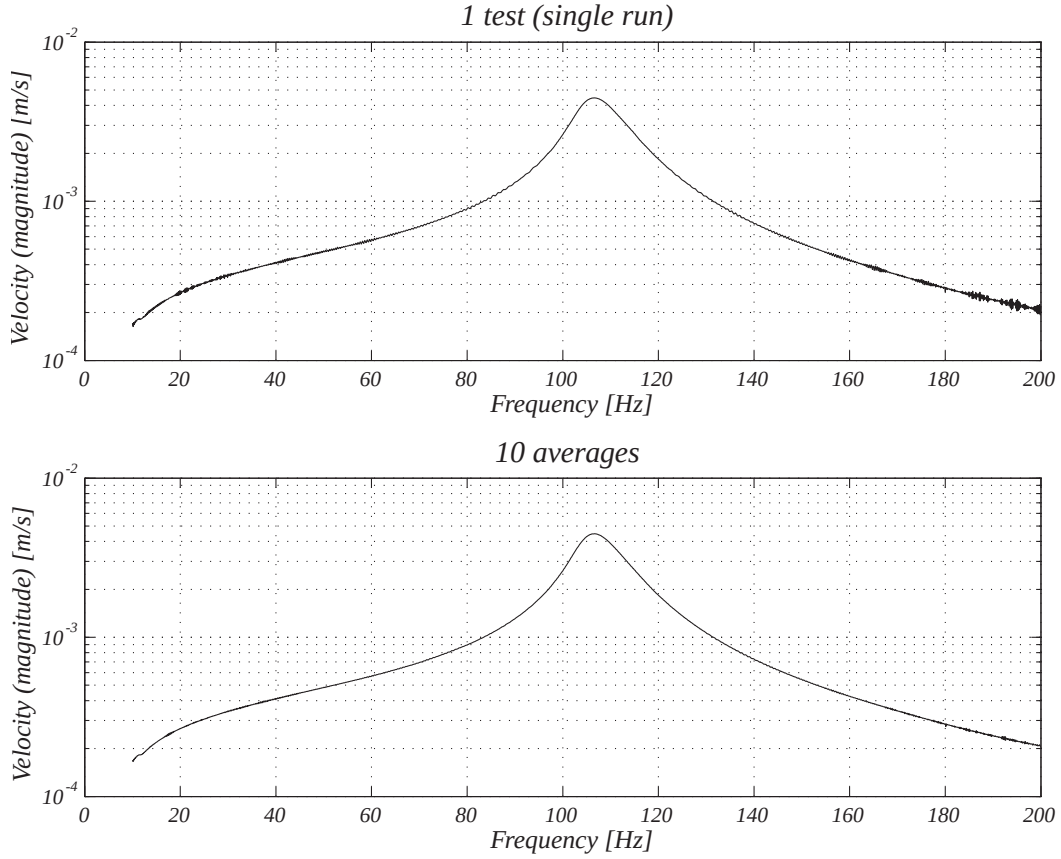


Figure 4-5: Effect of number of averages on the measured response during a frequency sweep (frequency domain). Results for 1 (*top*) and 10 (*bottom*) averages are shown. The deterministic excitation is very clean (negligible noise) and the measured response is very clean, even for the single run measurement.

The final consideration is the conversion of absolute motion to relative motion (refer to Figure 2-1). The relative (z), absolute (w), and base (w_B) motions are related through (for the displacements):

$$z(t) = w(t) - w_B(t) \quad (4.20)$$

For the base-excited single degree of freedom system (which is the approximation for the cantilevered structure used in this project), the transfer function, $H_{w|w_B}$ ¹,

¹The same will be true for the velocity and acceleration since $H_{w|w_B} = H_{\dot{w}|\dot{w}_B} = H_{\ddot{w}|\ddot{w}_B}$

relating the absolute and base displacements is:

$$H_{w|w_B} = |H_{w|w_B}| e^{-i\chi} \quad (4.21)$$

χ is the transfer function phase angle. For low damping, the absolute tip and base displacements are in phase ($\chi \approx 0$) below resonances² (to a close approximation), and the magnitude of the relative displacement is determined through:

$$z \approx w - w_B \quad (4.22)$$

Above resonance, the absolute tip and base displacements are nearly perfectly out of phase ($\chi \approx \pi$), and the magnitude of the relative motion is obtain from:

$$z \approx w + w_B \quad (4.23)$$

At, or near resonance, the phase is $\chi \approx \frac{\pi}{2}$ and the base displacement is small compared to the tip displacement. As a result, the relative displacement is approximately equal to the absolute displacement:

$$z \approx w \quad (4.24)$$

Using these relations for the different operating regimes, the relative motion of the device can be approximated from the absolute tip and base motions, noting that the same relations hold for the velocity and acceleration.

4.3.3 Summary of Performance Tests

From the modeling section (Section 3.3), two prospective operating points were identified for piezoelectric energy harvesters. For low damping and high piezoelectric coupling, these operating points coincided with the resonance and anti-resonance frequencies, respectively. Two tests were performed to analyze the response of the

²“Resonances” here refers to either the resonance or the anti-resonance frequency

system at and around these operating points and are described below. The last test is performed to analyze the device response away from the resonances.

First, open-circuit and short circuit conditions are applied to the electrodes of the active elements and the mechanical response for the frequency range of $10\text{ Hz} - 2\text{ kHz}$ is measured (nominally at frequency increments of 0.125 Hz). This bandwidth can later be reduced. The general procedure is described, before the actual settings used for the tests performed on Device 2 are given. The structure is excited using a “chirp” function. This function creates a sinusoidal signal of constant voltage magnitude with varying frequencies such that the final signal contains all the frequency components in the specified bandwidth (thus all frequencies are excited) and which is of a specified length. This length should be equal to the measurement window of the laser vibrometer. It should be noted that the base acceleration will not be constant at all frequencies for this test due to the frequency dependent dynamic response of the shaker. However, since the frequency of the resonance is of primary interest, this is not a concern.

The purpose of these measurements is to determine the resonance and anti-resonance frequencies of the device. The resonance frequency corresponds to the natural frequency of the device under short-circuit condition. More formally, the resonance frequency is defined as the device natural frequency at minimum impedance, [82, 84]. Conversely, the anti-resonance frequency is the natural frequency of the structure under open-circuit conditions (infinite resistance). The anti-resonance frequency is formally defined as the device natural frequency at maximum impedance, [82, 84].

For this test, the tip velocity of the cantilevered structure is used to measure the frequency response of the system. Please refer to Figure 4-6 for an example of the frequency response of the system under short- and open-circuit conditions, respectively. This measurement was taken by exciting the system using the “chirp” function with an output voltage magnitude of 0.6 V . The “chirp” function is a harmonic signal of constant magnitude (0.6 V) with frequency components across the bandwidth frequency range ($10 - 200\text{ Hz}$ for the current test). Since the response

of the electrostatic shaker is not flat across the frequencies under investigation, and the shaker dynamics are influenced by the device dynamics, the base acceleration was not constant at all frequencies for this test. However, since the primary interest is the frequencies, this non-uniform input was not a concern. The bandwidth (0.2 kHz) was set on the laser vibrometer, and 1600 FFT lines were used, resulting in a frequency resolution of 0.125 Hz . Furthermore, the laser vibrometer sensitivity was set at $25\frac{\text{mm}}{\text{s}}/\text{V}$, and the response of the device tip was measured. Resonance and anti-resonance frequencies ranging from $106 - 107\text{ Hz}$ and $112.5 - 113.5\text{ Hz}$, respectively, were measured. These frequencies are strongly dependent on the material properties, which in turn are temperature dependent. It is suspected that the variation in these frequencies is an artifact of varying environmental conditions, specifically tempera-

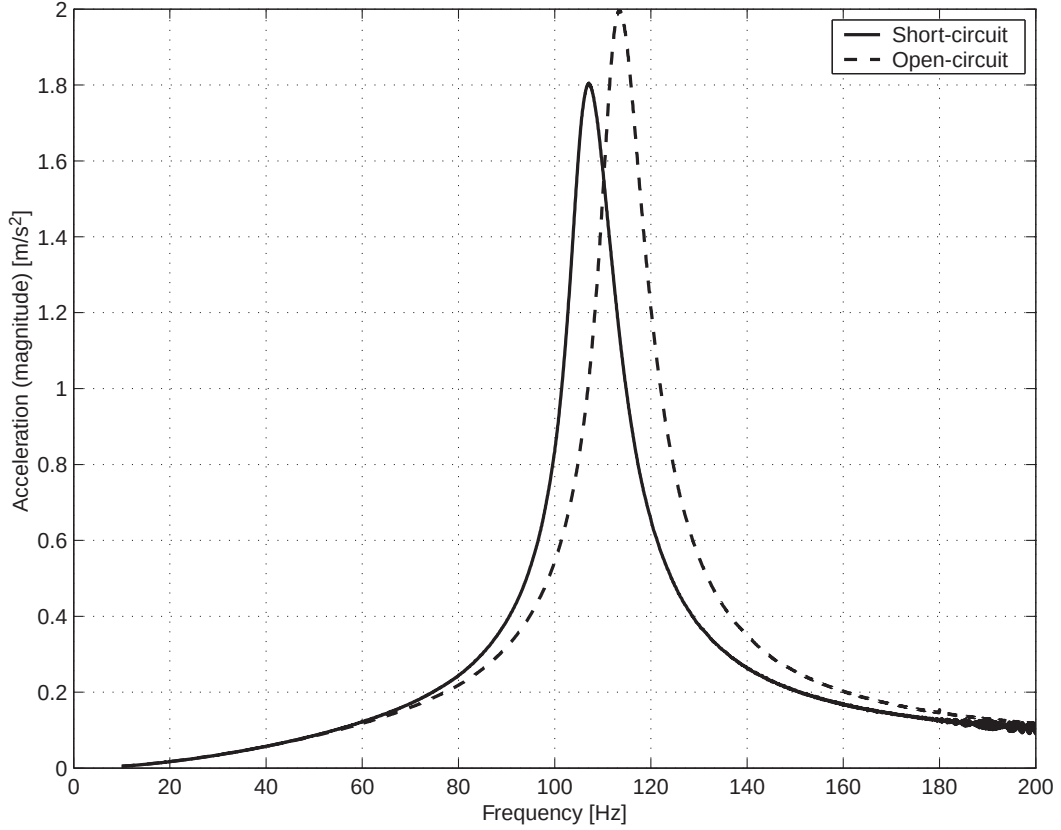


Figure 4-6: Example frequency sweeps at short- and open-circuit conditions. The resonance (corresponding to short-circuit) and the anti-resonance (open-circuit) frequencies are distinguishable.

ture. A secondary cause is material-related losses in the piezoelectric elements due to hysteresis.

The scanning mode of the vibrometer is also used here to measure the mode shape of the structure. A grid of scan points is specified on the device surface (see lower left of Figure 4-2) and a measurement is taken at each grid point. From the magnitude and phase data the mode shape is reconstructed (refer to Figure 4-3 for examples). This is primarily for visual verification, and to ensure that there are no torsional modes near the bending mode frequencies to complicate later measurements.

Next, the mechanical and electrical responses of the device, excited at either the resonance or anti-resonance frequencies, are measured for varying electric loads. The purpose of the test is to determine the maximum power generated and the optimal resistances at the resonance and anti-resonance frequencies, $R_{l,r}$ and $R_{l,ar}$. The system is driven with a sinusoidal signal with a frequency corresponding to either the resonance or the anti-resonance frequency. The magnitude of the drive signal is adjusted to ensure that a constant base acceleration (2.5 m/s^2) is obtained by monitoring this acceleration using the laser vibrometer. The laser vibrometer sensitivity was set at $10 \frac{\text{mm}}{\text{s}}/\text{V}$ for these tests since only the base acceleration was measured. The electrical load is varied using discrete resistors. Since the electrical load (damping) changes, the dynamics of the system change for each measurement, and the base acceleration (and drive signal) needs adjustment for each measurement. The optimal resistances are the resistances at which the maximum power is developed at the corresponding frequencies. The power is a function of the resistance and the voltage, so both of these values are required. Furthermore, the base acceleration has to be measured as explained earlier. Please refer to Figure 4-7 for an example power and voltage response.

The last performance test measures the device response away from the resonances. The same method as in Test 2 is employed, with the sole difference that the excitation frequencies do not coincide with the resonance and anti-resonance frequencies, respectively. The base acceleration is regulated to 2.5 m/s^2 in all cases by manually varying the shaker drive signal, based on laser vibrometer measurements of the base accel-

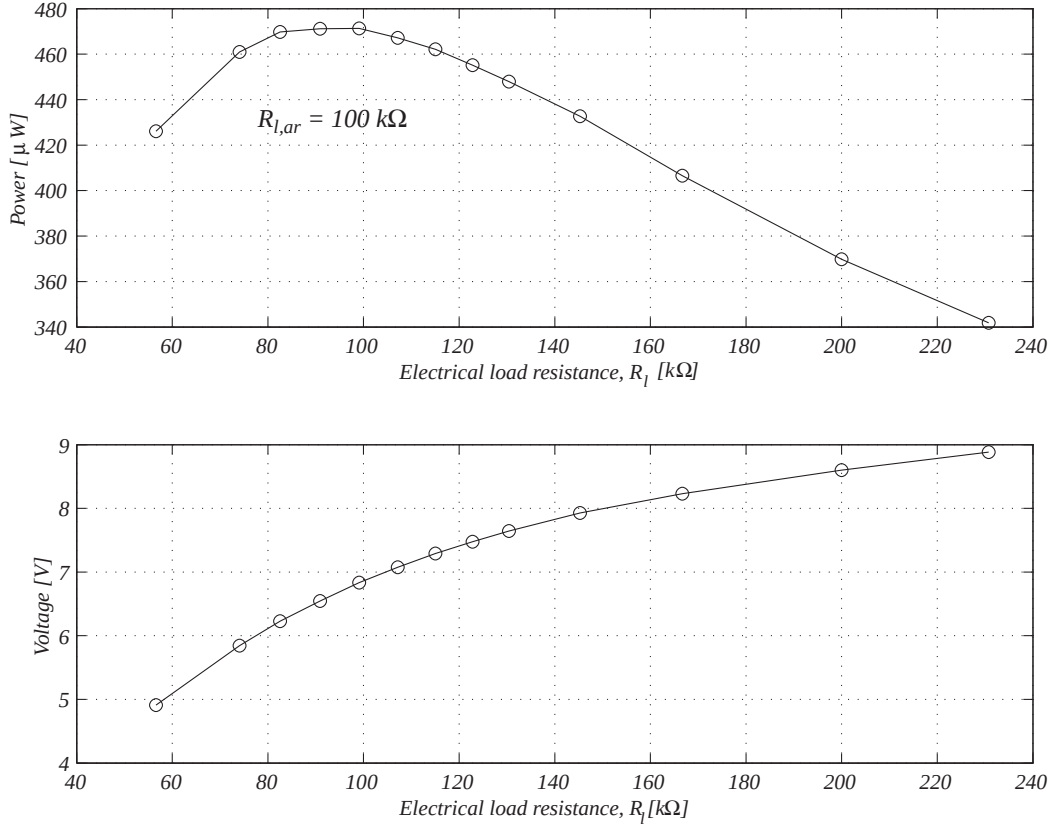


Figure 4-7: Example of (*top*) power and (*bottom*) voltage plotted against effective electrical load for Device 2 (Piezo Systems, Inc. bi-morph). These measurements were taken at the anti-resonance frequency. The electrical load, R_l is the effective electrical load (see Section 4.3.2 on the Power Measurement).

ation. The voltage and electrical load are recorded to obtain the power developed. Both the absolute tip and the base displacements are recorded, from which the relative motion is obtained. The laser vibrometer sensitivity was adjusted to $10 \frac{\text{mm}}{\text{s}}/\text{V}$, with the exception of the measurements at the resonance and anti-resonance frequencies, where a sensitivity of $25 \frac{\text{mm}}{\text{s}}/\text{V}$ was necessary. A response form similar to Figure 4-7 is obtained for the power and the voltage, though both parameters are significantly smaller away from resonance. Resolution on displacement and velocity is given in Table 4.2.

4.4 Summary of Experimental Results

In this chapter, the procedures for the measurements required for model implementation and validation/verification are presented. Two groups of measurements are necessary for model implementation: mechanical damping, and various device/material properties. The device mechanical damping is measured with a slight variation of the logarithmic decrement method. The material properties of interest are the elastic stiffness, the piezoelectric constant, and the permittivity. The results obtained for Device 2 are summarized in Table 4.2.

The performance of the harvester is measured using a combination of three parameters: voltage, power, and tip displacement. Three tests are defined: the resonance and anti-resonance frequency measurements, the voltage/power performance at these frequencies for varying electrical loads, and the off-resonance voltage/power measurements. The results will be discussed in Chapter 5 by comparison to the modeling results.

Chapter 5

Experimental Results and Model Verification

The experimentally measured performance for two harvesting devices is compared to results from the models presented in Chapter 3. Sodano *et.al* published experimental results for a bi-morph harvester device from Midé Technology Corporation: the QuickPack QP40N [55, 56], which will be referred to as “Device 1”. A model was also presented, showing good agreement with the measurements. As a preliminary validation, the device geometry and material properties are described and the model developed in Section 3.3 is implemented for the specific device. The second validation/verification was performed on a bi-morph device acquired from Piezo Systems, Inc.: the T226-A4-503X, and will be referred to as “Device 2”. The electrical performance (voltage and power) and the mechanical performance (lateral tip displacement) were measured in two operating regimes, at or near the resonance/anti-resonance frequencies, or away from this frequency. The procedures for these tests, including the mechanical damping measurement, are described in Chapter 4. Measured and predicted results are compared, both at and away from the natural frequencies. A consistent underprediction in electrical performance at resonant operation is addressed.

5.1 Bi-morph Harvester from the Open Literature

Experimental results were recently published for a bi-morph piezoelectric harvester device, the QuickPack QP40N, by Sodano *et al.* [49, 56]. Very little well documented experimental studies for model validation were found in the literature. A model is also reported in this prior work, though it is shown in Section 5.1.2 that there are some problems with the model as presented. Implementing the geometry with the model developed in Section 3.3, the reported experimental performance is compared to model results.

5.1.1 Geometry and Material Properties

The Quick Pack QP40N actuator from Midé Technology Corporation was modeled and tested by others [55, 56]. The Quick Pack is a composite plate device, consisting of four piezoelectric elements ($t_p = 0.254 \text{ mm}$) embedded in a Kapton and epoxy matrix ($t_s = 0.254 \text{ mm}$), with overall dimensions of $100.6 \text{ mm} \times 25.4 \text{ mm} \times 0.762 \text{ mm}$ (length $\times$ width $\times$ thickness). The electrode thicknesses were not reported, but are negligible. The electrical leads are non-uniform, and were lumped with the structural layer. Please refer to Figure 5-1 for an illustration of the device configuration. Each piezoelectric element has dimensions: $45.574 \text{ mm} \times 20.574 \text{ mm} \times 0.254 \text{ mm}$ (based on on device parameters published by Midé Technology Corporation [108]). The top and bottom piezoelectric elements are oppositely poled for the series connection configuration. The top and bottom electrodes for the two top elements are connected, and similarly for the two lower elements.

The following section outlines an interpretation of the device geometry for modeling in the current work. The device consists of four piezoelectric elements, where the two elements at the base are connected to the outer elements through a structural section. The structural section is reportedly much more compliant than the piezoelectric elements ($c_s = 2.5 \text{ GPa}$). With the compliant connection between the two sets of piezoelectric elements (see Figure 5-1) of length $\sim 4.33 \text{ mm}$ (approximately 4.7 % of the effective beam length), the bending strain transferred from the beam section

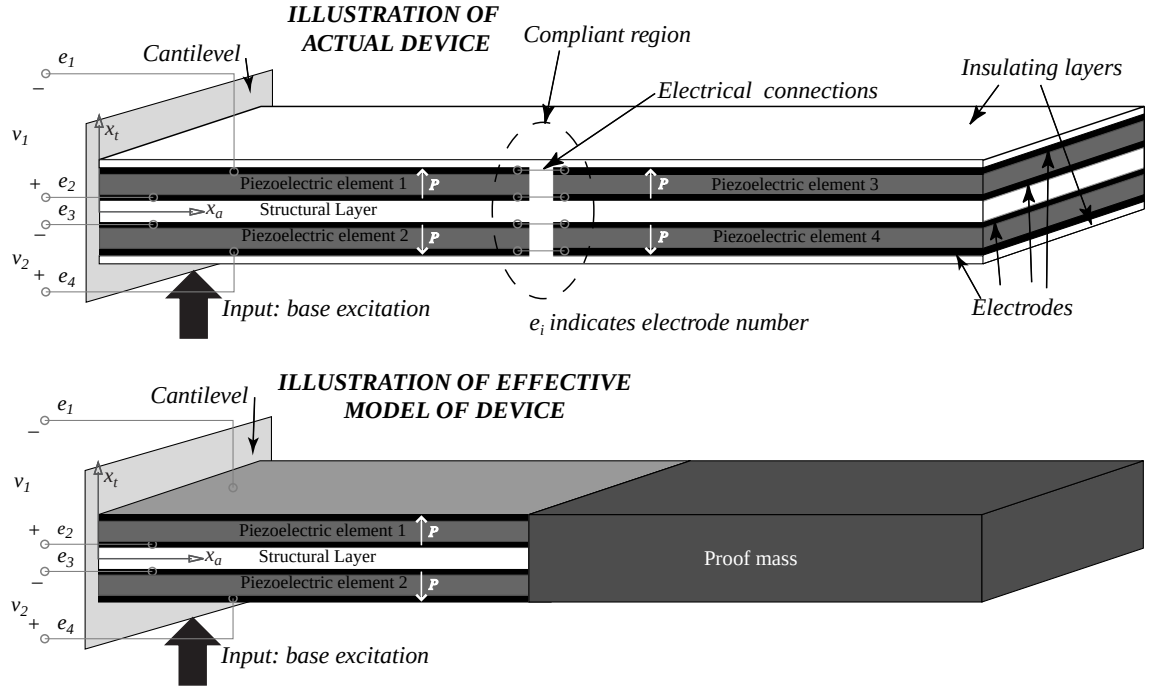


Figure 5-1: Device 1: Illustration of the (top) QuickPack QP40N bi-morph harvester and (bottom) the modeled device geometry.

to the outer set of piezoelectric elements will be negligible and the outer elements will rotate (approximately rigid body motion) about the compliant connection. Very little strain will develop and the electrical energy generated in the outer two elements will be negligible. Typically this device is bonded to a structure for actuation and the compliant region is of little consequence. Given this feature of the QuickPack device, the outer half of the device is modeled as a proof mass. It is assumed that the connection between the effective proof mass and the beam structure is rigid for the dynamic analysis, which is a zeroth order approximation. Furthermore, since the piezoelectric elements are much stiffer than the structural layers, the effect of the structure on the sides of the piezoelectric elements was ignored (the piezoelectric elements cover 81% of the beam width). The device width was simply modeled as the width of the piezoelectric elements (20.574 mm). The device was reportedly clamped 8 mm from the end, to have an effective length of 92.6 mm. Accounting for all of this, the harvester effective dimensions become: 40.1 mm $\times$ 20.574 mm $\times$ 762 μ m. A center structural

layer thickness of $100\ \mu m$ was assumed, based on cross-sectional measurements on a QuickPack QP25N (which has a similar geometry). The outer structural layers had thicknesses of $77\ \mu m$ each. The effect of these layers on the stiffness of the device was ignored since the stiffness of the piezoelectric elements is much higher than the structural layer, giving the device a total modeled thickness of $608\ \mu m$. The density of the structural layer, $\rho_s = 2150\ kg/m^3$, had to be adjusted (outer layers lumped with the center region) and an effective density of $5461\ kg/m^3$ was used for the structural layer. The proof mass has dimensions of $52.5\ mm \times 20.574\ mm \times 608\ \mu m$ and a mass of $4.87\ grams$ ($\rho_0 = 7415\ kg/m^3$). PZT-5A material properties [108] were used for the piezoelectric elements of the device. The material properties, device dimensions, and other input parameters for the model implementation are summarized in Table 5.1.

Some key input parameters were lacking in the published experimental results, including the mechanical damping ratio and the input base acceleration. A mechanical damping ratio of $\zeta_m = 0.01$ was assumed for the analysis, based on the mechanical damping ratio measured for Device 2 (refer to Section 5.3.2). A base acceleration of $9.81\ m/s^2$ or $1\ g$ was used, which was found to develop a maximum strain of $313\ \mu - strain$, which is within static design limits of the device materials. The maximum strain was limited to $\sim 500\ \mu - strain$ to avoid depoling and other non-linearities in the piezoelectric material. The geometry described above was implemented in the model developed in the current project, which is discussed next.

5.1.2 Model Implementation

In this section, the model implementation is described to illustrate how the model developed in Section 3.3 is to be applied for this specific geometry. In this case, the device is a symmetric bi-morph cantilevered structure. The device has two oppositely poled active elements and thus four electrodes (two pairs). However, in Section 3.3 it was assumed (for simplicity) that the device has only two electrodes since the equations of motion (eqs. 3.41 and 3.42) reduce to scalar equations. A scheme to convert this geometry into the single electrode pair setup is presented in Appendix

Table 5.1: Device dimensions and material properties for model of Device 1.

Property in model	Value
Device overall length, L [mm]	92.6
Modeled bi-morph length, L [mm]	40.1
Device width, b [mm]	20.6
Piezo layer thickness, t_p [μm]	254
Structure layer thickness, t_s [μm]	100
Device overall thickness, t_s [μm]	608
Device mass [grams]	8.59
Mass of proof mass [grams]	4.87
Proof mass length, L_0 [mm]	52.1
Proof mass width, b_0 [mm]	20.6
Proof mass thickness, t_0 [μm]	608
ρ_p [kg/m^3]	7700
c_{33}^{E*} [GPa] (beam stiffness)	63
d_{31} [m/V]	-179×10^{-12}
e_{31}^* [C/m ²] (beam piezoelectric constant)	$-11.3^\dagger$
ε_{33}^T [F/m]	$1800 \times \varepsilon_0$
ε_{33}^{S*} [F/m] (beam permittivity)	$1579 \times \varepsilon_0^\ddagger$
Beam stiffness for structural layer, c_s [GPa]	2.5
$\rho_s^\S$ [kg/m^3]	5461
ρ_0 [kg/m^3]	7415

[†] Calculated from eq. 3.74 for the beam geometry.

[‡] Calculated from eq. 3.75 for the beam geometry.

[§] Effective density includes outer structural layers lumped with the center layer.

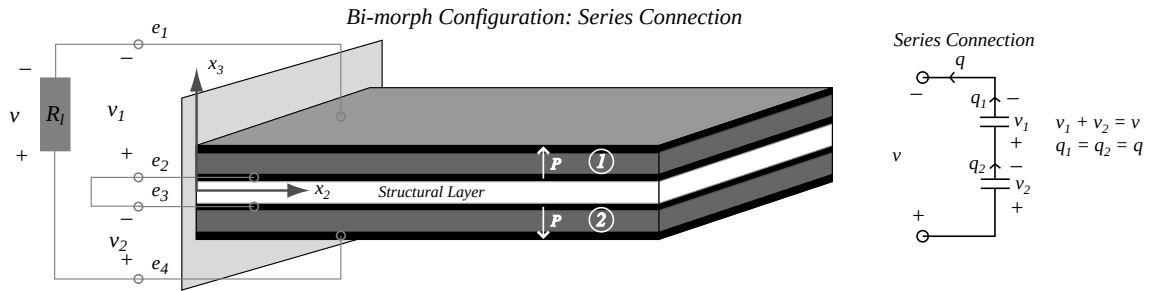


Figure 5-2: Device 1 electrical connections: (*left*) symmetric bi-morph configuration with oppositely poled active elements. The series connection (*right*) is illustrated with a simplified equivalent electrical circuit.

C. First, the effect of the piezoelectric element orientation needs to be discussed. In Section 3.3.1 it was briefly mentioned that the orientation of the piezoelectric element is defined by the poling of the element. Per convention, the material local coordinate x_3^* direction is always defined positive in the direction of poling. However, for the analysis, all parameters must be analyzed in the global coordinates, which can be chosen arbitrarily. When the local coordinates of the element local and the global coordinates do not align, adjustment of the constitutive relations are necessary. This is detailed in Appendix B. It is assumed here that the piezoelectric elements' local x_3^* directions are either parallel or antiparallel to the global x_3 direction. In this case, the piezoelectric constants can simply be multiplied with -1 when the element is poled in the $-x_3$ direction. The above assumption is slightly limiting, but the scheme is sufficient for the current analysis.

For the current configuration, the two active elements (elements 1 and 2, as illustrated in Figure 5-2) are oppositely poled, allowing for the series connection configuration. The two center electrodes (e_2 and e_3) are connected. When the active elements are represented as capacitors (for simplicity), the effective electrical circuit presented in Figure 5-2 is obtained. Since the developed strains are opposite above and below the neutral axis, the electric fields developed in the two elements will be in the same direction. The corresponding electric potential varies from 0 at the top of each piezoelectric element, to +1 at the bottom of each element:

$$\psi_v = \psi_{vI} = \begin{cases} \frac{-x_3 + (\frac{t_s}{2} + t_p)}{t_p} & \text{for } x_3 > 0 \quad (\text{corresponding to element 1}) \\ \frac{-x_3 - \frac{t_s}{2}}{t_p} & \text{for } x_3 < 0 \quad (\text{corresponding to element 2}) \end{cases} \quad (5.1)$$

The numbering of the active elements are indicated in Figure 5-2. The resulting electric fields are:

$$E = -\nabla \psi_v = \begin{cases} \frac{1}{t_p} & \text{for } x_3 > 0 \\ \frac{1}{t_p} & \text{for } x_3 < 0 \end{cases} \quad (5.2)$$

From Figure 5-2 it can be seen that the bottom element will have negative piezoelectric constants as the poling direction is opposite the global x_3 , or x_t , direction. Conversely, the top element will have positive piezoelectric constants. The coupling

and capacitance terms were defined in Section 3.3.1 as:

$$\Theta = \int_{V_p} (-x_t \psi_r'')^t \mathbf{e}^t (-\nabla \psi_v) dV_p \quad (5.3)$$

$$\mathbf{C}_p = \int_{V_p} (-\nabla \psi_v)^t \boldsymbol{\epsilon}^S (-\nabla \psi_v) dV_p \quad (5.4)$$

Note that the mode shape, ψ_r , is defined in eq. 3.67 and accounts for the proof mass contribution. Calculating these terms for the individual elements in the current geometry and for each element, the following is obtained:

$$\theta_1 = \theta_2 = e_{31}^* \psi_r'(L) \left(\frac{t_p + t_s}{2} \right) b \quad (5.5)$$

$$C_{p1} = C_{p2} = \frac{\varepsilon_{33}^S b L}{t_p} \quad (5.6)$$

Since there are four electrodes, the equations of motion are not scalar, which makes the analysis more tedious. Thus, an effective system is defined with one electrode pair as detailed in Appendix C, and the resulting effective coefficients (see eqs. (C.20) and (C.21)) for the system are: the coupling term of the effective system is equal to the coupling term of the individual elements, $\theta_e = \theta_1$. The effective capacitive term is one half the capacitance of the individual elements, $C_{p,e} = \frac{1}{2} C_{p1}$, which is expected for two identical capacitors connected in series. With these effective parameters, the equations of motion reduce to the scalar equations developed in Section 3.3 (eqs. (3.41) and (3.42)) and the scheme can be used to predict the performance of the system.

It should be stated briefly that a number of inconsistencies surfaced in the prior model presented with the experimental results [55]. All references to equations in this paragraph are from the group's article in Strain (2004). First, the external work term defined in Sodano *et al.* eq. (4) is written in terms of applied electrical work (applied charge) [85]. For a harvester system, electrical work is extracted and charge is generated. Thus, the second term in this equation should be positive to

account for the work extracted. This correction flows through to Sodano *et al.* eq. (18), where the right hand side term of the sensing equation should have a negative sign. This sign difference was compensated for by defining the current as $i = -\frac{dq}{dt}$ in Sodano *et al.* eq. (19). In the end, the correct governing equations are obtained: Sodano *et al.* equation (22). The second inconsistency is the developed electrical field in the two active elements. Midé Technology Corporation has confirmed that all QuickPack devices are oppositely poled and the series connection needs to be used for these devices (there is no way around this experimentally, given the device configuration). The resulting electrical field in the two elements should be in the same direction, as seen in eq. (5.2). However, in the Strain (2004) article, the electric fields are defined in opposite directions in Sodano *et al.* eq. (13), which is consistent with a parallel connection configuration. It is suspected that the opposing poling directions of the active elements were not accounted for (refer to Appendix B). In other words, it appears as though a bi-morph with the parallel connection was modeled to obtain the coefficients for the equations of motion. For the interconnections it is suspected that a series configuration was modeled and the device tested in the series configuration, since the parallel connection of a series configuration device would result in no electrical energy generation. Lastly, the beam was reportedly modeled with a uniform density and stiffness, suggesting that the device was modeled as a cantilevered bi-morph beam with all four piezoelectric elements generating power, rather than just two due to the compliant connection. Thus, even though there were a number of inconsistencies in the prior modeling, a correct set of governing equations in their final form was obtained.

5.1.3 Results Comparison: Preliminary Validation

The predicted results from the implemented model are compared to the experimental results presented by Sodana *et al.* [55] for Device 1. The measurements published were away from the resonance and/or anti-resonance frequencies. The actual resonance frequency of the device is not stated, nor the anti-resonance frequency. It is estimated from the frequency response plot of the system that the resonance fre-

Table 5.2: Device 1: Predicted and measured performance below the first resonance of $\sim 33 \text{ Hz}$

Frequency [Hz]	Resistance [Ω]	Predicted current [mA]	Measured current [mA]	Error [%]
25	100	0.104	0.105	0.6
25	10,000	0.106	0.104	-1.5
30	100	0.360	0.280	-22
30	10,000	0.295	0.279	-5.4

quency is between 30 and 35 Hz . This aligns very well with the predicted (from this work) resonance frequency of 33.5 Hz , and anti-resonance frequency of 35.4 Hz . The response of the system near the resonances¹ is summarized in Table 5.2.

From the comparison, it is clear that the model predicts the off-resonance response well (25 Hz), though the resonance response prediction is low. Given all the assumptions necessary for the obtained result, the comparison served only as an order of magnitude validation. Higher frequency results (50, 75, and 150 Hz) are also included in the published experimental data, though these frequencies are increasingly close to the second bending mode of the structure ($\sim 200 \text{ Hz}$), for which this model does not account (only the first bending mode is analyzed). As a result, poor correlation was obtained at these frequencies, which is to be expected.

These results were not enough to validate the model developed in Section 3.3. No other well-documented (for modeling purposes) experimental data was found in the open literature. Therefore, a macro-scale device was acquired and tested for model validation/verification, as discussed in the section to follow.

5.2 Device 2: Bi-morph Harvester

Device 2 was acquired from Piezo Systems, Inc.: the T226-A4-503X (see Figure 5-3). The device has a symmetric bi-morph configuration (Figure 5-2) with no proof mass, as can be seen in Figure 5-3. In an attempt to minimize uncertainty in the modeling,

¹In this section, and the sections to follow, reference to the “resonances” will imply both the resonance and anti-resonance frequencies.

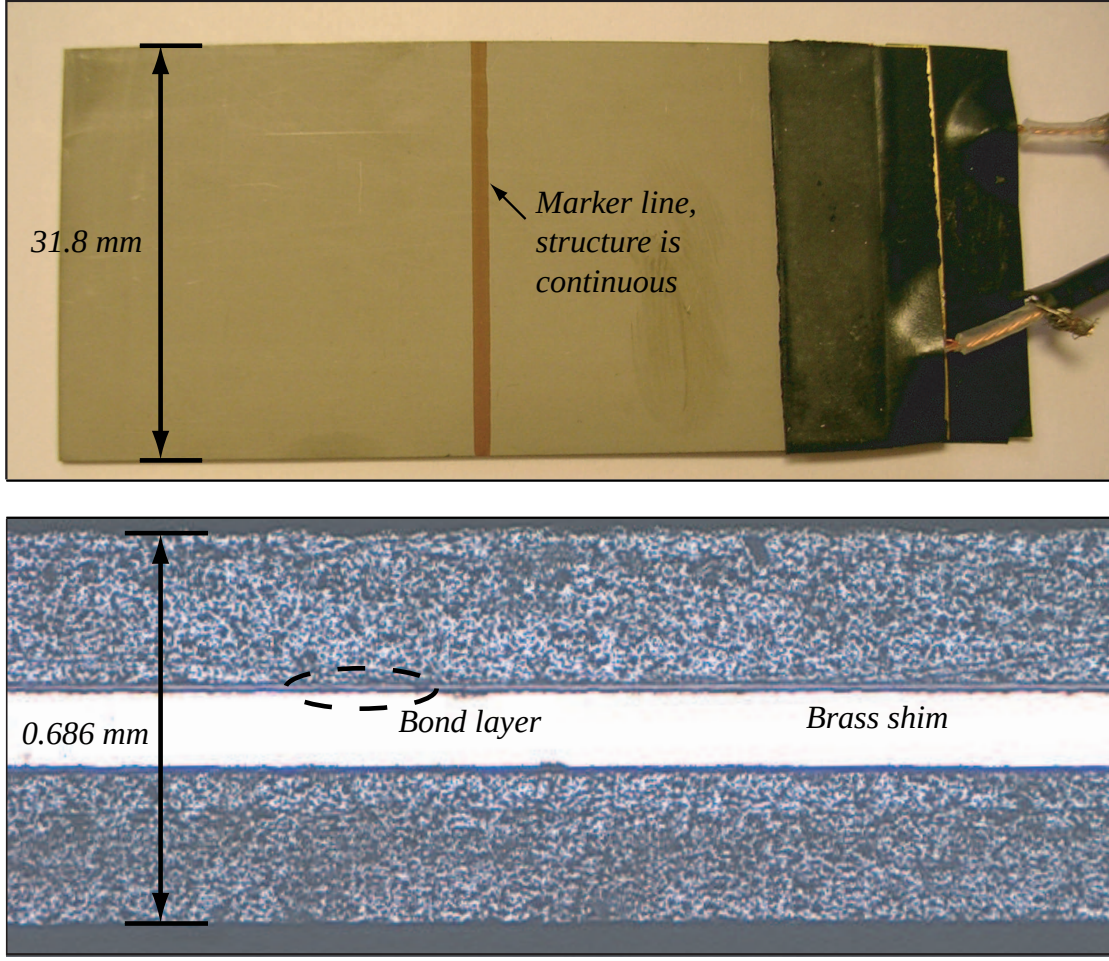


Figure 5-3: Device 2: Photos of (*top*) planform of device and (*bottom*) a cross-section of the individual layers of the device.

the geometric parameters of the device, as well as material properties were measured whenever possible. These parameters (summarized in Section 4.1, Table 4.2) were used in the model implementation, and the modeling results are compared to the experimental results in the section to follow.

Device 2 consists of two metallized (electroded) piezoelectric elements (PZT-5A), separated by a brass shim (see Figure 5-3). The piezoelectric elements are oppositely poled, allowing for series connection (as with Device 1). The brass shim serves as the electrical connect between electrodes e_2 and e_3 , as illustrated in Figure 5-2.

The as-acquired (un-clamped) device dimensions are $63.5 \text{ mm} \times 31.8 \text{ mm} \times 0.686 \text{ mm}$ (*length* $\times$ *width* $\times$ *thickness*). The device was inspected under a micro-

scope (at $10\times$ magnification) to ensure that no cracks are present on the surfaces. The thicknesses of the individual layers was measured using the Zeiss microscope ($t_p = 270 \mu m$ and $t_s = 140 \mu m$, accurate to within $\pm 5 \mu m$). The nickel electrode thicknesses are negligible.

The device did not come with electrical leads, and the procedure to attach the leads is discussed briefly here. The lead wires were connected to the piezoelectric elements using an intermediate brass pad. Initially, the lead wires were soldered directly onto the piezoelectric electrodes. However, the soldering process locally raises the temperature of the piezoelectric elements above the Curie temperature, resulting in local de-poling. The process was modified and the leads were first soldered to bronze pads before the pads were bonded to the piezoelectric element electrodes (e_1 and e_4 , respectively) using a conductive tape (3M 1182 Tape).

The device was mounted on an electrostatic shaker with an aluminum clamp of length 8.5 mm , width $\sim 45 \text{ mm}$, and thickness $\sim 5 \text{ mm}$ (see Figure 5-4). A clamping pressure of 12.5 MPa , or 1800 Psi , was applied (the clamping area was $8.5 \text{ mm} \times 31.8 \text{ mm}$), which was controlled with a torque wrench. The effective length of the clamped device was $L = 55.0 \text{ mm}$.

The leads were connected to the electrical load resistance as illustrated in Figure 5-2. The active elements are oppositely poled, allowing for the series connection configuration. The effective electrical circuit is also shown in Figure 5-2 (where the piezoelectric elements are simplistically represented as capacitors). Test procedures are detailed in Section 4.3.

5.3 Comparison and Discussion of Results

In this section, the predicted and measured performance for Device 2 are compared. Preliminary model validation suggested that the model predicted the off-resonance response of the device well, but not the resonance response. It is shown that the model does predict all the trends accurately, though the prediction (voltage and power) is

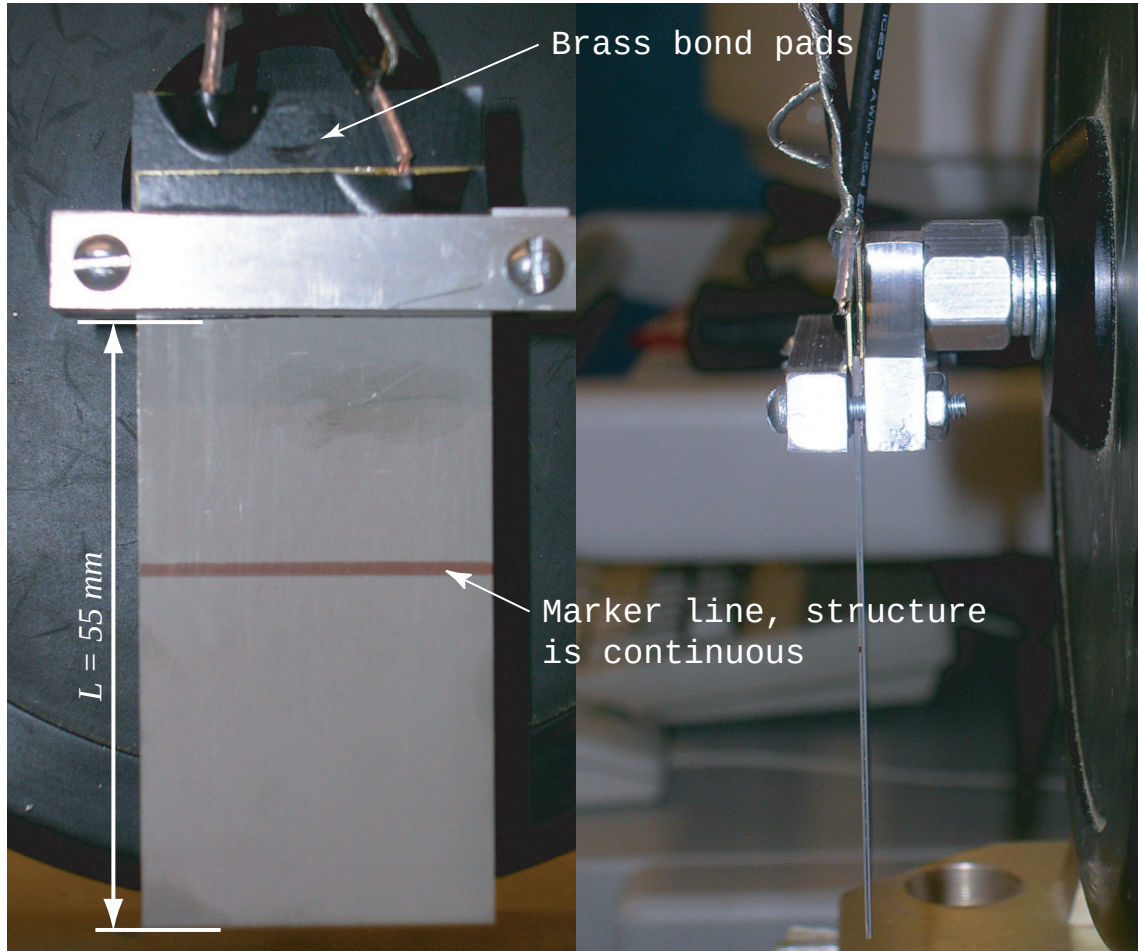


Figure 5-4: Device 2: photos of the clamped device mounted on the electrostatic shaker. The indicated marker line appears on the as-acquired device and is of no structural consequence.

consistently low at the resonances².

First, the positions of the resonance and anti-resonance frequencies are compared, as is the optimal electrical loadings (the electrical loading required for maximum power generation at resonance and anti-resonance respectively). Next, the overall response of the system is investigated. Consistent with the preliminary model validation, the correlation of the model to experimental results depend on the operating regime (*i.e.*, at or away from resonance). The off-resonance results are compared first. Lastly, the resonance results of the system is compared to the predictions, and

²Again, reference to “resonances” in this chapter implies both the resonance and anti-resonance frequencies.

the model underprediction is explained. Tabulated experimental data is provided in Appendix D to supplement the plots in this chapter.

5.3.1 Resonance Frequency, Anti-resonance Frequency, and Optimal Resistances

The first comparison between the simulation and the measurements is in the position of the resonance and anti-resonance frequencies. The measured results suggested the resonance and anti-resonance frequencies of $f_r = 106.5$ to 107.8 Hz and $f_{ar} = 112$ to 113.8 Hz , respectively. This is compared to the predictions of $f_r = 106.8$ Hz and $f_{ar} = 112.9$ Hz for the resonance and anti-resonance, respectively. Please refer to Figure 4.6 for the resonance test results. Given that published material properties were used for the piezoelectric material elastic stiffness, the results for the resonance frequency agree very well. The anti-resonance frequency is a function the elastic stiffness of the device and the piezoelectric coupling. This experimental value was used to calculate the piezoelectric constant, e_{31}^* , as discussed in Section 4.1.2.

The optimal electrical loading for maximum power extraction was determined by exciting the device at either the resonance or anti-resonance frequencies and sweeping through the electrical loads by using discrete resistors. As discussed in Chapter 4, the base acceleration of 2.5 m/s^2 was held constant during these tests by monitoring the base acceleration with the laser vibrometer. The measured values were $R_{l,r} = 11$ $k\Omega$ and $R_{l,ar} = 100$ $k\Omega$. These again compare very well with the predicted values of $R_{l,r} = 10.0$ $k\Omega$ and $R_{l,ar} = 108$ $k\Omega$. These results suggests that the model captures the overall response of the structure at and around the resonances. The next step is to compare the predicted and measured response (including voltage and power) at these resonances.

5.3.2 Overall Device Response

First, the overall response of the system is presented. As an illustration of the performance, the power is plotted vs. varying frequency ratio for different electrical loads.

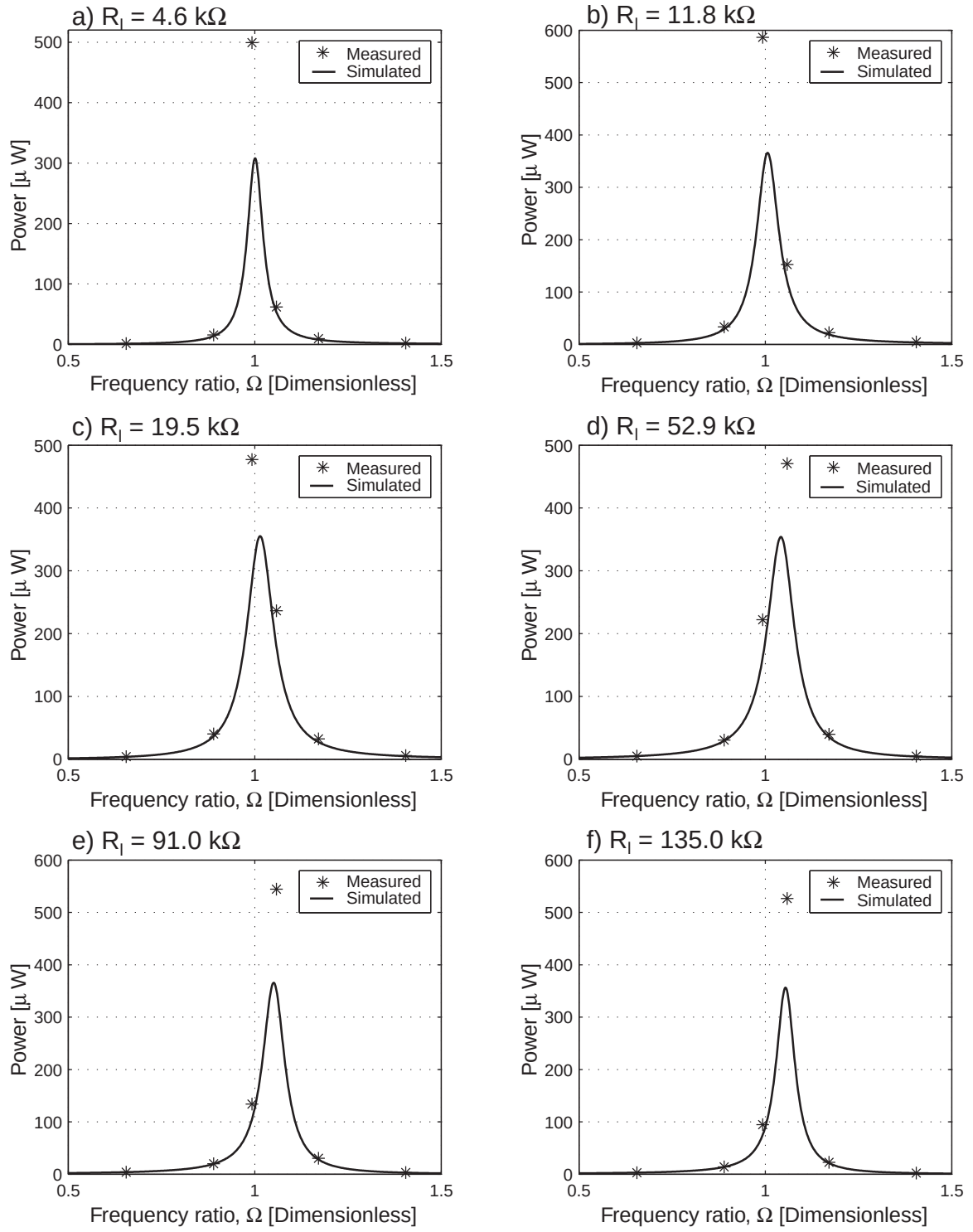


Figure 5-5: Predicted vs. measured power plotted vs. frequency for varying electrical loads. Base acceleration is held constant at 2.5 m/s^2 . $f_r = 107 \text{ Hz}$ and $f_{ar} = 113 \text{ Hz}$.

The results are presented in Figure 5-5. It is clear that the predicted results are in excellent agreement away from the resonances (in the small-strain regime), but there is significant deviation from the experimental results at the resonances. These two operating regimes will be investigated in detail in the sections to follow.

5.3.3 Off-resonance Response

From the previous section it is clear that the model underpredicts the electrical response of the system at the resonances, though the correlation away from the resonances is excellent. The next step was to measure the off-resonance electrical and mechanical response of the system in more detail, and compare this to simulations. A test matrix was set up to measure this response. The tabulated results are included in Appendix D for the voltage generated, the power generated, and the relative tip displacements. The voltage data is presented here first.

In Figure 5-6 it can be seen that the model predicts the device electrical response very well away from resonance (refer to plots a), b), e), and f)). In plot c), the driving frequency was the resonance frequency. At low electrical loads, the deviation between the measured and predicted response is pronounced. However, at higher electrical loads, the resistance approaches the open-circuit condition and the natural frequency of the device shifts to the anti-resonance frequency (therefore, small displacements and strains are developed here). Thus, the device is effectively operated away from the resonance, and the simulations and measured results align well. The converse is also true at anti-resonance, as seen in plot d). At low electrical loads, the natural frequency corresponds to the resonance frequency, and the device is effectively operated away from resonance. The simulated and predicted results align well there. However, at higher electrical loads, the device natural frequency coincides with the anti-resonance frequency, and the model consistently underpredicts the electrical response.

When comparing the power developed, as shown in Figure 5-7, the same trends in model agreement are observable. The power is calculated from the square of the voltage and resistance measurements and any errors in these measurements will accumulate (especially errors in the voltage measurements). It is observed that the

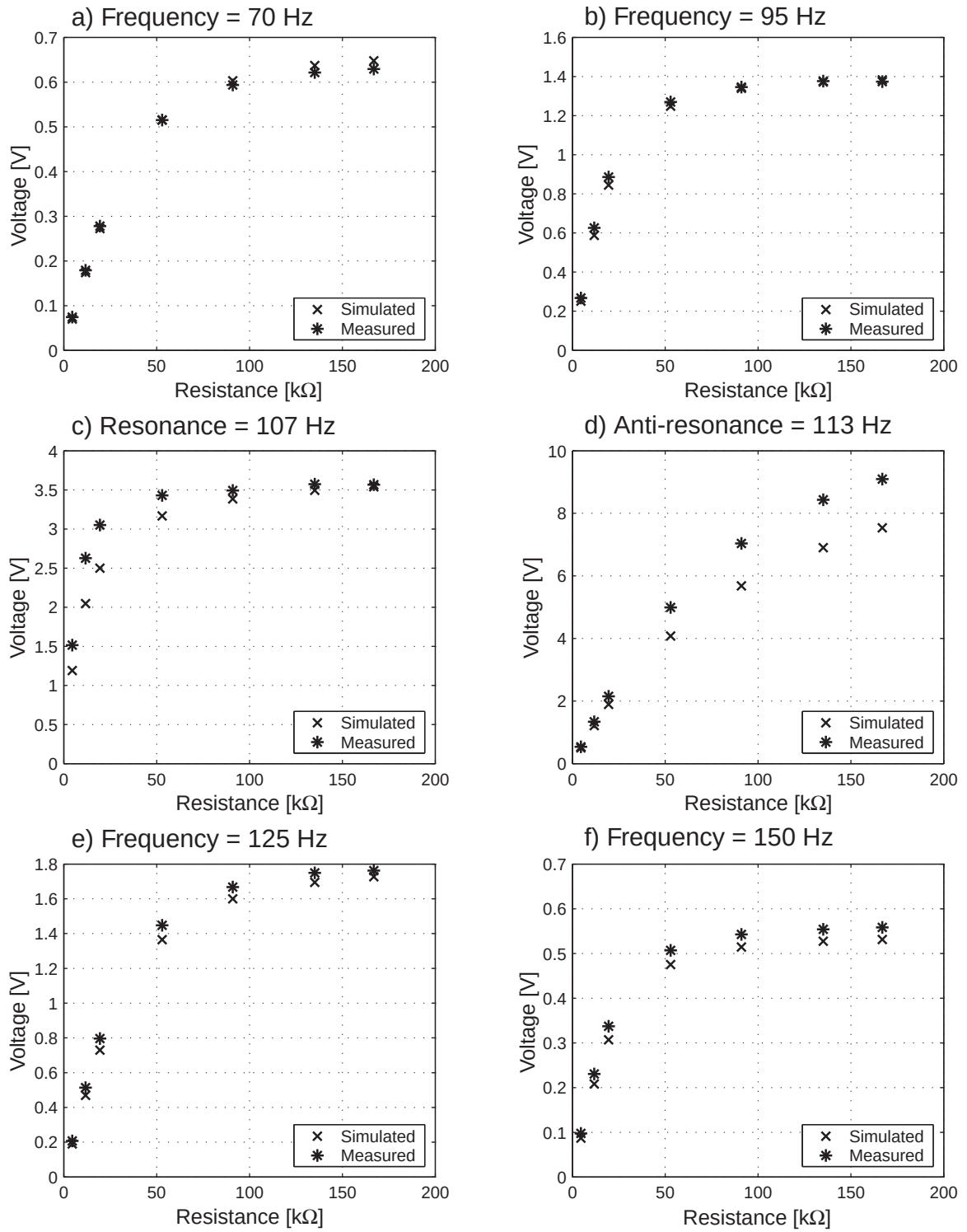


Figure 5-6: Predicted vs. measured response: voltage plotted against electrical load at various input frequencies, corresponding to both off-resonance and resonance/anti-resonance frequencies. Base acceleration was held constant at 2.5 m/s^2 .

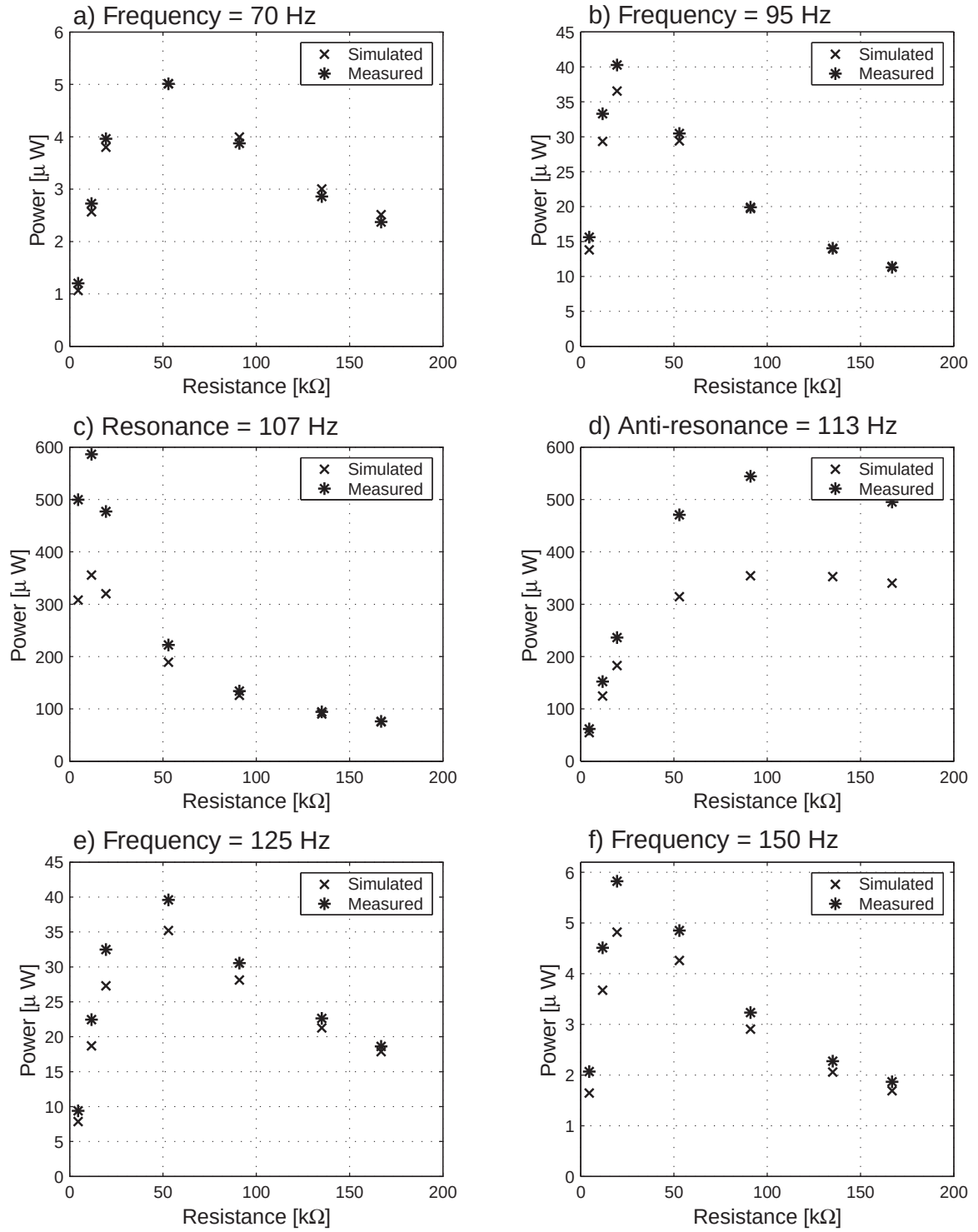


Figure 5-7: Predicted vs. measured response: power plotted against electrical load at various input frequencies, corresponding to both off-resonance and resonance/anti-resonance frequencies. Base acceleration was held constant at 2.5 m/s^2 .

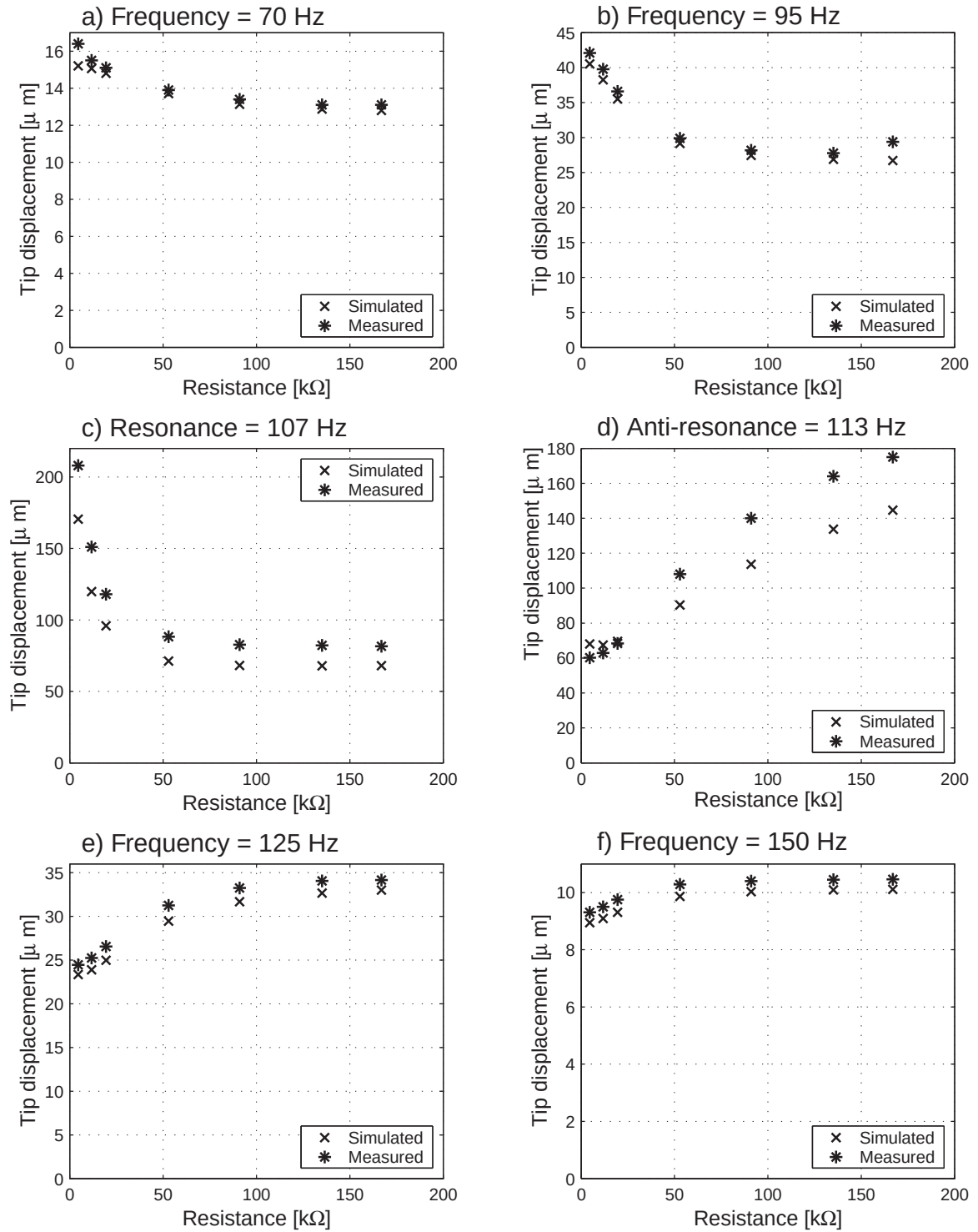


Figure 5-8: Predicted vs. measured response. Relative tip displacement plotted against electrical load at various input frequencies, corresponding to both off-resonance and resonance/anti-resonance frequencies. Base acceleration was held constant at 2.5 m/s^2 .

maximum power developed at the resonance frequency ($\sim 590 \mu W$) is slightly higher than at the anti-resonance frequency ($\sim 550 \mu W$).

Last, the predicted and measured mechanical response of the system is compared using the tip displacement in Figure 5-8. Again, the same trends as above are observed: good correlation away from the resonances, underprediction around the resonances. It would be expected that the predicted relative tip displacement at resonance frequency should line up well with the measured tip displacement at low electrical load. This is because at low electrical loads the short circuit electrical condition is approached. The reason for the discrepancy is that the lowest electrical load at which a measurement was taken was $4.61 k\Omega$. The mechanical damping was measured at zero electrical load (no piezoelectric coupling) and tip displacement aligns perfectly for this condition because the damping was determined using the measured tip and base displacement ratio (see Section 4.2).

5.3.4 Resonance Response

From the previous section it is clear that the simulated response differed from the measured response at or near the resonances. This phenomenon was investigated in more detail. The device was driven at the resonance and anti-resonance frequencies, respectively, while varying the electrical load. The driving signal magnitude was adjusted for each measurement to ensure that the base input acceleration was kept constant ($\ddot{w}_B = 2.5 m/s^2$) as before. The results at the resonance frequency are compared first.

The electrical load was varied from $4.6 \rightarrow 167 k\Omega$. The simulated and measured electrical responses are shown in Figure 5-9. It is clear that the model predicts the trends and locations of the power maxima correctly, but the measured voltages are consistently high as compared to the simulation. Since the power is a function of the voltage squared, the error in the power predictions are more substantial. It is interesting to note that again the simulation and measurements are in good agreement away from resonance (high electrical loading, since the natural frequency of the device corresponds to the anti-resonance frequency). The mechanical response in terms of

the tip displacement has been compared in Section 5.3.4 (see Figure 5-8), both at and away from the resonances.

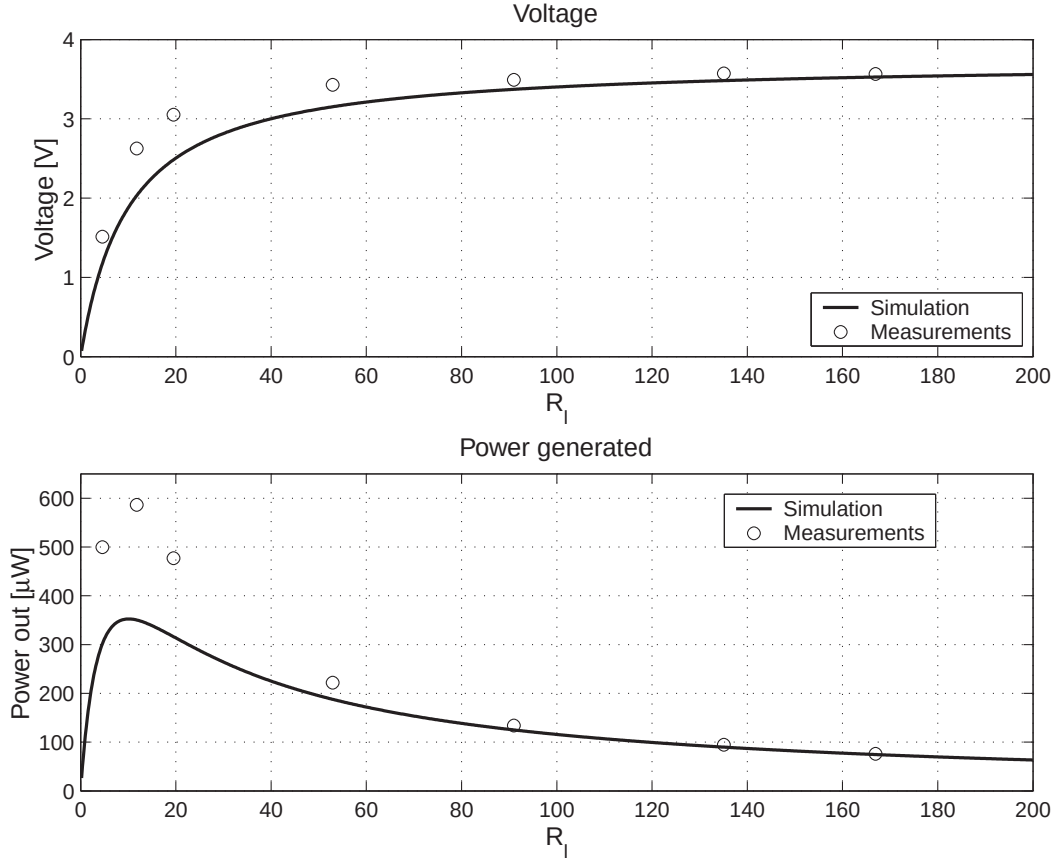


Figure 5-9: Predicted vs. measured response at the resonance frequency (107 Hz): (top) voltage and (bottom) power plotted vs. electrical load.

The simulated and measured results were also compared at the anti-resonance frequency. As with the resonance frequency operation, the model consistently under-predicts the performance of the device. However, away from the resonance (for low electrical loading), the correlation between the modeled and measured performance is again excellent. The measured maximum power at the anti-resonance frequency was slightly lower than at resonance. This was contrary to predictions that showed equal maximum power at the resonance and anti-resonance frequencies should be obtained. This deviation is discussed in Section 5.4 and is attributed to non-linear piezoelectric constitutive relations.

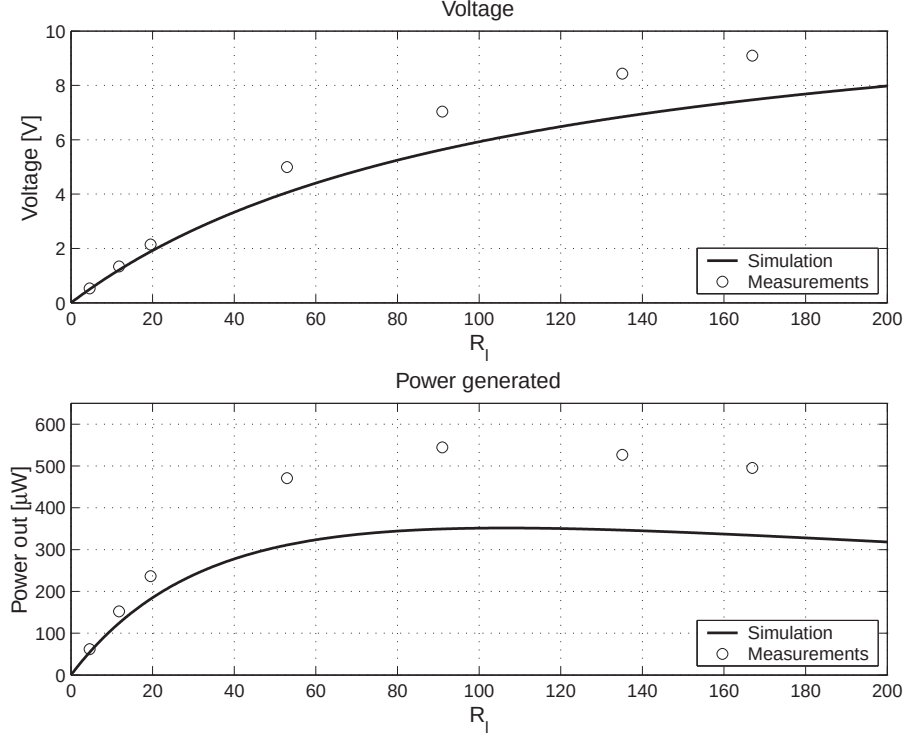


Figure 5-10: Predicted vs. measured response at the anti-resonance frequency (113 Hz): (*top*) voltage and (*bottom*) power plotted vs. electrical load.

Overall, the model predicts the response of the device at resonances well, except that the magnitude predictions are consistently low (power and voltage).

5.3.5 Un-modeled Piezoelectric Response

From the above discussion it is clear that the model predicts the response of the device away from resonance very well, but generally underpredicts the electrical and mechanical response at the resonances. It is suspected that this low prediction is an artifact of the linear small-signal model used to model the piezoelectric effect (in the constitutive relations). Crawley *et al.* experimentally documented the deviation of the piezoelectric constant from the linearized model at varying mechanical loadings (applied strain) [66]. It was shown that at low frequencies, the piezoelectric coupling deviates by as much as a factor of 70% *increase* at 100 μ -strain. At the resonances, the mechanical response of the device is much larger than away from resonance for

Device 2, and the developed strains in the elements are higher (*e.g.*, $\sim 5 \mu - strain$ at $R_l = 4.61 k\Omega$ and $70 Hz$, compared to $\sim 50 \mu - strain$ at $R_l = 11.8 k\Omega$ and at the resonance frequency). The maximum strain developed at the base of the structure is plotted against the electrical loading at both resonance and anti-resonance frequencies in Figure 5-11. The maximum strain developed is $\sim 50 \mu - strain$, suggesting that a higher electrical conversion is to be expected. Data on the effect of mechanical loading on the relative permittivity has not been obtained.

The deviation of the piezoelectric constant from the small-signal linear model value at varying mechanical loading also explains the lower maximum power developed at the anti-resonance frequency. At resonance, the maximum strain developed in the device is 10% higher than at the anti-resonance frequency. Thus, a higher piezoelectric constant can be expected (due to the non-linear relationship between applied strain and this constant). The voltage is proportional to the piezoelectric constant, and the the power is proportional to the voltage squared. Thus, this difference in strain can easily translate into the 3% decline in piezoelectric constant required to result in a 10% variation in the powers developed.

Thus, piezoelectric large-signal vs. small-signal constitutive response adequately explains the model underprediction at the resonances. Structural non-linearity is not an explanation, given the magnitude of the tip displacements at the resonances.

5.4 Validation and Verification

In the previous sections, the simulated and measured response for Device 2 were compared. First, the model predicted the overall response of the system very well, including the resonance and anti-resonance frequencies, and the corresponding optimal electrical resistances (for maximum power extraction). At the resonances, the simulations consistently predicted a lower electrical and mechanical performance. However, the model captured the response trends very well. The higher measured values are attributed to the non-linear response of the piezoelectric element to applied strain [66]. At higher applied strain conditions, the piezoelectric constant is higher than

assumed with the small-signal linear model. Thus, a higher electric field is induced, resulting in both higher voltages and higher power generation.

The power peaks at the resonances were predicted to be equal, though the measurements indicated that the resonance peak is slightly ($\sim 10\%$) higher than the anti-resonance peak. This is also explained by the non-linear response of the piezoelectric element to applied mechanical loads. From Figure 5-11 it is seen that the induced strain is around 10% higher at resonance than at anti-resonance. Combined with the non-linear response of the piezoelectric constant to applied strain, this lowered strain condition would explain the 3% change in piezoelectric constant from resonance to anti-resonance frequencies required to result in the 10% difference in power generated.

Lastly, the performance of Device 2 was analyzed away from resonance (in the small applied strain regime). In this regime, it was found that the model predicted

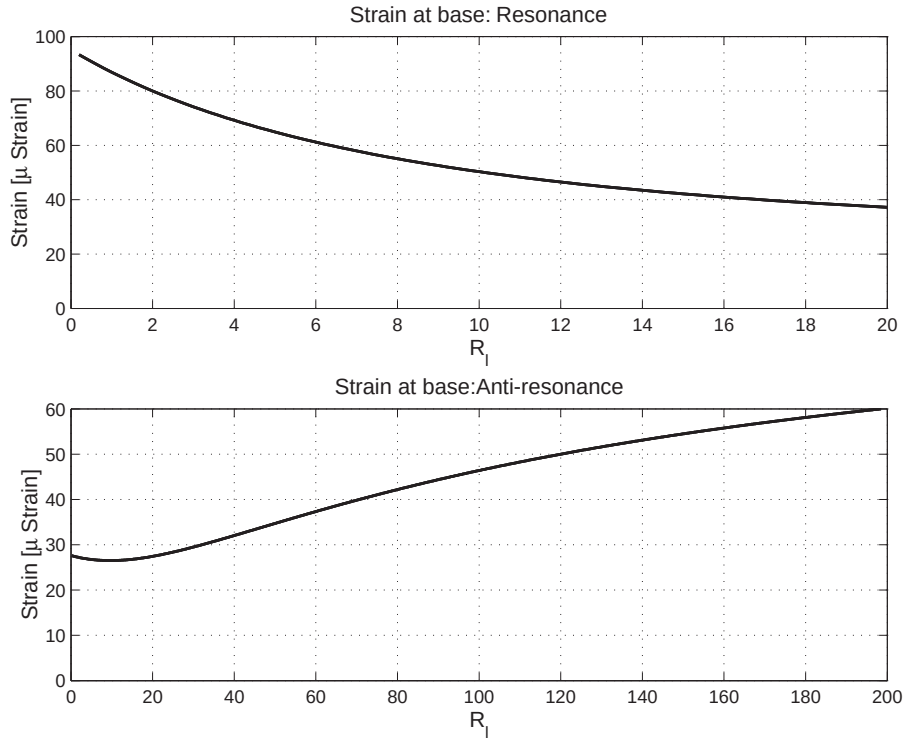


Figure 5-11: Predicted maximum strain at the base of the structure for varying electrical loads: (*top*) resonance and (*bottom*) anti-resonance response.

the device response very well. There is excellent agreement between both the electrical and mechanical response of the system. This supports the hypothesis that the better than predicted electrical performance around the resonances are in fact due to the non-linear response of piezoelectric materials to applied strain, rather than a problem with the model.

Based on these results, it was concluded that the model accurately represents the physical system away from resonance. Around the resonances, the model consistently underpredicts the mechanical and electrical performance of the device due to piezoelectric material non-linearity. Thus, when applying the model for design purposes, it can be expected that the device will produce more power (though only by a factor of < 2) at the resonances.

Chapter 6

Prototype Design and Fabrication

In this chapter, the model that was developed in Chapter 3 and validated in Chapter 5 is implemented as a design tool. A prototype device is designed to operate at 150 Hz and a base acceleration of 2.5 m/s^2 (0.25 gs) and thus targets *low-level* ambient vibrations. A MEMS-scale device was targeted to demonstrate that a viable device geometry is obtainable for such low operational frequency. When designing a MEMS-scale device, the fabrication processes determines the geometry of the device to a large extent. For this reason, a concurrent approach is to be employed where the device geometry and the fabrication process is developed simultaneously. Once a viable fabrication sequence is obtained, the device dimensions can be optimized, based on the constraints imposed by the fabrication processes. Packaging of the device, which is critical for manufacturing the power sub-system and important in device performance (due to damping, as discussed in Section 2.3), is not addressed. The packaging will be part of a chip-level device design, consisting of a system of individual harvesters. Only a single harvester is modeled and optimized here.

Before the device geometry is described, lessons learnt in the clean room, as well as from the power optimization in Section 3.3, are summarized. Based on these considerations, a basic device geometry is specified and a viable fabrication process constructed. The design dimensions are then optimized for the geometry. Lastly, a scheme for a device-level implementation of the individual harvester devices is presented.

6.1 Device Design Considerations

In Section 3.3.2, the power extracted from a resonant beam harvester was optimized, and a number of counter-intuitive results were obtained. These results have certain implications on the design of an energy harvester, which are discussed in the section to follow. A high-frequency prototype harvester device, the PMPG (Piezoelectric Micro Power Generator), has previously been developed, fabricated, and tested [39, 87]. In an attempt to replicate the fabrication of the device in this work, many valuable lessons were learnt with regards to the fabrication of such MEMS devices. These lessons are summarized in Section 6.1.2.

6.1.1 Modeling: Power Optimization

The power optimization undertaken in Section 3.3.2 showed that, when the harvester is operated at resonance (or anti-resonance) frequency under optimal electrical loads (*e.g.*, the maximum power is generated), the piezoelectric coupling cancels out of the equation. The only material properties that affect the power generated are the density and the bending modulus of the total device. The density should be maximized, while the elastic stiffness should be minimized. This simple result implies the following: for power generation, the mode of operation ($\{3-1\}$ vs. $\{3-3\}$) has very little effect on the power generated. The material density is independent of orientation, and the difference between the bending modulus, parallel and perpendicular to the poling direction, respectively, is small for common piezoelectrics/poled ferroelectrics, *e.g.*, PZT-5A vs. PZT-5H. Thus, the maximum power generated is largely independent of material selection and mode of operation. The same is not true for the voltage and current generated.

Both the material selection and the mode of operation will determine the voltages and currents developed in the device. The voltage developed at optimal power extraction is inversely proportional to the piezoelectric constant, e_{31}^* , whereas the current is proportional to this coefficient. Also, one of the advantages of using the $\{3-3\}$ mode of operation is that the output voltage can be controlled. This voltage is determined

by the spacing between the electrodes. Since interdigitated electrodes are used for this mode of operation (see Section 3.3.5), the pitch between the electrodes can be varied to obtain the required voltage. In the case of the {3-1} mode of operation, the thickness of the piezoelectric layers determines the voltage output. However, this thickness also influences dynamics of the structure, and is limited by the deposition process. This lack of flexibility in the control of the voltage in the individual {3-1} device is offset by the ease with which these devices can be interconnected on a die to form a system of harvesters. By connecting these devices in a combination of series and parallel connections¹, the electrical output may be tailored.

Another consideration is the interconnections of the elements of the individual bi-morph {3-1} energy harvester. For the series configuration, the voltages add, and the current developed is constant. For the parallel connection, the currents add and the voltages are constant (refer to Appendix C). This also gives some flexibility in controlling the electrical output from the individual harvesters. A more important consideration, however, is the ease with which these configurations can be implemented in a MEMS device. A parallel connection is preferred and this is discussed in more detail in Section 6.3.

Based on these considerations, a device utilizing the {3-1} mode of operation is selected since the marginal benefit of using the {3-3} mode of operation is offset by the added complexity in the fabrication of the device and/or reduced power density, as will be discussed next.

6.1.2 Prior MEMS Harvester Device Fabrication Sequence

In previous research undertaken at MIT, a high-frequency MEMS-scale vibration energy harvester (Piezoelectric Micro Power Generator, or PMPG) was designed, built, and tested. The PMPG device has a uni-morph configuration with interdigitated electrodes to utilize the {3-3} mode of operation. The PMPG device targeted high-

¹Series and parallel connection in this context refers to the interconnection of the individual harvesters, and is not the same as the series and parallel configurations for the bi-morph harvester structures described in Appendix C.

frequency, high-level vibrations, operating at 13.9 kHz with a base acceleration of $\sim 107\text{ m/s}^2$ ($\sim 11\text{ gs}$). In preparation for the fabrication process development for the low-frequency prototype, an attempt was made to replicate the PMPG device². This was unsuccessful, though many valuable lessons were learnt, and will be outlined in the following section. First, the process will be briefly described, before a number of specific observations are made. It is important to note that not all the processes are required, or compatible, with the current low-frequency prototype design, which is the topic of this chapter.

The reported fabrication sequence, developed in house [39, 87] at MIT, is based on a 3-mask, 5-layer design. The first layer is the structural layer consisting of Vertical Tube Reactor (VTR) silicon nitride (SiN_x) and Plasma-Enhanced Chemical Vapor Deposition (PECVD) silicon dioxide (SiO_2). The thickness ratio of these layers is varied to control the residual stress and curvature of the device. The structural layer ensures that the neutral axis of the asymmetric structure is below the piezoelectric layer, to prevent charge cancellation from bending. The structural layer feature of a {3-3} device, like the PMPG, allows the active layer to generate power at the expense of volume and complexity. The structural layer could potentially act as a diffusion barrier, but is not compatible with the piezoelectric layer due to diffusion into the silicon-based structural layers. A buffer layer of ZrO_2 and is deposited with a spin-on process. The layer is necessary to prevent charge diffusion from the piezoelectric element into the $\text{SiN}_x/\text{SiO}_2$ substrate. More importantly, the layer prevents the PZT from diffusing into the silicon-based structural layer. Since the ZrO_2 does not adhere well to the SiN_x , the SiO_2 layer is a necessary interface [87]. Third, a piezoelectric layer (PZT) is deposited with a spin-on process (material and process specifications supplied by Mitsubishi Materials Company) [109, 110, 111]. The first three layers (SiN_x , $\text{SiO}_2/\text{ZrO}_2$, and PZT) are patterned with the first mask, using reactive-ion etching (RIE). The electrode layers (Ti and Pt) are deposited with an electron-beam evaporation process and the electrodes are patterned with the second mask and a lift-off process. The last layer is an SU-8 proof mass, which is deposited with a spin-

²Jointly undertaken with Mr. Wonjae Choi, MIT, Mechanical Engineering

on process and is patterned with the third mask. An isotropic XeF_2 -vapor etch is used to release the structure. The isotropic etch will lengthen the structure due to undercut, which needs to be accounted for in the design.

The first consideration for high-aspect ratio cantilevered MEMS structures is residual stress. Residual stress is the stress present in a layer or material after the fabrication process. Residual stresses are described as comprised of two components: the intrinsic stresses caused by the deposition of the materials, and extrinsic thermal stresses caused by a mismatch of the thermal coefficients of expansion for the materials with a temperature change. When the cantilevered structure is released (separated from the substrate), this residual stress manifests itself most prominently as bending in the device. Compressive residual stresses causes an elongation of the layers upon release, while tensile stress results in a contraction. The forces developed in the individual layers (stress times the area of the layer) are offset from the neutral axis which results in a moment around the axis. Thus, depending on the magnitude of these stresses, the structure will bend when there is a moment imbalance. For the asymmetric design employed in the previous work, a moment imbalance is practically unavoidable, and the structure is very susceptible to residual stresses. The thickness of the SiN_x layer was varied to minimize bending from residual stress and obtain a flat structure. This issue can be largely eliminated by utilizing a more symmetric structure, since the individual layers will have similar residual stresses and a moment balance is much easier to obtain. Care has to be taken to ensure that the strain induced during the release does not cause the layers to crack or reduce the effective yield strain of the device (*i.e.*, the additional strain that can be induced during bending before failure).

The sol-gel spin on process used to deposit the piezoelectric layer (PZT) for the PMPG is discussed next. The PZT was deposited on a zirconia (ZrO_2) buffer layer in the PMPG device. This layer is necessary to prevent the PZT from diffusing into the structural layer (SiO_2 or SiN_x) and as an electrical insulator. The process has been successfully applied [97, 112]. In both these cases, a SiO_2 layer is included for adhesion. However, this process could not be implemented during the current

attempt at fabricating a replica PMPG without severe macroscopic cracking of the piezoelectric layer. Macroscopic cracking is common for this sol-gel PZT process, and is not an artifact of the thermal mismatch between the layers [113]. The macroscopic cracking is believed to be due to an increase in intrinsic stress in the PZT layer during the heating phase of the anneal/crystalization step. It was observed that the cracking severity is dependent on the thickness of the PZT film and the heating rate [113]. This is consistent with findings in this project. Three major schemes to prevent cracking have been reported: the first is the adjustment of the heat treatment step, and it is suspected that increasing the heating rate reduces cracking. A rapid thermal anneal (RTA) has been used by multiple groups and seems to be the preferred method [110, 114, 115]. However, when this anneal step was implemented here, it was found that the PZT solution severely outgassed, resulting in contamination of the heat source. This suggests a problem with the pyrolysis step used, and is still under investigation. The second scheme is to limit the PZT thickness deposited per layer ($\sim 30 - 50 \text{ nm}$ per layer), but there still remains a critical thickness ($\sim 1 \text{ }\mu\text{m}$) where macroscopic cracks initiate [116]. The third scheme is to include organic polymers in the solvents [109, 113, 117]. This is an exciting result, though lowered piezoelectric coupling is reported [117]. This last scheme was not explored in this work.

A PECVD process was used for the fabrication of the PMPG device. Due to wafer level incompatibility with the machine originally used on the PMPG (Concept1 in ICL has been upgraded to 6 inch wafer configuration), an alternative (STS-CVD in TRL) had to be used here. However, the obtained oxide layer was not compatible with the zirconia and PZT layers. With the inclusion of the oxide layer, bubbles (likely hydrogen) form underneath the PZT and zirconia layers during the subsequent anneal steps. These bubbles cause the deposited layers to delaminate. PECVD oxide is hydrogen rich [80] and the layer can be annealed, typically at temperatures above 700°C , to release the hydrogen. This process was attempted, but even after prolonged ($\sim 3 \text{ hours}$) annealing at 950°C , there was still sufficient hydrogen present to cause the top layers to delaminate. The proposed solution is to switch to a thermal oxide layer. To accomplish this, a poly-silicon layer has to be grown on the nitride layer,

and then the layer can be oxidized. This process was not implemented, but will be investigated in the future.

Furthermore, the reported PZT etch step has proven unsuccessful. A reactive ion etch (RIE) process was previously used with thick photo resist as masking layer³. However, when the process was implemented, the photo resist was burnt. Instead, a wet etch (Buffered Oxide Etch, or BOE) was explored. Using a thin resist as an etch mask, severe undercut was observed. A thick resist was used instead, which reduced the undercut, though the exact reason for this is unknown. The PZT was successfully etched with this method.

Lastly, the PMPG structure was previously released with a XeF_2 etch. This is an isotropic etch and the structure was undercut during the etch of the silicon. This undercut changed the clamping conditions of the structure, effectively elongating the structure and reducing the resonance frequency. This lack of control over the length of the structure is extremely undesirable when the objective of the process is to realize a device with a resonant frequency aligned to a specific frequency.

Though the repeated fabrication of the high-frequency MEMS-scale harvester is still ongoing, valuable knowledge was obtained to date, which is applied to the fabrication scheme and device geometric design in the sections to follow.

6.2 Prototype Geometry

In Section 6.1, design considerations were discussed, both from the model developed in Section 3.3, and the replica high-frequency prototype fabrication (PMPG) that was undertaken. Based on these results, it was decided to create an MPVEH device utilizing the $\{3-1\}$ mode of operation since a larger proportion of the structure can be made from PZT, which is a relatively compliant, high density material (as compared to silicon). Both of these properties are beneficial for power generation. Another device geometry could well be justified for different electrical output specifications. Also, from the literature, it was deduced that the PZT deposition process was better

³Personal correspondence with Mr. Wonjae Choi, MIT Department of Mechanical Engineering

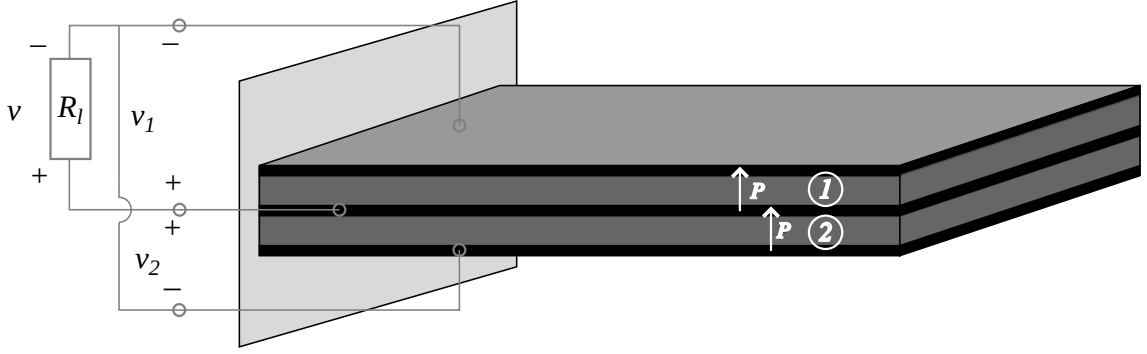


Figure 6-1: Prototype MPVEH device design: symmetric bi-morph configuration utilizing the $\{3-1\}$ mode of operation and parallel connection. A proof mass can be added to the tip as necessary.

developed for the case where the PZT is deposited on a metal electrode, instead of on a zirconia buffer layer. From the residual stress analysis, it was concluded that a symmetric structure would reduce device curvature upon release. This is very important in the low-frequency prototype since a very high aspect ratio (length: thickness of $\sim 1000 : 1$) is necessary to achieve the low resonance frequencies required. A symmetric bi-morph structure consisting of two piezoelectric layers, two outer electrodes, and a center electrode was designed, as illustrated in Figure 6-1.

A cantilever beam configuration was chosen for its simplicity, compatibility with MEMS manufacturing processes, and its low structural stiffness. A low resonant frequency is desired since the ambient vibration measurements (see Section 2.4 and Appendix A) have shown that the majority of ambient sources have significant vibration components below 300 Hz . However, designing a MEMS device with the resonant frequency below 100 Hz can be problematic [5]. For these reasons, a target frequency range of $100 - 300\text{ Hz}$ was chosen. For this specific design, an input frequency of 150 Hz was assumed, with a base acceleration of 2.5 m/s^2 (approximately that of a microwave side panel).

For an individual device, either the parallel or series configuration (see Appendix C) is feasible. However, upon analysis of the fabrication scheme, the implementation of the parallel connection is easier to achieve when a system of the individual harvesters is analyzed, due to the required interconnections between the harvesters. For

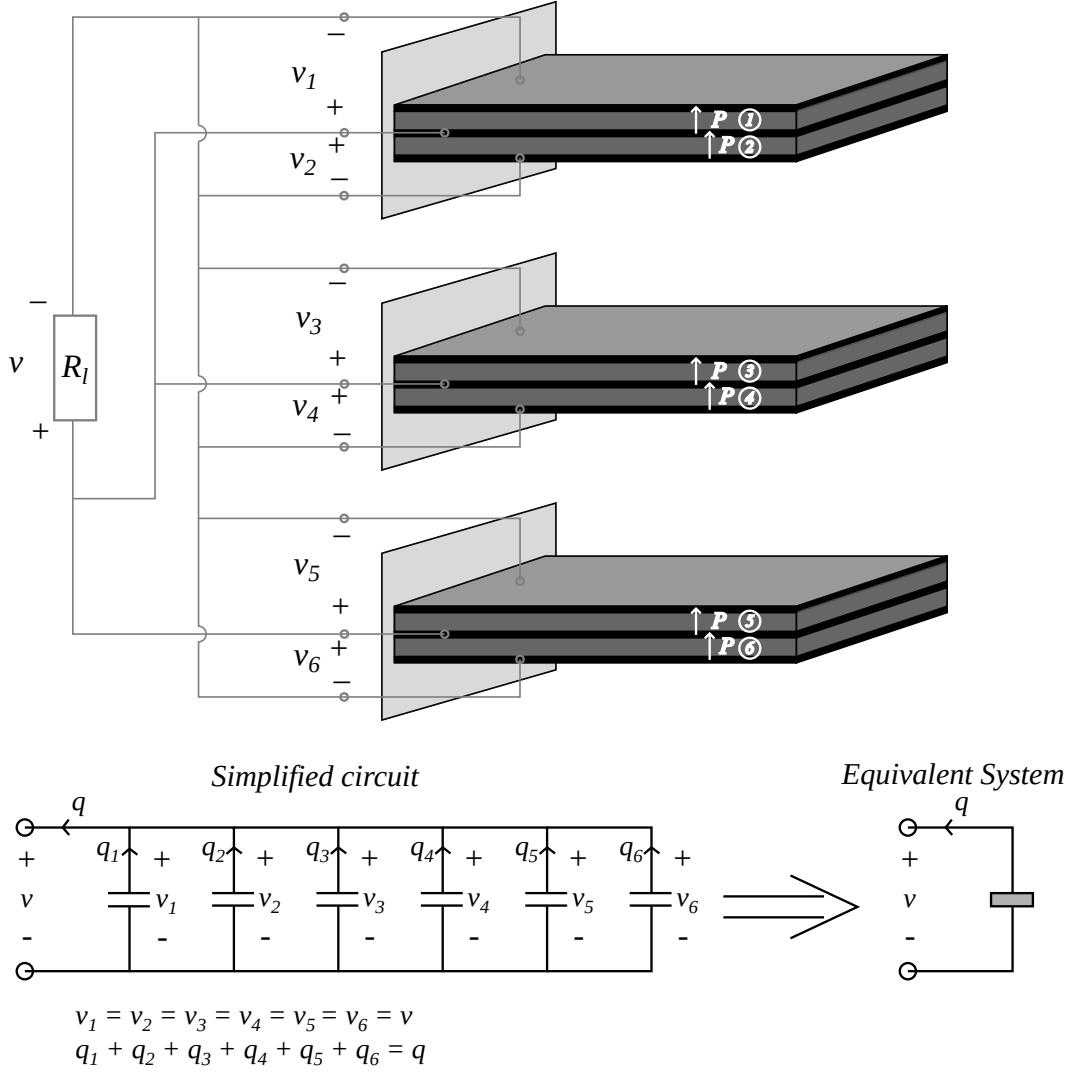


Figure 6-2: Device made up of different parallel-connected individual harvesters (*top*), interconnected to additively collect current (charge). A simplified parallel circuit representation (*bottom*) for the 3-harvester cluster is also shown.

the parallel connection, electrodes on the same layer are connected between individual harvesters (see illustration in Figure 6-2). Different layers need to be connected for the series connection. This requires extra metal deposition for the vias, masking, and insulating steps.

The voltage developed for an individual harvester for the parallel connection is half of the voltage developed for the series connection (refer to Appendix C), assuming the structure operates symmetrically. The converse is true for the current; the parallel

connection develops double the current compared to the series connection. When a number of these harvesters are interconnected (see Figure 6-2), the parallel connection will result in a high-current device. For the series connection, the device will deliver a higher voltage. However, there are certain specifications on the electrical output of the total device, especially the voltage output, which are known to be $\sim 3\text{ V}$. From Section 3.2, a higher voltage is developed at the anti-resonance frequency. Thus, it was decided to align the anti-resonance frequency of the device with the frequency of vibration input (150 Hz) given that the parallel interconnection reduces the voltage developed, relative to the series connection. The proposed fabrication scheme to achieve this geometry (parallel configuration single devices connected in parallel to form clusters) is described below.

6.3 Proposed Fabrication Sequence

In this section a viable fabrication recipe is developed, though it should be noted that this process should only be used as a guideline. There are many recent and continuing fabrication advances, especially in the deposition of the piezoelectric elements (PZT is used in this project). An updated and optimized process design will be necessary for the realization of a physical device. The various specialized processes are discussed, before the actual fabrication sequence is described.

6.3.1 Applicable Specialized MEMS Fabrication Processes

The first required process is the deposition of an insulating layer to electrically insulate the harvester from the silicon substrate. Silicon oxide and nitride are two options. Since residual stresses are a concern for the cantilevered structure, small residual stress is preferred. In the proposed process (described later), the insulating layer will be removed (from the structure) when the harvester is released, and residual stress is of less concern. For now, an oxide layer is used. Of the processes commonly available

(Thermal⁴, LTO⁵, TEOS⁶, HTO⁷, and PECVD⁸) [80], only LTO and PECVD satisfies the stress requirement. Both methods produce hydrogen-rich oxides, though the LTO oxide is thermally stable. Since this process will be the first, process compatibility is not a problem. However, LTO deposits a conformal film which will cover both the front and backside of the substrate. Since backside processing is required, the PECVD oxide is preferred as the oxide is deposited only on the front of the substrate. The layer should be annealed (RTA) to outgas the hydrogen as much as possible [118, 119], though the oxide obtained is reportedly compatible with the rest of the process [114, 115]. It should be stated that the incompatibility of the PECVD oxide in the fabrication of the replica PMPG device (Section 5.1.2) is likely a characteristic of the specific CVD machine used, and not the general process.

Next, electrodes need to be deposited. In general, platinum electrodes are used with a titanium layer between the platinum and the oxide, which serves as a diffusion barrier and an adhesion layer. The titanium reportedly diffuses into the platinum and forms nucleation sites for the PZT, during PZT deposition [111]. For this application, a high-density, low-stiffness structure is preferred, and platinum will be used. Platinum has the drawback that it is hard to etch, though two etching processes are available: aqua regia and ion milling. These will be discussed presently. Two of the available deposition processes are electron-beam (e-beam) evaporation, and sputtering. The process of choice will be determined by residual stresses in the layers. For sputtered platinum, Zakar *et al.* report a residual stress of -284 MPa [115]. Zhang *et al.* investigated the effect of the subsequent heat treatments of the PZT on the bottom platinum electrode layer, and it is recommended that the electrode layer be annealed. This anneal step is necessary both for residual stress relaxation and to allow for the titanium to diffuse into the platinum. This anneal step caused the residual stress to switch from compressive to tensile [120]. This result is consistent with

⁴Thermally grown SiO_2 , from silicon or poly-silicon layer, used for growing layers below $1 \mu\text{m}$ thickness.

⁵Low Temperature Oxide

⁶Oxidation with tetraethylorthosilicate

⁷High Temperature Oxide

⁸Plasma Enhanced Chemical Vapor Deposition

other reports for a sputtering deposition with a measured stress of 858 MPa after an anneal step (RTA) [115]. For e-beam evaporation, the residual stress is dependent on the deposition rate, and residual stresses as low as $+230 \text{ MPa}$ have been reported [121]. Furthermore, e-beam deposits higher purity materials, but has the drawback that lower thickness uniformity is obtained. E-beam evaporation is the process of choice and was previously employed of the PMPG device.

The piezoelectric layers (PZT) are to be deposited using a sol-gel process. The specific material used will depend on availability and enhancements of the process, but an example material is the PZT A6 solution, supplied by Misubishi Materials Co.. The resulting PZT has a composition (weight fraction) of $\text{Pb}/(\text{Zr} + \text{Ti})$ of (118/100) and Zr/Ti of (52/48). In Section 6.1.2, it was reported that many problems arose from the deposition of PZT on zirconia, but most of these problems can be avoided since the PZT is deposited on the platinum layer. As mentioned, the titanium reportedly diffuses into the platinum and forms nucleation sites for the perovskite phase to initiate. The perovskite phase is the required phase for strong piezoelectric coupling [111]. Furthermore, a $\text{Pb}_{1.25}\text{TiO}_{3.25}$ (PT) seeding method can be used to prevent delamination of the PZT layer [109]. A major difficulty for the sol-gel deposited PZT is the macroscopic cracks that form due to the increase in intrinsic stress during the heating up stage of the annealing process. This difficulty has put a definite limit on the film thicknesses achievable. The common approach to alleviate the problem has been to limit the thickness of the layer per deposition ($\sim 30 - 50 \text{ nm}$ per deposition) and to repeat the deposition. Not only is this approach not very compatible with mass production, but there still exist a critical thickness of PZT film ($\sim 1 \text{ }\mu\text{m}$) beyond which cracking occurs [116]. An alternative method to increase these thicknesses is with the inclusion of organic polymers to act as precursors [109, 116, 117]. Hu *et al.* added ethylene glycol to act as a cross-linking agent [116]. These agents reportedly forms long chains between the organic components to relax the intrinsic stress. A total thickness of $3.8 \text{ }\mu\text{m}$ has been obtained. Park *et al.* reportedly used polyvinylpyrrolidone (PVP) as an additive to achieve $\sim 500 \text{ nm}$ thick layers per spin [117]. For this design, it is assumed that a maximum layer thickness of $1 \text{ }\mu\text{m}$ is

obtainable.

An etch step is required to define the structures on the substrate. One of the materials to be etched is platinum, which is a chemically inert metal and will govern the etch selection. There is one wet etch available for platinum, aqua regia, but the PZT layers will also be attacked, and the etch is not an option. Furthermore, under-etching and delamination are problems associated with this method. The only remaining etch available is ion milling. Ion milling physically removes material and has been successfully implemented for platinum etching [112, 114, 122]. Gas mixtures of argon (Ar), tetrachloromethane (CCl_4), and tetrafluoromethane (CF_4) are normally used [122]. Etch rates of up to 90 nm/min are reported for platinum [122]. Since the etching process is mostly physical, PZT will also be etched and an etch rate of 22 nm/min has been reported [112]. Thus, a single etch step can be used to define the harvester planform. The ion milling does affect the piezoelectric properties of the PZT, but the properties can be recovered with an appropriate anneal step [112]. Since the process is physical, etch stops are not effective, and the process will have to be timed to achieve the correct etch depth.

The silicon proof mass of the device will be defined with a backside etch. In order to obtain the correct proof mass thickness, the substrate will be polished down. The thickness of the post-processed substrate will determine the thickness of the proof mass. Chemical Mechanical Polishing (CMP) will be used for this purpose. The limit on the thickness of the remaining substrate is set by the practical thickness which allows the substrate to be handled without breaking. A thickness of $200 \mu\text{m}$ was chosen. CMP polish rates of around 100 nm/min are common, with a surface roughness less than 1 nm .

Lastly, a Deep Reactive Ion Etch (DRIE) is used to define the proof mass and to release the harvester structure from the substrate. The etch is performed on the backside of the wafer. A high frequency DRIE process can be employed. Reported etch rate for silicon is $1,500 \text{ nm/min}$, and 9.5 nm/min for PECVD oxide [123]. The process does not etch platinum, thus the platinum/titanium bottom layer will act as an etch stop and the oxide layer will be etched. With this process, a sym-

metric cantilevered structure can be obtained since the insulating oxide layer will be removed in the cantilevered region. Alternative methods to obtain a proof mass include SU-8 deposition (front-side) and patterning, but SU-8 has a lower density ($\sim \rho = 1200 \text{ kg/m}^3$) than silicon and a larger volume will be required. Also, KOH etch could be used to define the proof mass, but will result in a proof mass with a larger footprint, which will effectively limit the length of the harvesting structure.

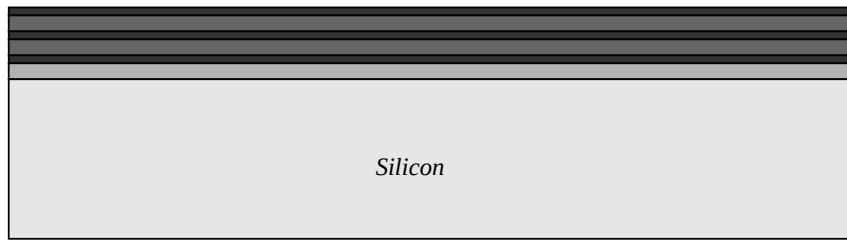
The specialized processes required for the device has been described above, focussing on the available processes and the motivation for the selection. It is important to note that MEMS fabrication is a rapidly developing field, and this recipe should be revised at the time of implementation.

6.3.2 Baseline Fabrication Scheme

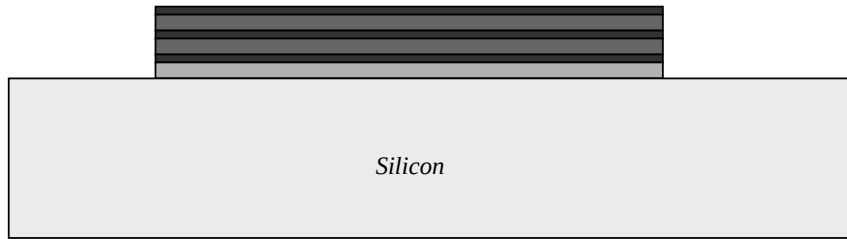
Below, the proposed fabrication scheme for a prototype MEMS-scale piezoelectric vibration energy harvester is described. A total of four masks are required for the process. The substrate used for the process is an N-type $< 100 >$, 6 inch silicon wafer ($\sim 500 \text{ }\mu\text{m}$ thick). Standard substrate specifications include: double sided polished, total thickness variation $< 3 \text{ }\mu\text{m}$, bow and wrap $< 10 \text{ }\mu\text{m}$. The outline of the proposed fabrication scheme is presented next. Also refer to Figure 6-3 for a graphical illustration of the process steps.

1. Create global alignment marks and die saw marks.
2. Clean wafer with a standard RCA clean step.
3. Deposit insulating oxide layer with a PECVD process.
4. Anneal (RTA) oxide layer for the outgassing of hydrogen at 900°C for 15 s [119].
5. E-beam evaporation deposit bottom electrode: titanium and platinum layer.
6. Anneal electrode layer above $\sim 600^\circ\text{C}$ for 30 min in N_2/O_2 environment [124].
7. Sol-gel spin deposit first PZT layer, pyrolyse, and anneal (RTA, 700°C for 1 minute). Repeat as necessary to obtain required thickness.

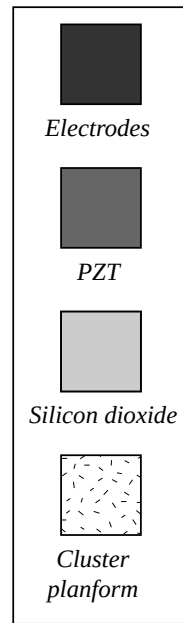
After step 12: oxide, three electrode layers, and two PZT layers.



After step 15: Structure defined through ion milling.



After step 25: Structured released with DRIE backside etch.



Planform of cluster of 3 harvesters after step 25 (wire-bond connections also indicated).

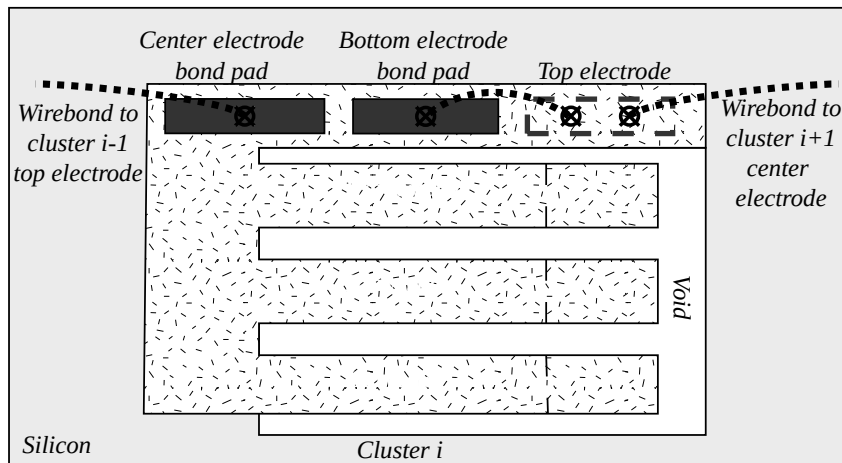


Figure 6-3: Illustration of fabrication scheme: (*top*) device cross-sections at various fabrication steps and (*bottom*) planform of 3-harvester cluster, including wire-bond connections to adjacent clusters.

8. E-beam evaporation deposit center electrode: titanium, platinum, and titanium layer (for now the second titanium layer is included to ensure symmetry, but may be eliminated if the titanium is found to diffuse uniformly into platinum layer from below).
9. Anneal electrode layer above $\sim 600^{\circ}\text{C}$ for 30 *min* in N_2/O_2 environment [124].
10. Sol-gel spin deposit second PZT layer, pyrolyse, and anneal (RTA, 700°C for 1 *minute*). Repeat as necessary to obtain required thickness.
11. E-beam evaporation deposit top electrode: platinum and titanium layer.
12. Anneal electrode layer above $\sim 600^{\circ}\text{C}$ for 30 *min* in N_2/O_2 environment [124].
13. Photolithography: deposit thick photo resist to define the harvester planform (length and width), using Mask 1. Refer to Figure 6-3 (bottom illustration) for a planform of a cluster of harvesters.
14. Ion milling, timed etch of structure using an appropriate mixture of Ar, CCl_4 , and CF_4 (up to silicon substrate).
15. Remove excess photo resist.
16. Photolithography: deposit thick photo resist and define the first bond pad (expose bottom electrode), using Mask 2.
17. Ion milling, timed etch of structure using an appropriate mixture of Ar, CCl_4 , and CF_4 (up to bottom electrode).
18. Remove excess photo resist.
19. Photolithography: deposit thick photo resist and define second bond pad, using Mask 3.
20. Ion milling, timed etch of structure using an appropriate mixture of Ar, CCl_4 , and CF_4 (expose center electrode). Keep photo resist as protection.

21. Turn over wafer for backside processing.
22. Chemical Mechanical Polishing: polish the silicon substrate down to $200\ \mu m$ thickness.
23. Photolithography: deposit thick photo resist on backside and define the proof mass and release holes, using Mask 4. For DRIE, the etch selectivity between silicon and photo resist is 50:1. Selectivity between the oxide and photo resist is around 0.32:1.
24. DRIE etch: etch from backside to define the proof mass and to release the structure, and to remove the oxide layer on the structure. The titanium/platinum electrodes are used as an etch stop.
25. Remove excess photo resist from both sides of substrate to obtain the released structures.

Post-processing steps include the non-trivial separation of the dies to form devices. One consequence of the chemical mechanical polishing step is that a relatively thin and perhaps fragile device is obtained. Furthermore, the harvester clusters will have to be interconnected using bond-pads created above to form a device (refer to Figure 6-3, bottom). All the devices in a cluster are interconnected as required (a consequence of the process developed). Wire-bonding is proposed to achieve cluster interconnections. When the device is eventually packaged, the substrate will have to be supported. The final device will have two connections to be connected to the electrical load.

6.4 Prototype Device Design Optimization

The biggest constraint imposed by the MEMS manufacturing processes is the limitation on the cantilevered structure length to around $1\ mm$ (length to thickness ratio, $\sim 1000 : 1$), such that a proof mass is needed to reduce the natural frequency of the device further. Schemes for eliminating the proof mass will be investigated in future work. Both the power and volume are proportional to the width, so the width does

not affect the power density of the individual harvester. This variable does however affect the current developed, as well as the power developed per harvester. The width is approximately limited to 2 mm.

In Section 3.3 it was shown that there exist two operating points for resonant piezoelectric energy harvesters: resonance and anti-resonance frequency. This has been confirmed with experimental results of Device 2. For this prototype design, the anti-resonance frequency of the device will be aligned with the vibration source frequency as an illustrative design. This operating point was chosen to increase the voltage output from the device. It should be stated further that, though an optimization scheme for a single device is used, the device design is dependent on numerous factors, importantly the fabrication recipe used. This recipe will impose definite limits on the device geometry. Thus, the design process is iterative between finding an optimal design and feasible fabrication sequence.

In the initial analysis, only static device failure will be considered. A maximum allowable strain of $500 \mu - strain$ has been assumed. This limit is imposed to both prevent depoling of the piezoelectric elements [110], and to prevent static failure. Fatigue in MEMS devices is normally negligible, [47, 125], however, fatigue will be considered in future work.

A MATLAB optimization function, “fmincon.m”, was used to maximize the power output from a single device. In order to perform an optimization, several constraints were imposed. First, the basic geometry (cantilevered, symmetric bi-morph configuration with the parallel connection rectangular plate with a proof mass) had to be specified (as described in Section 6.2). The free variables for the optimization were the structure length, L , the thickness of the PZT layer, t_p , and the proof mass length, L_0 . The constraints imposed on these variables (as described below) are summarized in Table 6.1. Material properties are required for the device layers. For the piezoelectric layers, PZT-5A properties were assumed, and are summarized in Table 6.2, along with the properties for the other materials. It was assumed that the two PZT layers are of equal thickness. The same assumption was made for the titanium ($t_{ti} = 0.02 \mu m$) and platinum layers thicknesses ($t_{pt} = 0.1 \mu m$). Lastly, the proof

Table 6.1: Constraints for MPVEH prototype device design optimization.

Constraint	Motivation
Geometric constraints	
$0 < L < 1.0 \text{ mm}$	The cantilevered structure length was limited due to fabrication considerations.
$0.1 < t_p < 1.0 \text{ }\mu\text{m}$	Limits enforced by spin-on process used for deposition.
$0 < L_0 < 1.0 \text{ mm}$	Proof mass length limit from practical considerations.
Material constraints	
$S_1 < 500 \text{ }\mu - \text{strain}$	Limited to prevent depoling of the piezoelectric element [110].
Device level constraints	
$149 < f_{ar} < 150 \text{ Hz}$	Anti-resonance frequency of 150 Hz is required.
$L + L_0 < 1.0 \text{ mm}$	Total structure length limited.

Table 6.2: Material properties (plate values) used for MPVEH prototype device design optimization.

Property	Value	Reference
Piezoelectric material properties: PZT-5A		
$s_{11}^E [m^2/N]$	16.4×10^{-12}	[67]
$s_{12}^E [m^2/N]$	-5.74×10^{12}	[67]
$c_{11}^{E*} [N/m^2]$	69.5×10^9	From eq. (4.2)
$d_{31} [N/m^2]$	-171×10^{-12}	[67]
$e_{31}^* [C/m^2]$	-16	From eq. (4.3)
$\varepsilon_{33}^T [F/m]$	$1700 \times \varepsilon_0$	[67]
$\varepsilon_{33}^{S*} [F/m]$	$1080 \times \varepsilon_0$	From eq. (4.4)
$\rho_p [kg/m^3]$	7750	[67]
Electrode material properties: Platinum (pt)		
$c_{pt} [N/m^2]$	170×10^9	[126]
ν_{pt}	0.39	[100]
$\rho_{pt} [kg/m^3]$	21,440	[100]
$c_{pt,pl} [N/m^2]$	200.5×10^9	Plate stiffness
Electrode material properties: Titanium (ti)		
$E_{ti} [N/m^2]$	110×10^9	[126]
ν_{ti}	0.34	[100]
$\rho_{ti} [kg/m^3]$	4510	[100]
$c_{ti,pl} [N/m^2]$	124.4×10^9	Plate stiffness
Proof mass material properties: Silicon		
$\rho_0 [kg/m^3]$	2330	[100]

mass will consist of silicon for the current manufacturing scheme.

Table 6.3: Optimized single MPVEH harvester design and predicted performance.

Geometric Parameter	Value	Performance Parameter	Value
Structure length, L [mm]	0.860	Resonance frequency [Hz]	145.8
Structure width, b [mm]	2.000	Anti-resonance frequency [Hz]	150.2
Piezo thickness, t_p [μm]	0.360	Maximum power [μW]	0.126
Platinum thickness, t_{pt} [μm]	0.100	Optimal electrical load [$k\Omega$]	280.9
Titanium thickness, t_{ti} [μm]	0.020	Voltage developed [V]	± 0.188
Proof mass length, L_0 [mm]	0.139	Current developed [μA]	0.670
Proof mass width, b_0 [mm]	2.000	Maximum strain [$\mu - strain$]	± 499.7
Proof mass height, t_0 [μm]	200.0	Tip displacement [μm]	± 298.6
Foot-print area [†] [mm^2]	2.000	Power density [†] [$\mu W/cm^2$]	6.300
Harvester volume [‡] [mm^3]	0.402	Power density [‡] [$\mu W/cm^3$]	313.2
Operating Volume [§] [mm^3]	1.679	Power density [§] [$\mu W/cm^3$]	75.04

[†] Calculated from the foot-print area of a single device.

[‡] Calculated from volume of a single device.

[§] Calculated from operating volume of a single device.

Using these material properties, and imposing the constraints on the optimization, the power was maximized. It should be noted that the limiting constraint on the developed power (and power density) and the mechanical response of the device was the maximum allowable strain. Since the limiting factor was the strain, the optimal design is a function of both the mechanical damping (or quality factor) and the input vibration. The optimal design obtained is given in Table 6.3, including a summary of the predicted performance at the resonance frequency. A base acceleration of $2.5 m/s^2$ and a mechanical damping ratio of $\zeta_m = 0.005$ has been assumed. The design will have to be repeated for different operating conditions.

6.5 Chip-level Device Design

In the previous section, a prototype design for the preferred geometry and fabrication sequence of a single device was presented. A power density of $\sim 75 \mu W/cm^3$ was obtained, based on the device operating volume. It was stated in Section 6.1.1 that the electrical output of the chip-level device (consisting of a number of individual harvesters) can be controlled through the interconnection of the individual harvesters.

However, the individual harvester response (mechanical and electrical) is influenced by the effective coupling (θ_e) of the system of individual harvesters, which in turn is dependent on the interconnections between the harvesters. This is analogous to the scheme presented in Appendix C where the interconnections for two piezoelectric elements in a bi-morph configuration was considered. The system response is dependent on the effective coupling (for example, the effective coupling determines the anti-resonance frequency position, Ω_{ar}). As a result, the optimization will have to be repeated for each configuration of interconnected clusters. This optimization was not undertaken in the current work and only the concept is presented. Interconnections to increase voltage and current in this section *assumes* that the individual harvesters operate in phase. This assumption needs to be investigated experimentally as the random nature of input vibration, fabrication non-uniformity, etc. may affect this.

As was seen in Section 6.3, many factors influence the electrical output of the individual harvesters. These factors include: the piezoelectric material selected, the dimensions of the device, the piezoelectric mode of operation, and the interconnections of the elements in each bi-morph harvester. Some of these factors will be chosen, as done here (material, mode of operation), while others will be governed by the fabrication process (mode of operation, bi-morph configuration). The required electrical output will in turn be governed by the specific application. For example, charging a storage device requires a high current, while diodes and transistors require high voltages. A scheme is presented below that allows the designer to tailor the output for the total device by appropriately connecting individual harvesters.

For the parallel configuration that has been implemented in this scheme, the current can be controlled by adding more devices in a cluster of harvesters. One such cluster (3 devices) is illustrated in Figure 6-2. For each cluster, all the electrode layers are connected (*e.g.*, the bottom electrode layers are connected, etc.), and access to each electrode layer is required to connect these clusters to form a device. The voltage of the total device can be controlled by connecting a number of these clusters in series. This concept is illustrated in Figure 6-4.

Each of these clusters can be represented as an equivalent harvester. The same is

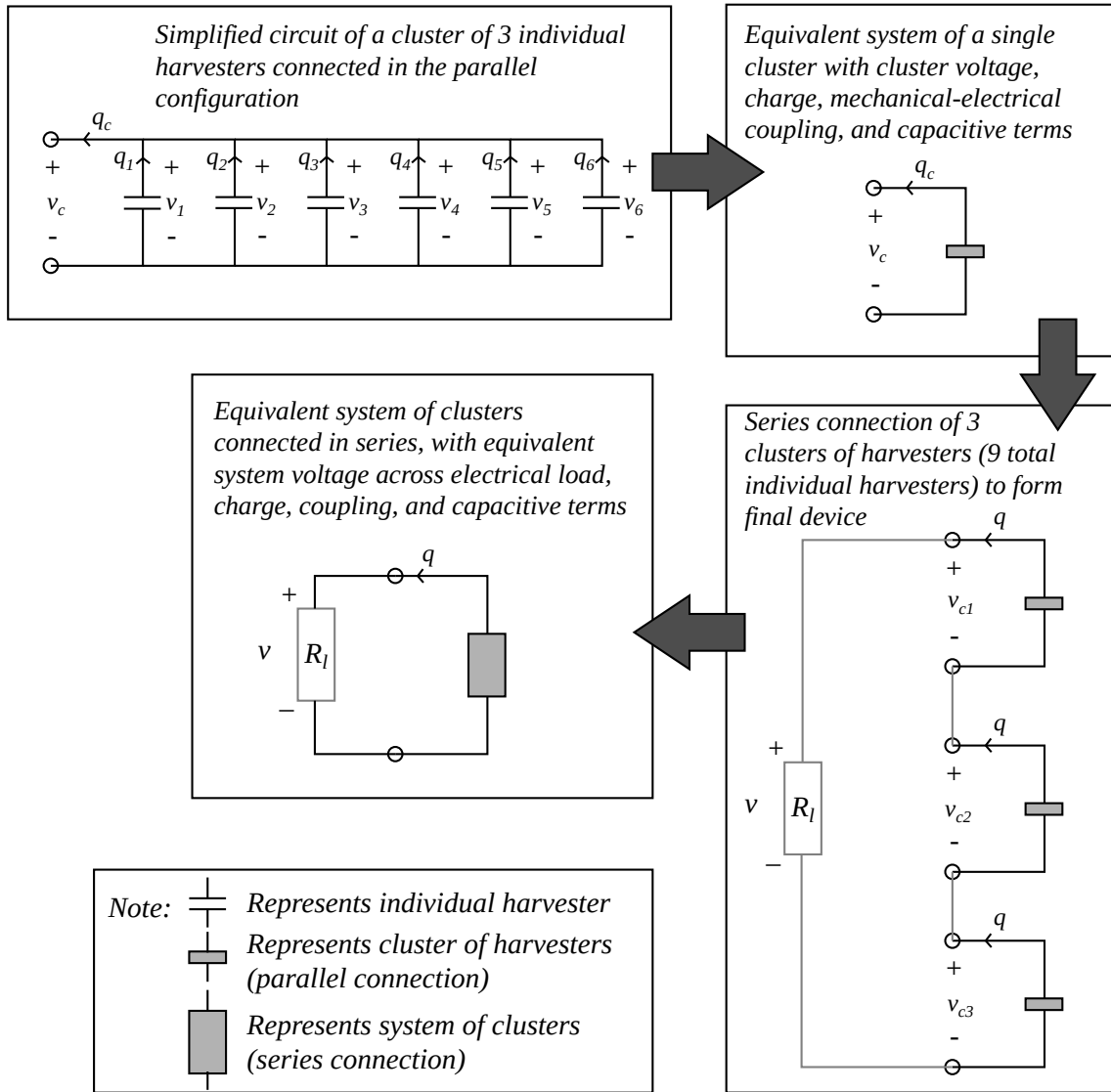


Figure 6-4: Simplified circuit representation to illustrate the concept of harvester clusters with equivalent properties, and the series connection of these clusters to control the voltage generated by the chip-level device.

true for the series connections of these arms, and the scalar equations developed in Section 3.3 are applicable. This is an extension of the scheme required to reduce the vector governing equations for a bi-morph structure to a scalar system with equivalent coefficients. As mentioned above, the optimal overall device design will be dependent on the effective system coupling, which is dependent on the interconnections of the device. The complete chip-level device has to be optimized, not just the individ-

ual harvesters since the effective system electromechanical coupling and capacitance change for these designs. This optimization will not be undertaken here.

6.6 Summary

In this chapter, the design of a prototype low-level vibration energy harvester was undertaken. First, the design implications from the developed model was presented, followed by a summary of the lessons learnt during the attempted microfabrication of a previously developed high-frequency prototype harvester. Next, the device geometry and a fabrication scheme is presented for the low-frequency prototype device, which was concurrently developed with a suitable device geometry. Based on the limitations imposed by the fabrication scheme, a 3-variable single device dimensional optimization was performed. It was found that the device performance and design is largely governed by the maximum allowable strain in the device. The design optimization is dependent on the quality factor and the vibration input parameters. Thus, the design optimization will be necessary when these conditions or the device configuration change. The predicted power density of the single MPVEH prototype harvester is $75 \mu W/cm^3$ (normalized by the operating volume, not the device volume) at $0.38 V$ peak-to-peak from a base acceleration of $2.5 m/s^2$ at $150 Hz$. Last, a scheme to control the electrical output of the chip-level harvester device (consisting of a cluster of individual harvesters) is presented but not implemented.

Chapter 7

Conclusions and Recommendations

In this thesis, the modeling and design of a MEMS-scale piezoelectric vibration energy harvester (MPVEH) was investigated. The focus of the project was on the development of a validated/verified electromechanically coupled model of a cantilevered resonant harvester. Also, the design and microfabrication of an MPVEH for harvesting *low-level* ambient vibrations was investigated. This harvester forms part of the power sub-system of the wireless sensor node. The contributions made towards this goal are presented next, before recommendations for future research efforts are made.

7.1 Contributions

Contributions from this project towards realizing an MPVEH can be divided into 8 areas:

1. Ambient vibration sources were measured and it was found that vibration levels suitable for harvesting exist in the frequency range below 300 Hz . A simple dissipative model was developed to interpret the vibration spectra. Based on the dominant damping terms of the structure, optimal input vibration frequencies (operating points) are identified (for maximum power harvesting), to which the resonant energy harvester resonance frequencies¹ are aligned. Damping

¹As in prior chapters, “resonances” or “resonance frequencies” refer to both the resonance and the anti-resonance frequencies of the device.

dependency on frequency is carefully considered. The operating point selection result is new and in contrast to the findings of others, as described in Section 2.3. The selected operating point will depend on the device size (micro- vs. macro-scale) and the operating environment (*e.g.*, vacuum or atmospheric), since the dominant damping components differ for these conditions.

2. Coupled electromechanical models are developed, based on a Rayleigh-Ritz modal decomposition of the system response and the small-signal linear piezoelectric material model (constitutive relations). The modal analysis for a cantilevered structure with a large proof mass is presented. Uni-morph and bi-morph harvesters are considered. Two power-optimal operating points and associated electrical loads, corresponding to the resonance and anti-resonance frequencies, respectively, are identified. Equal power is predicted at these peaks under power-optimal conditions, whereas the electrical (voltage and current) and mechanical response will not be equal due to different electrical loading conditions which influences system damping. Purely resistive electrical loads have been assumed. It is shown that the piezoelectric coupling cancels from the power equation at the optimums (see Section 3.3.2). This result suggests that the only piezoelectric material properties that affect the maximum power generated are the elastic stiffness and density. Since these properties vary little for typical piezoelectric ceramics, the choice of material will have little effect on the maximum power extracted. Furthermore, the piezoelectric mode of operation (*i.e.*, {3-1} vs. {3-3}) has negligible effect on the maximum power extracted. However, the electrical response (voltage and current) is dependent on the piezoelectric coupling. The piezoelectric material choice and mode of operation will have a significant effect on the voltage/current performance and need to be considered once application-specific electrical requirements are imposed.
3. A scheme is presented to transform a multi-electrical mode problem (multiple piezoelectric elements) into a single electrical mode problem to obtain scalar governing equations that allow for straightforward power optimization. The

different interconnection configurations for a bi-morph harvester are clearly presented, together with the sign conventions used in piezoelectricity theory. The developed model is implemented for each of these configurations.

4. Experimental data is required to validate/verify the proposed model, but suitable resonant response data for a piezoelectric harvester was not found in the open literature. As a result, an experimental setup was developed and a macro-scale validation device (Device 2) was tested under controlled base-excitation conditions. The validation device has a symmetric bi-morph configuration and utilizes the series connection. The experimental results, both near and away from the resonances, are presented in tabulated and graphical form. Measurements include: voltage, resistance, absolute tip motion, and absolute base motion (the latter motions are captured via a laser vibrometer). Power and relative tip motion are calculated from the above measurements.
5. The model is verified in the small-signal (linear) range of the piezoelectric material, corresponding to operation away from resonances. Excellent correlation is achieved in this regime. At resonances, the model consistently underpredicts the mechanical and electrical response of the device. Relatively large strains are developed during resonant operation, and the small-signal linear model assumption is no longer valid. This deviation does not invalidate the model since the trends of the response are accurately predicted. At resonances, the predicted mechanical and electrical response is low by a factor of less than 2. This deviation, including all trends at resonances, is satisfactorily explained by the large-strain (non-linear) piezoelectric response (Section 5.3.4).
6. A baseline microfabrication sequence is concurrently developed with device geometry specification for a low-level, low-frequency prototype harvester. The fabrication sequence was developed based largely on experience gained in an attempt to replicate a previous high-acceleration input, high-frequency prototype harvester (PMPG) with an existing recipe. The attempt has been unsuccessful to date, but valuable insight was obtained into the requirements for fabricating

a high-aspect ratio cantilevered harvester structure. Based on this microfabrication experience and insights gained from modeling, a viable microfabrication scheme for low-level, low-frequency prototype device is presented. The proposed fabrication scheme is a four mask process to realize a cantilevered, symmetric bi-morph piezoelectric harvester (parallel-connection configuration) with a silicon proof mass.

7. Based on the baseline fabrication sequence developed and design considerations from the modeling, a suitable harvester geometry is identified. A single device design is optimized (3-parameter optimization), subjected to constraints imposed by the fabrication scheme and material considerations. An input vibration of acceleration 2.5 m/s^2 at 150 Hz is targeted, aligned with the device anti-resonance frequency. A mechanical quality factor of 100 is assumed, based on prior work (quality factor = 0.0056 for PMPG device [87]). The performance of the optimized harvester design is predicted as: $75.0 \text{ } \mu\text{W/cm}^3$ (based on the operating volume), $313 \text{ } \mu\text{W/cm}^3$ (based on the device volume normally reported in literature), and $6.3 \text{ } \mu\text{W/cm}^2$ (based on the footprint area). A peak-to-peak voltage of 0.38 V is predicted. Considering the non-linear piezoelectric material response at resonances observed during device testing (see part 5 of this section), the power and voltage predictions are likely conservative by no more than a factor of 2.
8. A scheme for constructing a chip-level device, consisting of a number of interconnected clusters of individual microfabricated harvesters, is presented (but not implemented). The electrical output (voltage and current) of the chip-level device can be controlled through the interconnections of the individual devices.

7.2 Recommendations

Based on the contributions and the conclusions drawn from the current research, the following future research areas are identified:

1. Low-level, low-frequency vibrations in the ambient have been targeted for harvesting in this project. The resonant frequencies of the harvester need to be aligned to this low frequency. However, this low frequency places the MPVEH prototype device dimensions at the upper bounds of current microfabrication process capabilities. Alternative low-stiffness device configurations with more uniform strain distributions should be developed. Alternatively, higher frequency vibrations (from sources like engines) may be targeted since microfabrication is better suited for the resulting high-frequency devices. The response of the resonant harvester to “random” sources, instead of idealized sources, should be investigated (for example, strain cancellation due to the excitation of higher-order bending modes).
2. High quality factors are achievable with MEMS resonators. However, for high quality factors, very narrow response peaks are obtained, which need to be aligned with the dominant frequency component of the vibration source. Given the variability of ambient sources and microfabrication processes, it is likely desirable to incorporate a frequency-tuning mechanism into the harvester design. Such a mechanism will need to be investigated in the future.
3. The model developed in this project used the small-signal linear piezoelectric constitutive model, though it was shown to underpredict performance when the device is operated at the resonances (large-strain regime). The development and use of a non-linear material model may be justified if a more accurate prediction of the resonant response magnitudes (resonance frequencies and power-optimal resistances are well predicted) of the system is required. This non-linear material model will have to be validated experimentally, especially for piezoelectric thin-films (see below).
4. Material selection was preliminarily investigated in this project. It was shown that the piezoelectric coupling cancels from the optimal power equation, and that the material selection has little effect on the maximum power extracted. This will have to be verified experimentally in the future. The material choice

affects the electrical response of the system and will need to be considered once a specific application (with specific electrical requirements) is targeted. The material selection will also be influenced by the available materials for integration with the microfabrication scheme. Other material related issues to investigate include the fatigue properties of piezoelectric thin-films and the validity of both the linear and non-linear piezoelectric material models for (textured) piezoelectric thin-films. Lastly, the development of in-plane textured PZT films for harvesting purposes should be investigated.

5. The existing fabrication sequence investigated herein (for the high-level, high-frequency PMPG device from prior work) identified some process difficulties, especially associated with the PZT deposition process. This process will have to be refined, either through adjustment of the heat-treatment steps, or through the inclusion of precursors in the PZT solution to create a uni-morph {3-3} mode device. Alternatively, {3-1} bi-morphs or other piezoelectric material deposition processes, such as sputtering or pulsed laser deposition (PLD), may be investigated.
6. Packaging of the final chip-level devices to make up the harvesting system of the power sub-system of a wireless sensor node will need to be considered for future prototypes as the packaging scheme will likely influence the final fabrication sequence. One probable future packaging requirement is vacuum operation of the device (to increase the quality factor).
7. This study has focussed on the design and modeling of a single harvester, which is a component of the power sub-system of the wireless node. The next step is to implement the harvester design with the rest of the power sub-system, consisting of conditioning circuitry and a storage device (battery), among others. The first step has been taken in this research with a scheme for controlling the electrical output of the chip-level device by interconnecting a number of individual harvesters to form clusters and systems of clusters. The concept has been presented, but the scheme will have to be implemented in future research,

both in terms of modeling and the realization of actual devices.

8. A macro-scale device was tested to verify the model in this project, but such a verification will have to be extended to the micro-scale to ensure that the model is applicable in this regime. This will require the fabrication and testing (using the setup developed, especially the laser vibrometer) of a micro-scale device.

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Appendix A

Ambient Vibration Measurements

14 conditions of low-level vibrations were investigated to establish levels and frequency ranges for low-level “ambient” vibrations. The objective of the measurements was to get a quantitative indication of the frequency range and magnitude of vibrations from these sources. The purpose was not to investigate the vibration characteristics of a specific object in detail, as this investigation is to be undertaken when the final application for the MEMS Piezoelectric Vibration Energy Harvester (MPVEH) is identified. 14 conditions of 8 separate sources were analyzed in total, for the frequency range of $10 - 1000 \text{ Hz}$. Please refer to Table A.1 for a description of the vibration sources investigated as well as the environmental conditions during the measurements. Interpretation schemes, utilizing a simple but effective 1-D dynamic model for optimal harvesting point selection, were developed and it was found that macro- and micro-systems require different schemes. This necessity arises from the different dominant damping mechanisms for these systems. This is in contrast to previous findings where the forms of the damping were ignored and/or assumed independent of frequency. Furthermore, for micro devices, the operating environment (vacuum vs. atmospheric) will further influence the selection of a vibration peak to target. From the sources studied, harvestable levels of vibrations (acceleration $> 10^{-2} \text{ m/s}^2$, or 10^{-3} gs , in the frequency range of $100 - 200 \text{ Hz}$) were identified. This conclusion is in agreement with published data on ambient vibration sources [5, 8, 72]. Furthermore, ambient vibration sources generally exhibit multiple peaks of significant power, often

Table A.1: Ambient vibration source details.

#	Source	Mounting position	Comments	Date time	Weather Cond.	File name
1	A/C duct: center, low	Center of nozzle	Low fan speed	4/23/04 4:00 pm	Indoors	meas1.mat
2	A/C duct: side, high	Side of nozzle	High fan speed	4/23/04 4:05 pm	Indoors	meas2.mat
3	A/C duct: center, high	Center of nozzle	High fan speed	4/23/04 4:10 pm	Indoors	meas3.mat
4	Computer side panel	Center, side panel	-	4/23/04 4:17 pm	Indoors	meas4.mat
5	Microwave oven: top	Center, top panel	Old microwave	4/23/04 4:30 pm	Indoors	meas5.mat
6	Microwave oven: side	Center, side panel	Old microwave	4/23/04 4:36 pm	Indoors	meas6.mat
7	Office desk	Desktop mounting	PC running on desk	4/24/04 1.27 pm	Indoors	meas7.mat
8	Harvard bridge railing	Vertical mounting on barrier railing between road and sidewalk, $\frac{1}{3}$ span from MIT	Light traffic, fluctuating	4/24/04 1:51 pm	Sunny, light wind	meas8.mat
9	Parking meter: Perp.	On meter head, perpendicular to road	Light traffic, fluctuating	4/24/04 2:05 pm	Sunny, light wind	meas9.mat
10	Parking meter: Par.	On meter head, parallel to road	Light traffic, fluctuating	4/24/04 2:08 pm	Sunny, light wind	meas10.mat
11	Car hood: 750 rpm	Center of hood	~ 750 rpm parked car	5/20/04 2:47 pm	Indoors	meas11.mat
12	Car hood: 3000 rpm	Center of hood	~ 3000 rpm parked car	5/20/04 2:51 pm	Indoors	meas12.mat
13	Medium tree	Branch, dia. 25 cm	Next to Charles River	5/20/04 3:12 pm	Sunny, moderate wind	meas13.mat
14	Small tree	Branch, dia. 15 cm	Building 33 courtyard	5/20/04 3:27 pm	Sunny, light wind	meas14.mat

at much higher frequencies. This motivates an investigation of the effect of other device resonance modes on the power generation of the system presented in Section 2.5.

A.1 Experimental Setup

A portable vibration measurement set-up was assembled and consisted of the following components:

1. TELAC Laptop: Dell Inspiron 5100. Pentium 3, CPU 700 *MHz*.
2. SigLab Model 20 – 42 (Serial number: 11371) data acquisition unit.
3. PCB Single axis ICP accelerometer: X333A51 (Serial Number: 6394) from The Modal Shop (website: www.modalshop.com and www.pcb.com). Manufacturer provided specifications of:
 - Calibration factor: $966.03 \text{ mV/gs} = 98.474 \text{ mV} \cdot \text{s}^2/\text{m}$
 - Sensitivity: 1000 mV/gs
 - Frequency range: $0.5 - 3000 \text{ Hz}$
 - Amplitude range: $\pm 5 \text{ gs}$
 - Mass: 6.8 grams
 - Resolution: $0.00005 - 0.00008 \text{ gs}_{rms}$
 - Resonant frequency: $> 20 \text{ kHz}$
4. 480C02 ICP Signal Conditioner (Serial Number: 5339).

The accelerometer was chosen for its high sensitivity and fine resolution for measuring very low-level vibrations, while having a small mass. The low mass is to ensure that the sensor does not significantly influence the source dynamics. This corresponds to an infinite source assumption, which is common in the analysis of systems of this type (*e.g.*, a small sensor or harvester on a larger structure). The sensor is connected

to the Signal Conditioner, which is in turn connected to input channel 1 on SigLab. SigLab interfaces with the computer via a SCSI cable, through a PCMCIA SCSI card. The SigLab data acquisition unit is controlled with “virtual instruments”, *VI*’s, which are graphical interfaces within the MATLAB environment. Two *VI*’s were used: *VNA* (Dynamic Signal Analyzer) and *VCAP* (Long Record Capture). The *VNA VI* has the capability of analyzing and displaying the frequency-acceleration data in real time. For the frequency domain analysis, the signal is averaged by *VNA*. As a result, only the averaged time data is available (*e.g.*, one block of averaged data) from *VNA*. This was a concern as some of the sources to be investigated are highly fluctuating and the total time signal was required to enable signal conditioning. So it was decided to use the *VCAP VI*, with *VNA* as the parent *VI* (a technicality required by SigLab). *VCAP* gives the raw voltage-time data that can be converted to acceleration-time data with the appropriate calibration factor during post-processing (done here with MATLAB).

Since the levels of vibrations that were measured were very low, the acquisition system noise level was investigated. Initially, the system exhibited considerable electrical noise. Peaks appeared at both odd and even multiples of 60 *Hz*, attributed to external electrical noise. The main source of the electrical noise was found to be the portable power supply that was used to power the signal conditioner for the sensor. The portable power supply was a parallel set of 12 *V* batteries connected to an inverter to convert the voltage to the required 120 *V*. To eliminate the problem, a DC-powered signal conditioner was used, eliminating the need for the inverter. The upgraded system noise level was measured by letting the sensor hang in mid air via the output cable, effectively damping out all but very low frequency (~ 1 *Hz*) vibrations. The noise level was well below the system resolution and sensitivity level, even though the electrical pickup was still present. Refer to Figure A-1 for a comparison of an average measurement to the system noise level. Note that measurements of sources without electrical components (*e.g.*, bridges and trees) generally do not show any electrical pickup (see Section A-4 for plots).

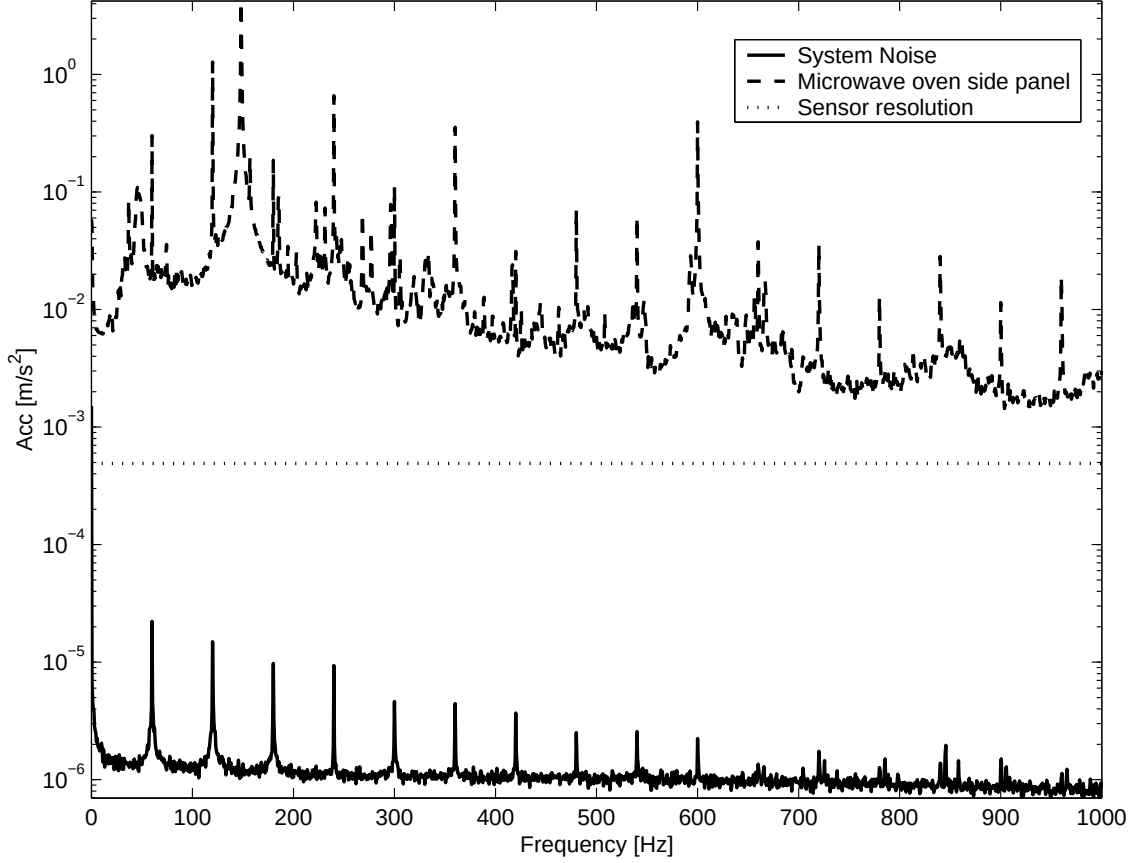


Figure A-1: Frequency domain results from system noise experiment.

System noise levels of the order 10^{-5} m/s^2 ($\sim 1 \text{ } \mu\text{gs}$) were measured. Typical source accelerations were around 10^{-3} m/s^2 , with peaks being no lower than 10^{-4} m/s^2 . The stated sensor resolution was around 10^{-4} m/s^2 ($10 \text{ } \mu\text{gs}$). Therefore, the experimental set-up is considered adequate for the measurement of the ambient sources of interest.

A.2 Ambient Vibration Testing

Using the set-up described, the data was acquired with the saved *VNA* configuration “*Acc test.vna*”. *VNA* settings included:

- Sampling Frequency: 2560 Hz
- Band Width: 1000 Hz

- Block Length (for Fast Fourier Transform, or FFT, analysis): 4096 data-points
- Frequency increments (for frequency domain analysis): 0.625 Hz
- Number of averages: 50. These parameters give a total signal length of 204,800 data points, or 80 s of data.
- Trigger: Free-run (technical requirement for use of *VCAP*)

It is important to note that only the sampling frequency setting influences the raw data taken and reduced. However, the other settings are required for *VNA* to run and have been included for that reason. Steps in the test procedure are:

1. Accelerometer placement:
 - (a) Attach the accelerometer to the source of interest. Bee's wax was used as a temporary adhesive.
 - (b) Tape the accelerometer cable to surface near the accelerometer. This will prevent damage if the accelerometer detaches and reduces load on the accelerometer from the cable.
2. Acquisition unit:
 - (a) Switch on SigLab (ensure that it is connected to the computer) and then switch on the computer.
 - (b) Start MATLAB and enter command "*VNA*;"
 - (c) Enter command "*VCAP*;" . This must occur after 2(b), as *VNA* is the parent *VI* for *VCAP*.
 - (d) Open *Acc test.vna* in *VNA* window. This will load all the acquisition settings as described above.
 - (e) Hit "Inst" button in *VNA*-plot window. This will show measurements graphically in real time.
 - (f) In the *VCAP* window, make sure that the acquisition length is set to 50 averages. Start acquiring.

- (g) Once acquisition is complete, go to the “File” menu and select “Dump Buffer to Workspace”. This will dump two variables into the MATLAB workspace: *VCAP DATA* [Volts] and *VCAP SAMPLERATE* [Hertz]. Given the system settings, each measurement file contains 204,800 *VCAP DATA* data points.
- (h) The raw time data can then be processed in the MATLAB environment.

A.3 Data Reduction and Interpretation

When interpreting vibration data, acceleration data in both the time and frequency domains and the “Power Spectral Density”, or PSD, function are of interest. These were all plotted, in addition to displacement in the frequency domain. Each plot will be discussed briefly, using plots for the vibrations measured on the top panel of a microwave (source numbering indicated in Table A.1) as an example.

A.3.1 Data Reduction Example and Discussion

The data obtained from the acquisition unit is the voltage-time data and must be converted into acceleration-time data with the relevant calibration factor. For the microwave example, Figure A-2a is the acceleration-time plot for a single block of data (4096 data-points). Plots A-2b-d are for the full-length signal (80 s of data, or 99 overlapping blocks, averaged). Next, the acceleration data is converted from the time domain to the frequency domain by means of Fast Fourier Transforms (FFT), see Figure A-2b. As mentioned earlier, the data acquired are grouped into blocks of length 4096 samples (this translates into 1.6 s time blocks). The block-length will determine the frequency increments for the frequency domain analysis. Each of these blocks can be analyzed separately, however to obtain an aggregate value, multiple blocks are averaged and then evaluated. As a rule of thumb in analyzing vibration data in the frequency domain, at least 50 averages are desirable [127]. Furthermore, the blocks overlapped by half the block-length (2048 data points). With the overlapping blocks, an effective 99 blocks are obtained for the full-length signal. The effect of using a

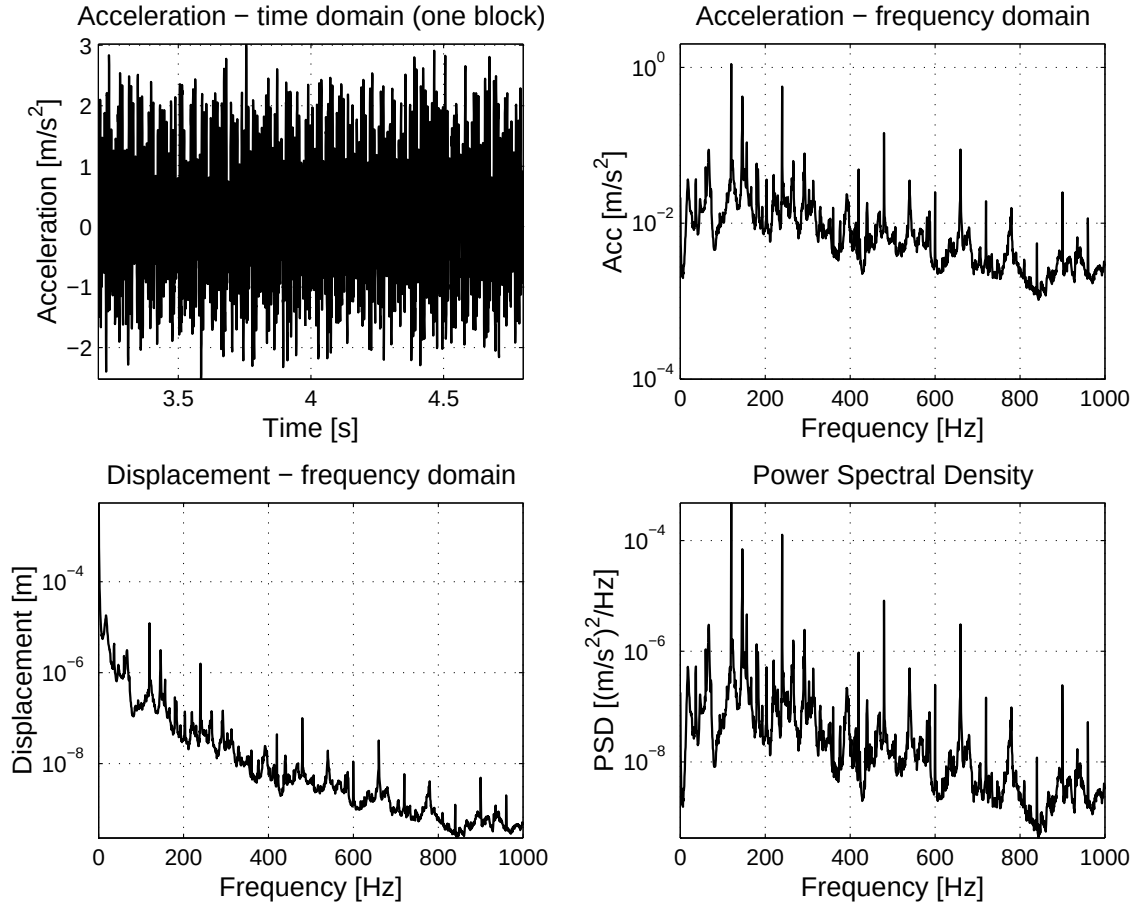


Figure A-2: Data for a microwave top panel: a) acceleration in time domain (1 block); b) acceleration in frequency domain; c) displacement in frequency domain; d) Power Spectral Density (PSD). Plots b, c, d are for full 80 s signal.

single data block versus 19 blocks versus the total signal (99 blocks) for averaging is shown in Figure A-3. Displacement, acceleration, and PSD in the frequency domain for all sources/conditions are provided graphically in Section A.4. The results in Figure A-3 are typical and show that the average of 99 blocks gives a convergent and representative acceleration spectrum for the “random” source.

The acceleration is directly related to the displacement in the frequency domain. The displacements are obtained by dividing the acceleration by the local frequency (in rad/s) squared (Figure A-2c). In the analysis of random vibrations, it is common practice to use the PSD function. The PSD function gives a measure of the “power content” of a signal at a specific frequency. The name “Power Spectral Density”

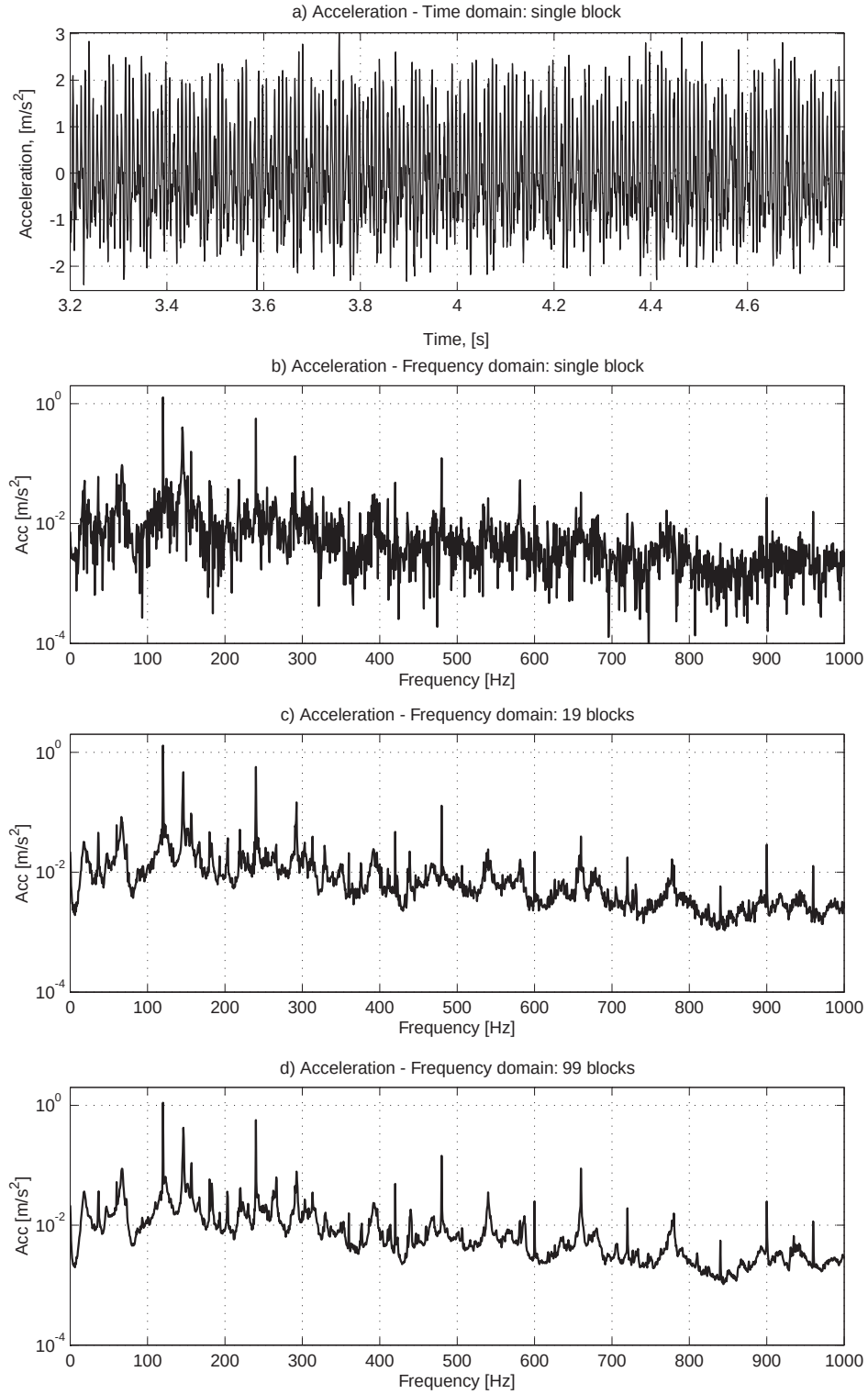


Figure A-3: Acceleration determined for a single data block vs. using multiple blocks of data for the top panel of a microwave (see sources in Table A.1): a) acceleration - time data, b) 1 block, c) 19 blocks, d) 99 blocks of data.

is misleading, as it is only applicable to a very specific case (where the signal is measured in volts, the PSD gives the power that will be dissipated in a $1\ \Omega$ resistor). A more appropriate description is the “Mean Square Spectral Density” because the integration of the PSD function over a specified frequency range gives the mean square value of the signal in that range. The PSD function has three main purposes [71]:

1. To determine the transfer function of a system, *e.g.*, measure the input and output and determine the transfer function.
2. To determine the response of a system. If the transfer function is known and the input PSD is known (can be measured), the output can be determined.
3. To determine the mean squared value of a signal. This is accomplished by integrating over the total frequency band.

Since none of these is necessary for the current investigation, the PSD function will not be used to interpret the measured data. All the information required is obtainable from the conversion of the acceleration data to the frequency domain. Furthermore, the acceleration-frequency plot is more intuitive to interpret as the PSD function has units of $[(m/s^2)^2/Hz]$. The PSD-function was determined using the “pwelch.m” function in MATLAB, (Figure A-2d). This MATLAB function calculates PSD as the square of the acceleration, divided by the local frequency.

A.3.2 Interpreting the Results

It has been shown in Section 2.2 that the maximum power generated by a 1-D resonant vibration energy harvester in terms of the model parameters is given by:

$$|P_e|_{opt} = \frac{m\ddot{w}_B^2}{16\zeta_{e,opt}\omega_N} \quad (A.1)$$

When a MEMS-scale device is operated at atmospheric conditions, the optimal damping ratio ($\zeta_{e,opt}$) is inversely proportional to the frequency. The influence of the

input vibrations on the generated power is given by:

$$|P_e|_{opt} \propto \ddot{w}_B^2 \quad (\text{A.2})$$

and is not dependent on the frequency of operation. The current device is to be operated under these atmospheric conditions. When selecting the vibration peak (in terms of acceleration and frequency) to design the MPVEH for maximum power generation, one should consider the maximum value of the input acceleration squared, or equivalently $\omega_N^4 w_B^2$, in terms of the base input frequency and the displacement. To facilitate this, the acceleration plots (in the frequency domain) include lines of constant “reference power”. Reference power is defined as $|P_e|_{opt}$ in eq. (A.1), at the acceleration and frequency of the reference peak (highest acceleration peak above 100 Hz). The upper line defines the constant power line at the reference peak and the subsequent lines (from top to bottom) give a 50% and 90% reduction in the reference input power contribution. Please refer to Figure A-4 for an illustration of these lines. These lines of constant reference power indicate the maximum contribution to the power generated from the vibration input, assuming damping ratio-optimized resonant harvesters of equal mass. Lastly, for most sources considered here, it is possible to identify a peak of comparable power to the reference peak. This secondary peak is defined as the “alternate peak”.

For the interpretation of the measured data, another consideration is the lower limit in natural frequency obtainable with a MEMS device. Consistent with our conclusions, others (*e.g.*, [5]) have found it difficult to design a MEMS device with a resonant frequency below 100 Hz (as size scales down, resonant frequency scales up). The lower limit for viable vibration peaks has been set to 100 Hz for the current investigation, thus defining the “accessible region” to be above 100 Hz (See Figure A-4 and Figure A-5).

When the MEMS-scale device is operated in vacuum, the damping ratio is independent of the frequency, and the ratio of acceleration squared to the frequency should be used to interpret the measurement data. Refer to eq. (A.3) and Figure

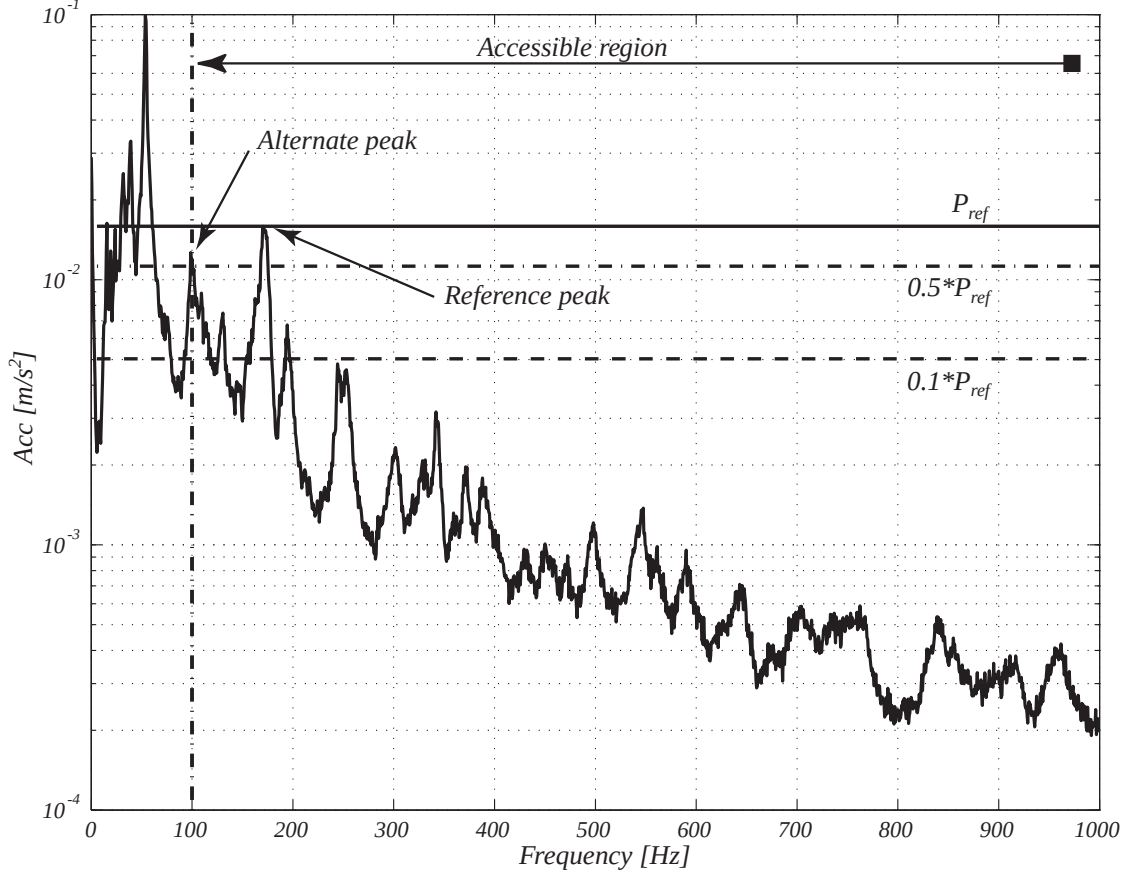


Figure A-4: Interpreting spectra to determine target acceleration peak under atmospheric conditions: microwave top panel example (see source 5 in Table A.1).

A-5 for constant power lines under vacuum conditions (compare to Figure A-4 for atmospheric operation).

$$|P_e|_{opt} \propto \frac{\ddot{w}_B^2}{\omega_N} \quad (\text{A.3})$$

From Figure A-5 it can be seen that in some cases, the optimal operating peak can have a lower acceleration than other peaks. Two other schemes for the interpretation of the measurement data have been put forward by others. In the most typical scheme (*e.g.*, [5, 30]), the generated power is written in terms of the mechanical and electrical damping ratios, ζ_m and ζ_e respectively. Using this scheme the generated power is given by:

$$|P_e|_{opt} = \frac{1}{4} \frac{\zeta_e}{(\zeta_e + \zeta_m)^2} m \frac{\ddot{w}_B^2}{\omega_N} \quad (\text{A.4})$$

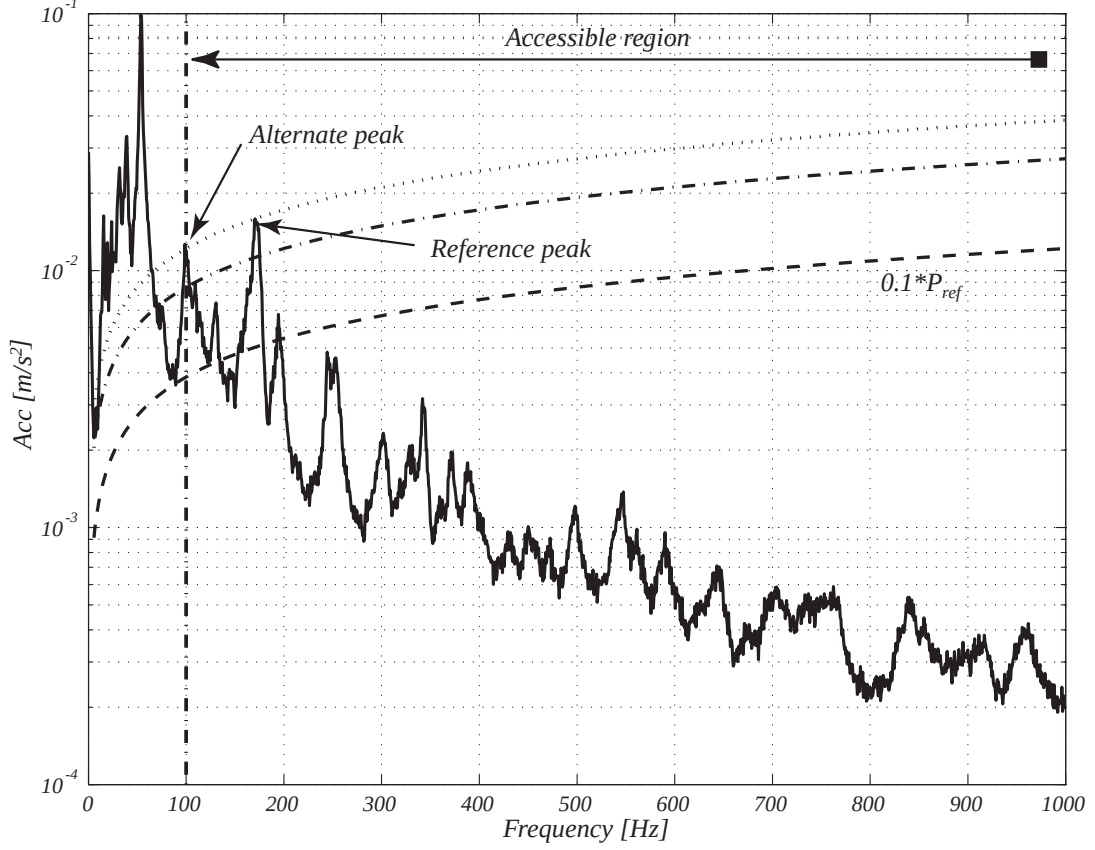


Figure A-5: Interpreting spectra to determine target acceleration peak under vacuum conditions: microwave top panel example (see source 5 in Table A.1).

Viewing eq. (A.4), the authors concluded that the contribution of the input vibrations to the power generated is $|P_e|_{opt} \propto \omega_N^3 w_B^2 = \frac{\ddot{w}_B^2}{\omega_N}$. This result corresponds to a MEMS device operated in vacuum, however, this is not stated in prior work, nor can it be inferred from the published results. In fact, the devices analyzed in that prior work [30, 5] operate under atmospheric conditions. In the second scheme (*e.g.*, [72]), the power is written both in terms of the base input displacement, w_B , and the relative displacement of the proof mass to the base, z . The relation between these parameters and the generated power is given by:

$$|P_e|_{opt} = \frac{1}{2} \frac{b_e}{(b_e + b_m)^2} m \omega_N^3 w_{BZ} = \frac{1}{2} \frac{b_e}{(b_e + b_m)^2} M \omega_N \ddot{w}_{BZ} \quad (\text{A.5})$$

From this analysis the authors conclude that the contribution of the input vibra-

tions to the generated power is given by $|P_e|_{opt} \propto \omega_N^3 w_{BZ} = \omega_N \ddot{w}_{BZ}$, and for maximum power generated, the frequency-acceleration product must be maximized. However, because the relative displacement is a function of the input vibration, eq. (A.5) can be simplified to obtain eq. (A.1).

Therefore, the recommended interpretation for MEMS-scale atmospheric or vacuum operation are given by eqs. (A.2) and (A.3) respectively. In summary, the optimal operating point for power harvesting is a function of the harvesting system and is strongly influenced by the dominant damping mechanism in the system. For microscopic devices, as is commonly known, the volume scales down as length cubed and the surface area scales down as length squared. As a result, surface-fluid interactions become dominant over inertial effects for microscopic devices. These fluidic-damping mechanisms are generally dependent on frequency, which must be accounted for when analyzing the generated power. On the other hand, surface-fluid interactions are negligible for macroscopic systems, and the dominant damping mechanisms (structural damping and support losses) are generally independent of frequency. Therefore, for macro-scale devices, the operating point should be selected using the relation in eq. (A.3).

A.3.3 Summary of Reduced Data

The interpretation scheme for a device operated under *atmospheric* conditions, discussed in the previous section, was used to identify three acceleration peaks for each source. The first peak has the maximum power content (*e.g.*, the highest acceleration squared) and is referred to as the “Highest Power Peak”, or HPP. The “Reference Peak”, RP, is the highest power peak in the accessible region (*e.g.*, above 100 *Hz*). For 6 of the 14 sources, the HPP and the RP are the same. The “Alternate Peak”, AP, is a secondary peak in the accessible region. These peaks are summarized in Table A.2.

The range and levels of ambient vibrations differ greatly from source to source, but much insight can be gained by inspection of Table A.2. The first observation is the acceleration levels measured. For HPP, the levels varied from 10^{-3} m/s^2 to

Table A.2: Ambient vibration sources: quantitative comparison for harvester operated in atmospheric conditions.

#	Source	HPP		RP		AP	
		Acc. [m/s^2]	Freq. [Hz]	Acc. [m/s^2]	Freq. [Hz]	Acc. [m/s^2]	Freq. [Hz]
1	A/C duct: center, low	0.0328	15.7	0.0254	171.9	0.0130	113.8
2	A/C duct: side, high	0.0990	53.8	0.0159	170.6	0.0127	98.8
3	A/C duct: center, high	0.0398	55.0	0.0366	173.1	0.0186	108.3
4	Computer side panel	0.0402	276.3	0.0402	276.3	0.0360	120.0
5	Microwave oven: top	1.11	120.0	1.11	120.0	0.570	240.0
6	Microwave oven: side	4.21	148.1	4.21	148.1	1.276	120.0
7	Office desk	0.0879	120.0	0.0879	120.0	0.00516	546.3
8	Harvard bridge railing	0.0215	171.3	0.0215	171.3	0.0193	136.3
9	Parking meter: Perp.	0.0327	13.8	0.00172	120.0	0.000697	148.1
10	Parking meter: Par.	0.0355	13.8	0.00207	923.8	0.000509	977.5
11	Car hood: 750 <i>rpm</i>	0.0744	35.6	0.0143	148.8	0.0103	510.6
12	Car hood: 3000 <i>rpm</i>	0.257	147.5	0.257	147.5	0.102	880.6
13	Medium tree	0.000985	16.3	0.000229	115.3	0.000226	240.0
14	Small tree	0.003	30.0	0.000465	293.1	0.000425	99.4

around 4 m/s^2 . However, not all these peaks are accessible, and RP values range from 10^{-4} m/s^2 to 4 m/s^2 . The frequencies of the significant power peaks varied greatly and 8 of the 14 measurements had the maximum power peaks below 100 Hz (*e.g.*, outside of the accessible region). Six of the sources exhibited HPP's in the $100 - 300 \text{ Hz}$ frequency range. The RP distribution for the 14 sources studied is represented in Figure A-6. The RP for 13 of the 14 sources lie inside the $100 - 300 \text{ Hz}$ range, even though the acceleration magnitudes differ over four orders of magnitude. These results show good agreement with published ambient vibration data. Table A.3 has been adapted from [5]. As a basis of comparison, both groups measured the vibrations from a microwave oven, and the magnitudes compare well (2.5 m/s^2 previously, compared to $1.1 - 4.2 \text{ m/s}^2$ for current investigation).

Upon comparing RP and AP values (see Table A.2), three important observations

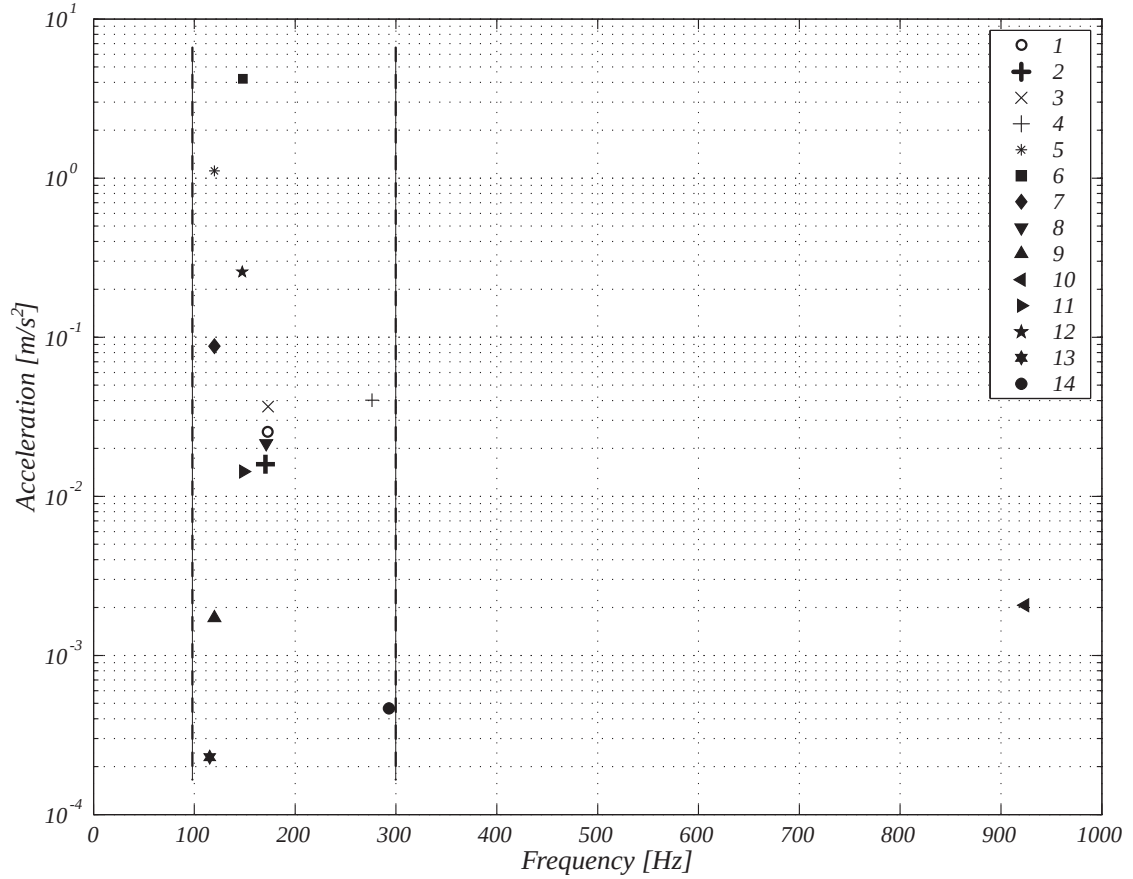


Figure A-6: RP acceleration vs. Frequency plot for all sources under atmospheric conditions. Legend indicates source/condition number in Tables A.1 and A.2.

can be made. First, some sources exhibit peaks with comparable power content at much higher frequencies. See for example source 12 (car hood at 3000 *rpm*) in Table A.2. The higher peak can excite a second or third resonance mode of the structure and strain cancellation (and therefore power loss) in the harvester device is likely. The effect of higher resonance modes on strain distribution for a harvester device has been investigated in Section 2.5.1. Second, comparable power at higher frequencies may suggest a more favorable operating point than RP due to frequency scaling with length. Thirdly, in 7 of the 14 sources/conditions investigated, an AP was identified at a lower frequency than the RP. The significance of this becomes clear when the device is operated in vacuum. As is illustrated in Figure A-5, the constant power lines for a device operated in vacuum drop to zero as the frequency decreases. This is because

Table A.3: Published data on ambient vibration sources [5].

Source	Acceleration [m/s^2]	Frequency [Hz]
Car engine compartment	12	200
Base of 3-axis machine tool	10	70
Blender casing	6.4	121
Clothes dryer	3.5	121
Car instrument panel	3	13
Door frame just after door closes	3	125
Small microwave oven	2.5	121
HVAC vents in office building	0.2-1.5	60
Windows next to busy road	0.7	100
CD on notebook computer	0.6	75
Second story floor of busy office	0.2	100

the power generated is inversely proportional to the vibration frequency. From the example it is clear that the AP will have the same power content as the RP. Depending on operating environment, the optimal harvesting point will not correspond to the highest vibration level peak (RP as defined here), but may be at a lower level and frequency.

A.4 Results: Graphical Representation

Measured displacement, acceleration, and PSD function frequency domain plots for all 14 sources/ conditions of ambient vibrations (see Table A.1) are presented here graphically for frequencies up to 1 kHz .

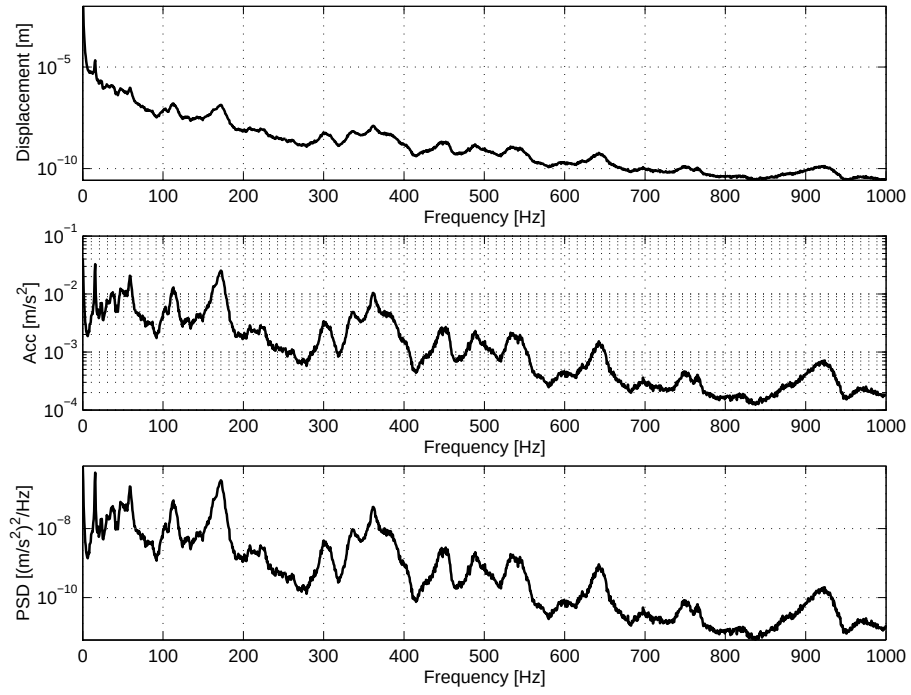


Figure A-7: 1. AC duct: center, low.

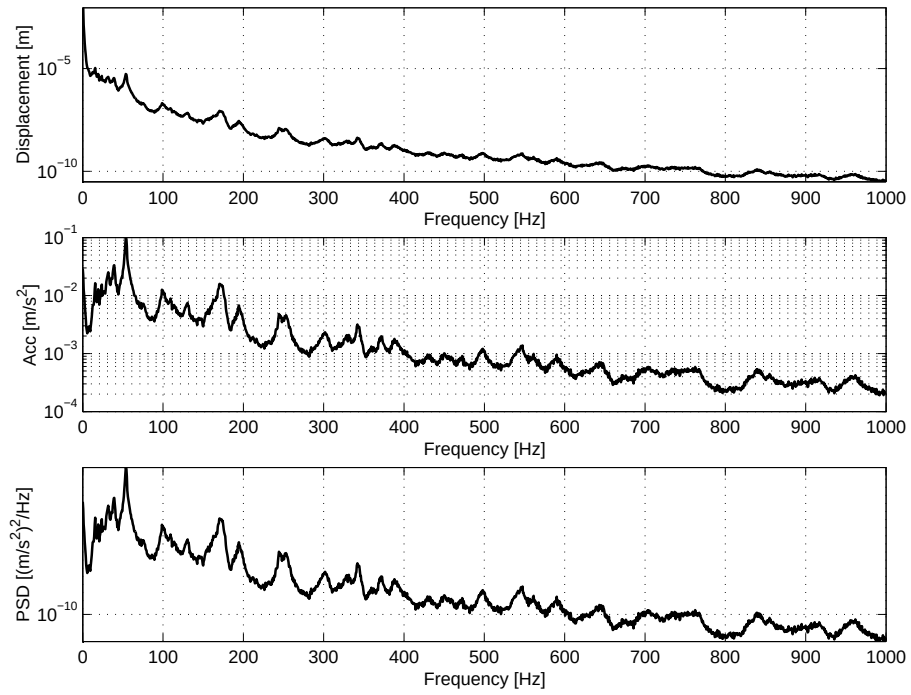


Figure A-8: 2. AC duct: side, high.

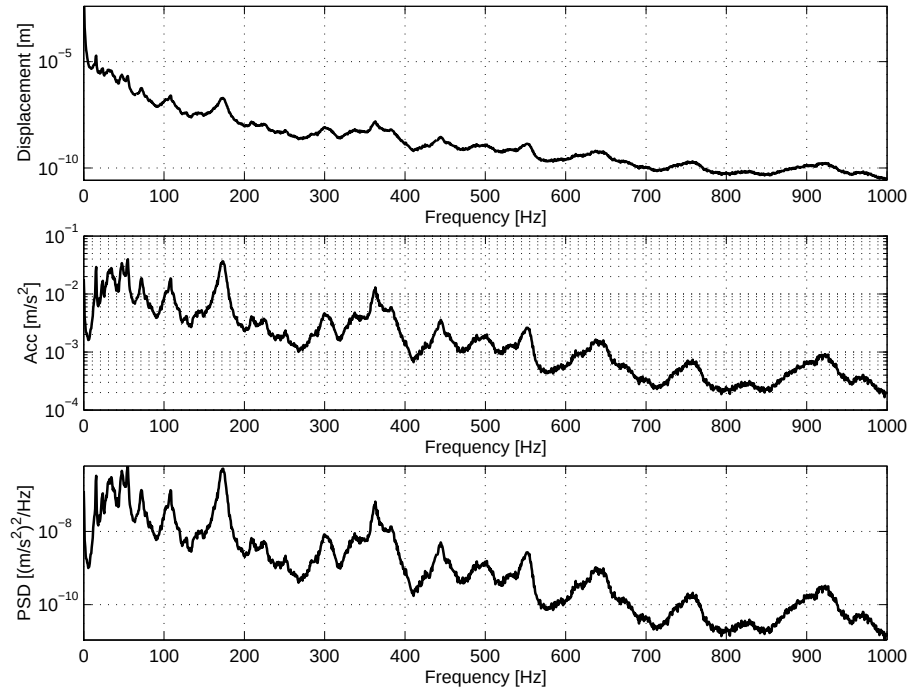


Figure A-9: 3. AC duct: center, high.

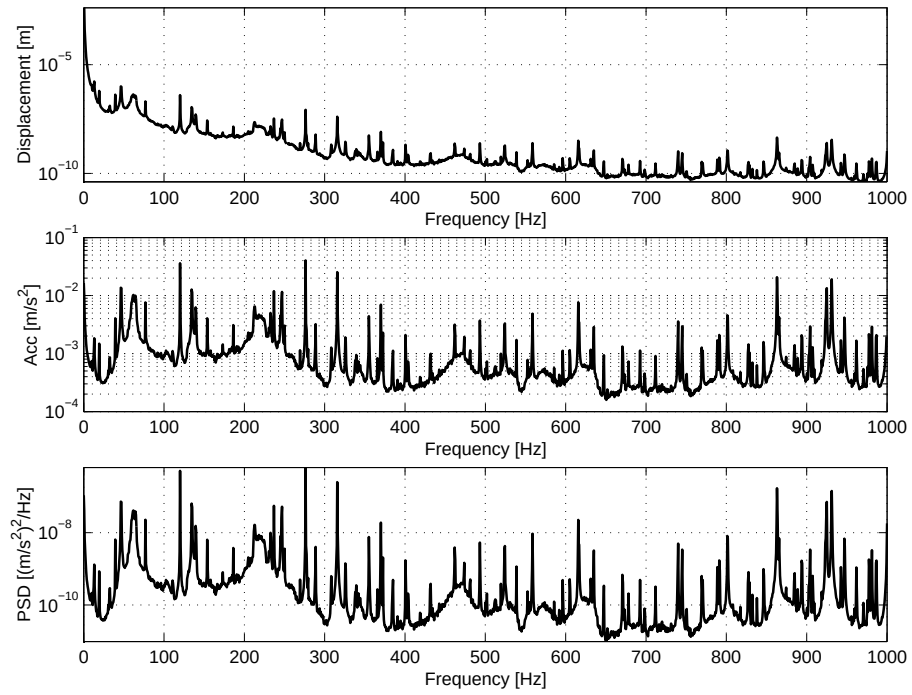


Figure A-10: 4. Computer side panel.

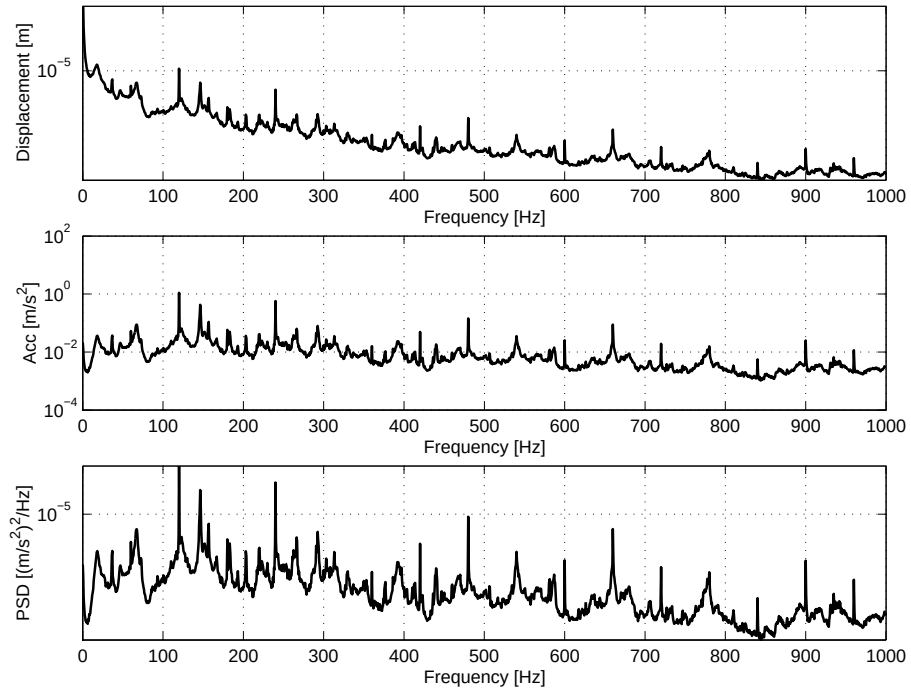


Figure A-11: 5. Microwave oven: top.

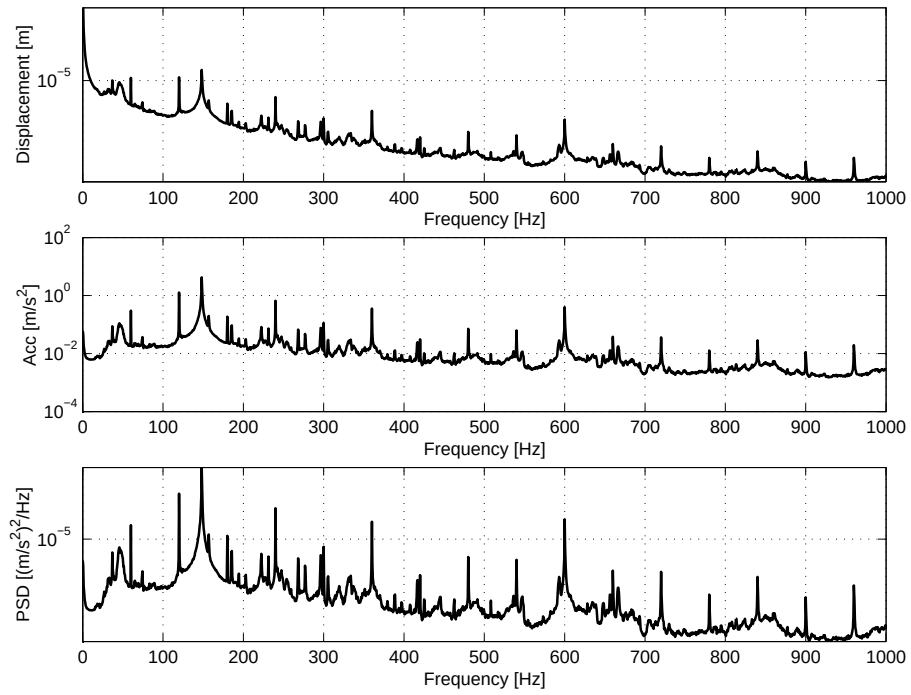


Figure A-12: 6. Microwave oven: side.

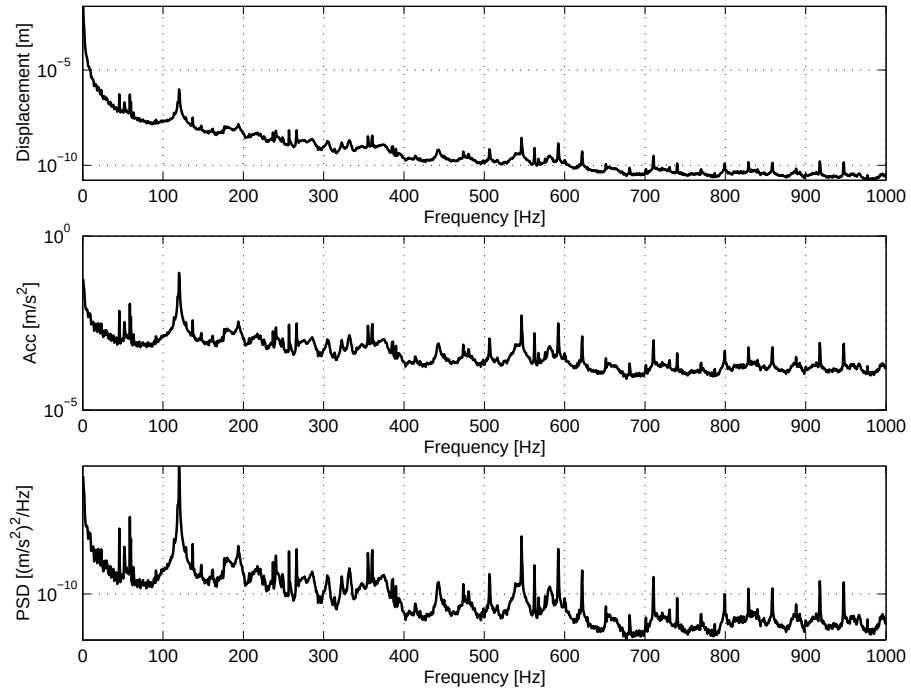


Figure A-13: 7. Office desk.

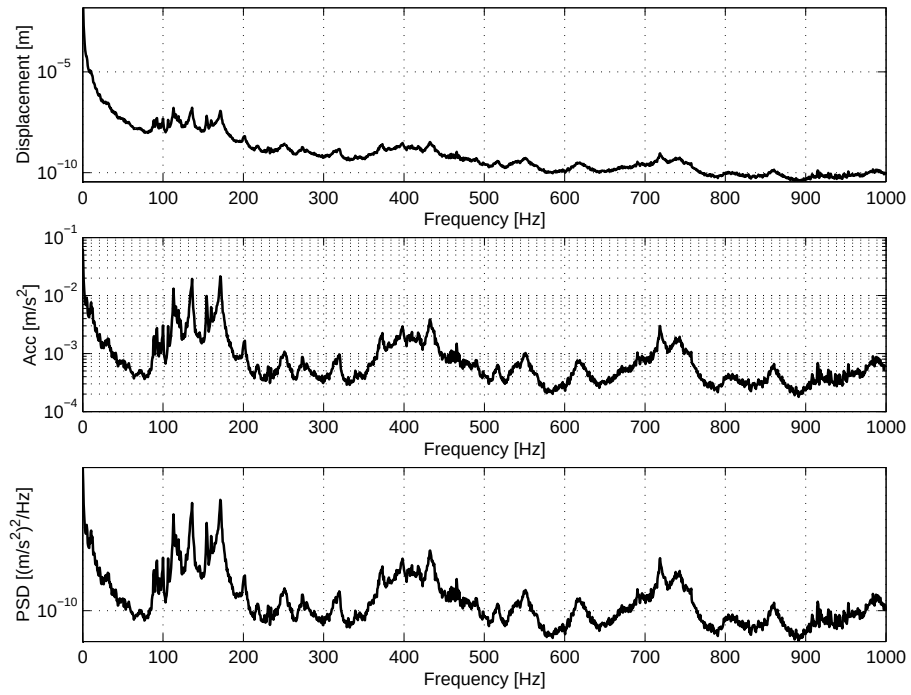


Figure A-14: 8. Harvard bridge railing.

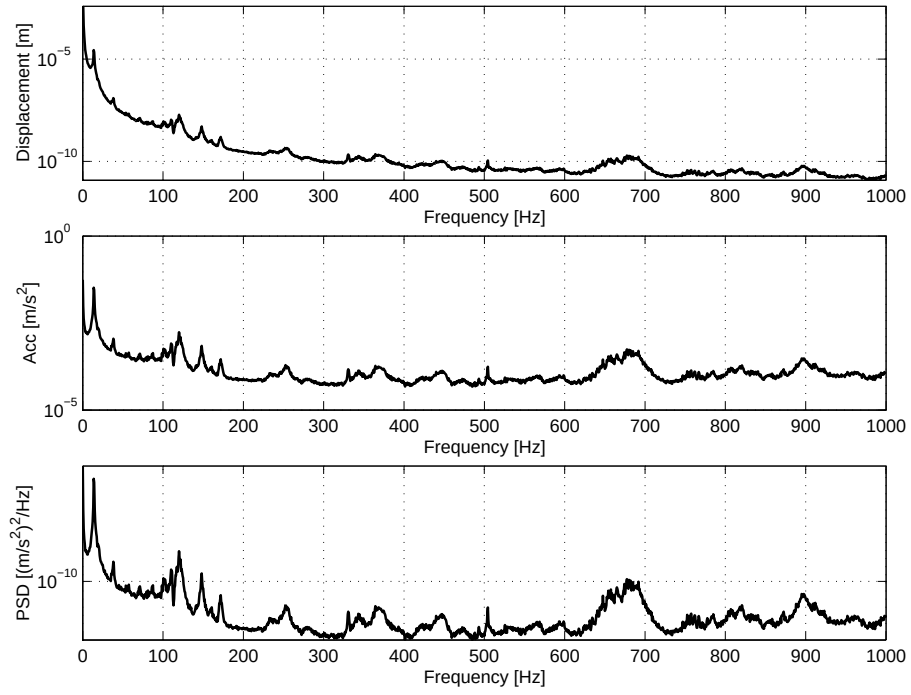


Figure A-15: 9. Parking meter: Perpendicular.

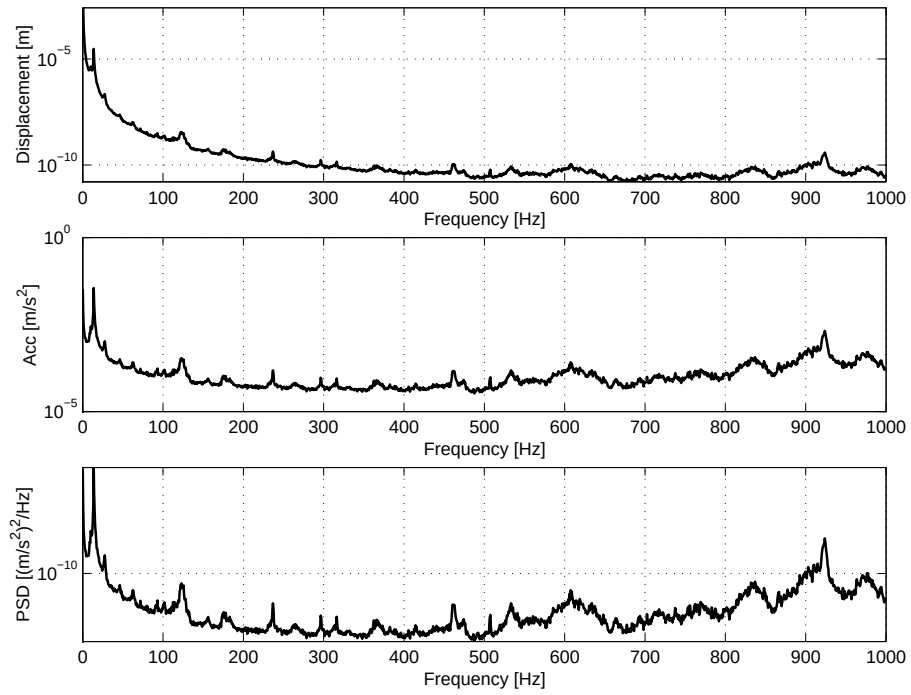


Figure A-16: 10. Parking meter: Parallel.

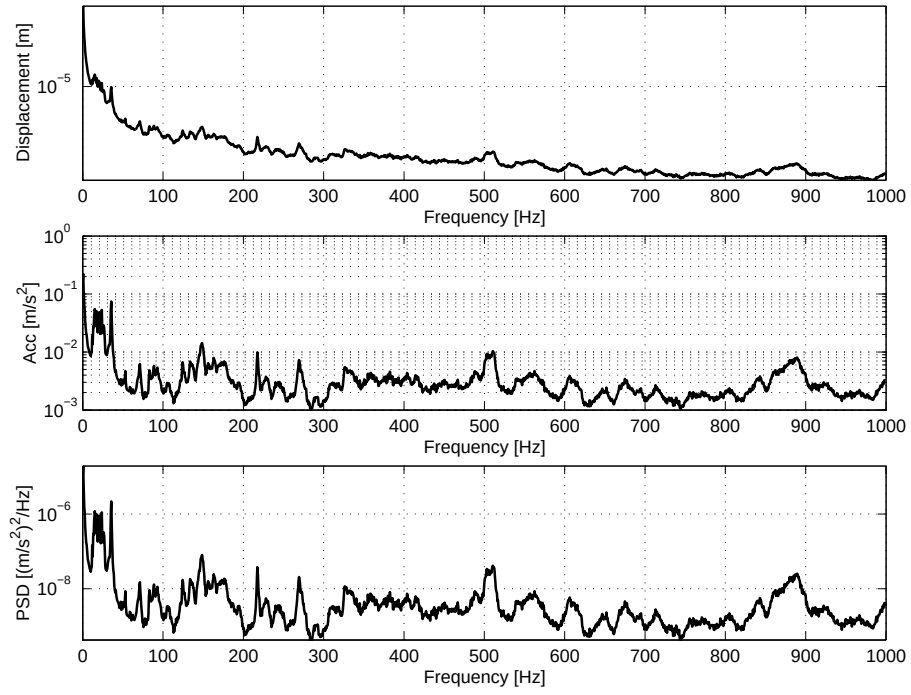


Figure A-17: 11. Car hood: 750 *rpm*.

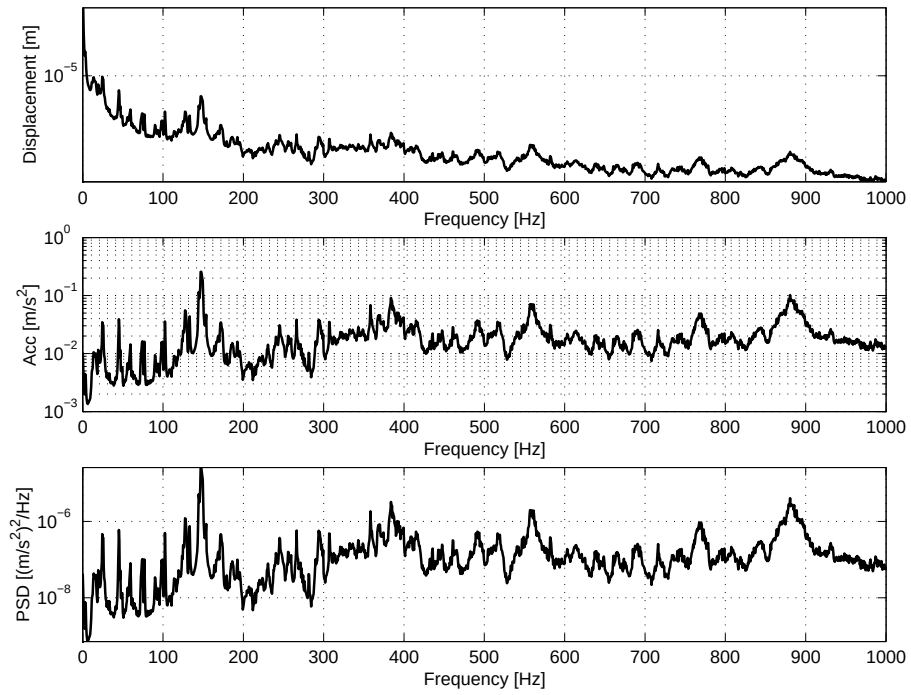


Figure A-18: 12. Car hood: 3000 *rpm*.

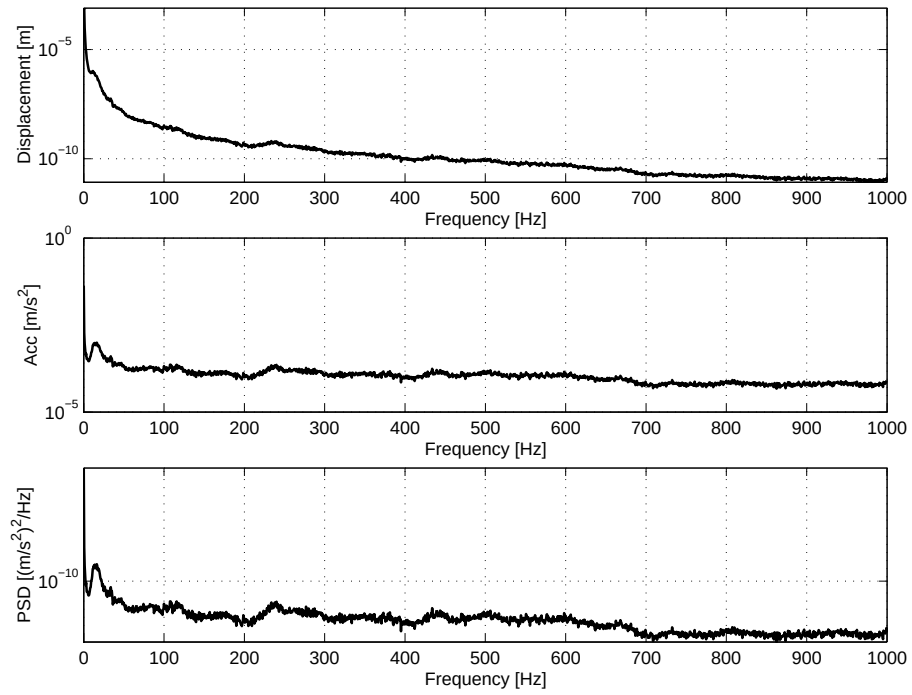


Figure A-19: 13. Medium tree.

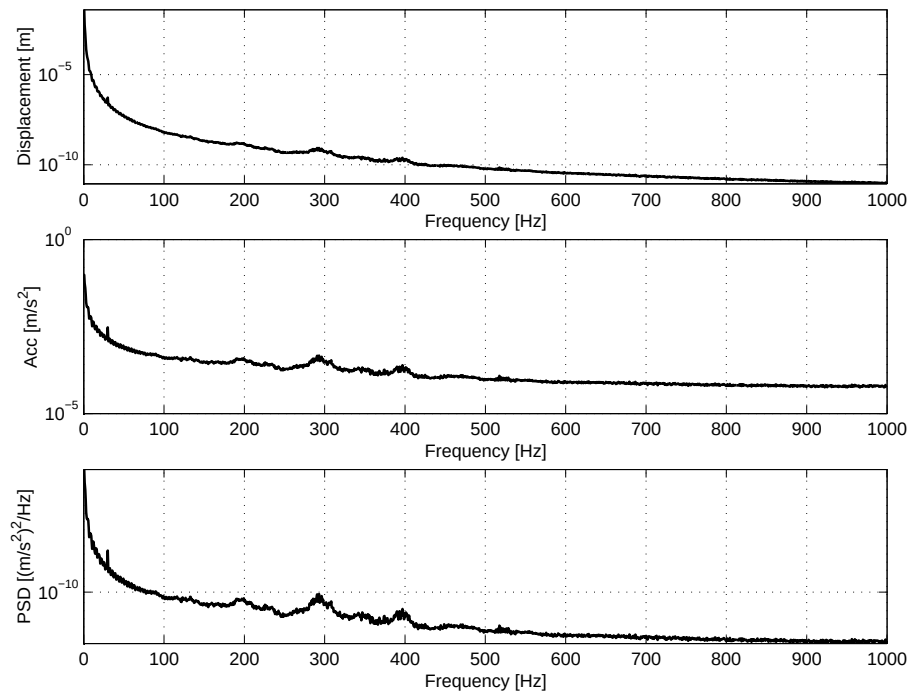


Figure A-20: 14. Small tree.

Appendix B

Poling Direction and the Constitutive Relations

It was stated briefly in Chapter 3 that the poling direction of the piezoelectric element has to be accounted for in the constitutive relations. The piezoelectric constants are positive (as per definition in the constitutive relations) when the poling direction (x_3^*) coincides with the global x_3 direction (refer to Figure B-1). For each element, the local coordinates are defined such that the local x_3^* direction is always in the poling direction. Hagood *et al.* presented a universal scheme where rotation matrices are used to accommodate any orientation of the element [85]. Smits *et al.* presented a simplified scheme [86] where it is assumed that the local x_3^* for each element is either in the global x_3 direction (parallel), or in the exact opposite (antiparallel) direction ($-x_3$). The latter scheme is adapted for the current project. For clarity, a 1-D crystallographic-based model is used to illustrate the effect of the poling directions on the constitutive relations in Sections B.1 and B.2. However, the sign conventions for electric field, voltage, and electric displacement need to be discussed first.

The sign convention can be established by looking at a charge difference between two electrodes, as illustrated in Figure B-1. The electrical engineering (EE) discipline standard has charge, q , flowing from positive to negatively charged electrodes as shown. The direction of charge flow determines the electric displacement, D . In the EE convention, the electric field is defined as positive from positive to negative charge

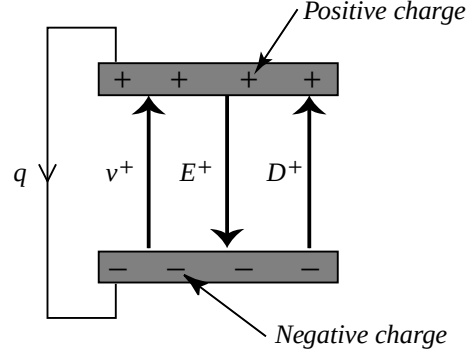


Figure B-1: Sign conventions for electrical engineering discipline.

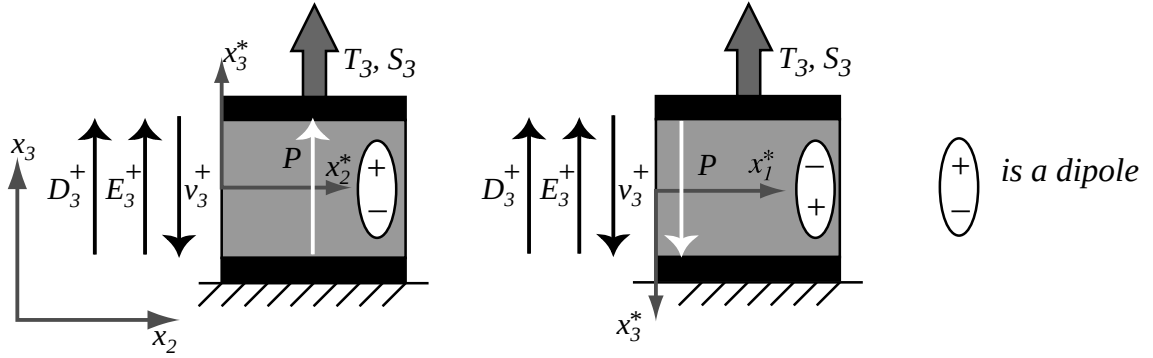


Figure B-2: Poling direction, local coordinates, and global coordinates: positively poled (*left*) and negatively poled (*right*).

[101]. The sign on voltage is defined in circuit theory with regards to a reference axis, but is commonly taken as $E = -\nabla v$ in piezoelectric theory and is adopted here. With the EE convention, the constitutive relations for a positively poled piezoelectric crystal (see [84]) give the proper signs on stress and strain when a positive electric field is applied (under constant stress or strain, respectively). This convention is consistent with the constitutive relations when implemented with the 1-D model as will be shown presently. This implementation is illustrated in Figure B-2.

B.1 Constitutive Relations

The constitutive relations in two forms are presented for the simplified 1-D case (poled in the x_3^* direction) in eq. (B.1) [84]. First, the stress and the electric displacement are

the dependent variables (T-D form). Second, the strain and the electric displacement are the dependent variables (S-D form). All constants in eqs. (B.1) are positive, *e.g.*, d_{33} is a positive number. In either case, the independent mechanical variable is zeroed (clamped in the case of the strain, and un-constrained in the case of stress) and a positive electric field applied (as defined in the alternative electrical convention) to establish the correct sign convention. Positive poling is described first.

$$\begin{array}{cc}
 \text{T-D form} & \text{S-D form} \\
 \left\{ \begin{array}{c} T_3 \\ D_3 \end{array} \right\} = \begin{bmatrix} c_{33}^E & -e_{33} \\ e_{33} & \varepsilon_{33}^S \end{bmatrix} \left\{ \begin{array}{c} S_3 \\ E_3 \end{array} \right\} & \left\{ \begin{array}{c} S_3 \\ D_3 \end{array} \right\} = \begin{bmatrix} s_{33}^E & d_{33} \\ d_{33} & \varepsilon_{33}^T \end{bmatrix} \left\{ \begin{array}{c} T_3 \\ E_3 \end{array} \right\} \quad (\text{B.1})
 \end{array}$$

B.2 Positive Poling

First, the element is positively poled, as shown in Figure B-2 on the left, using the EE sign convention. Analyzing the S-D form of the constitutive relations, for a positive electric field and zero stress, positive strain is expected through $S_3 = d_{33}E_3$ (as is indicated in the standard [84] defining d_{33} positively). Positive electric displacement is also implied by the S-D constitutive laws as ε_{33}^T is a positive constant: $D_3 = \varepsilon_{33}^T E_3$.

The same result is obtained when the T-D form of the constitutive relations are used: for the zero strain condition, the device has to be clamped in the 3 direction. When a positive electric field is applied, the developed stress should be negative since the rigid clamp will induce a compressive stress in the material. This is easiest thought of as positive strain induced by the applied electric field, free expansion, followed by the application of zero strain constraint, which induces a compressive stress. This result is obtained with positive piezoelectric constants, e_{33} : $T_3 = -e_{33}E_3$. For the electrical displacement, a positive ε_{33}^S value is also expected and obtained.

B.3 Negative Poling

When the element is negatively poled (the poling is in the $-x_3$ direction), the constitutive relations need adjustment (see Figure B-2, right). Again, the constitutive relations in the S-D form are analyzed first. When a positive electric field is applied with zero stress, the developed strain is negative. This is achieved with a negative sign applied to the piezoelectric constant (refer to eq. (B.2)). The electric displacement should still be positive to be consistent with the positive poling case.

$$S_3 = -(d_{33})E_3 \quad (\text{B.2})$$

The same result is obtained when the constitutive relations in the T-D form are analyzed. When a positive electric field is applied, the element will want to contract, but is clamped due to the zero strain (S_3) constraint. Thus, a tensile stress will develop in the element. This tensile stress is again obtained when a negative sign on the piezoelectric constant is introduced:

$$T_3 = -(-e_{33})E_3 \quad (\text{B.3})$$

The electric displacement will still be positive. Thus, the constitutive relations for the negatively poled element (see Figure B-2) are:

T-D form	S-D form
----------	----------

$$\left\{ \begin{array}{c} T_3 \\ D_3 \end{array} \right\} = \left[\begin{array}{cc} c_{33}^E & e_{33} \\ -e_{33} & \varepsilon_{33}^S \end{array} \right] \left\{ \begin{array}{c} S_3 \\ E_3 \end{array} \right\} \quad \left\{ \begin{array}{c} S_3 \\ D_3 \end{array} \right\} = \left[\begin{array}{cc} s_{33}^E & -d_{33} \\ -d_{33} & \varepsilon_{33}^T \end{array} \right] \left\{ \begin{array}{c} T_3 \\ E_3 \end{array} \right\} \quad (\text{B.4})$$

Eqs. (B.4) are simply the positively poled case (see eqs. (B.1)) with the sign of all the piezoelectric constants reversed. Note that the T-D form or the S-D form of the constitutive relations can be found by rewriting each in terms of their dependent variables.

B.4 Conclusion

The poling direction of the element relative to the global coordinates has to be accounted for in the models introduced in Chapter 3. The conventional (electrical engineering) sign convention for the electric field and voltage is consistent with the constitutive relations for piezoelectrics: for positive poling, positive piezoelectric constants are used. For negative poling, the negative of the standard piezoelectric constants are used.

Appendix C

Inter-Element Connections for Cantilevered Harvesters

In this appendix, the equations of motion for a general cantilevered harvester are analyzed to show how these equations are to be applied for the different electrical connections possible. In Section 3.3 it was assumed that there is only one electrode pair (as for a uni-morph) since the scalar equations can then be easily optimized to maximize the power extracted from the system. For bi-morphs this is not the case, and a scheme is presented to reduce the two-dimensional sensing equation to a scalar equation with effective coefficients to account for the specific geometry and interconnection of the piezoelectric elements.

C.1 Equations of Motion

In Section 3.3, the equations of motion (eqs. (3.41) and (3.42)) were derived for the cantilevered energy harvester under investigation in this project. In general, these equations are written in matrix form to account for multiple beam modes and multiple piezoelectric elements. The equations are repeated for convenience:

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} - \mathbf{\Theta}\mathbf{v} = -\mathbf{B}_f\ddot{\mathbf{w}}_B \quad (\text{C.1})$$

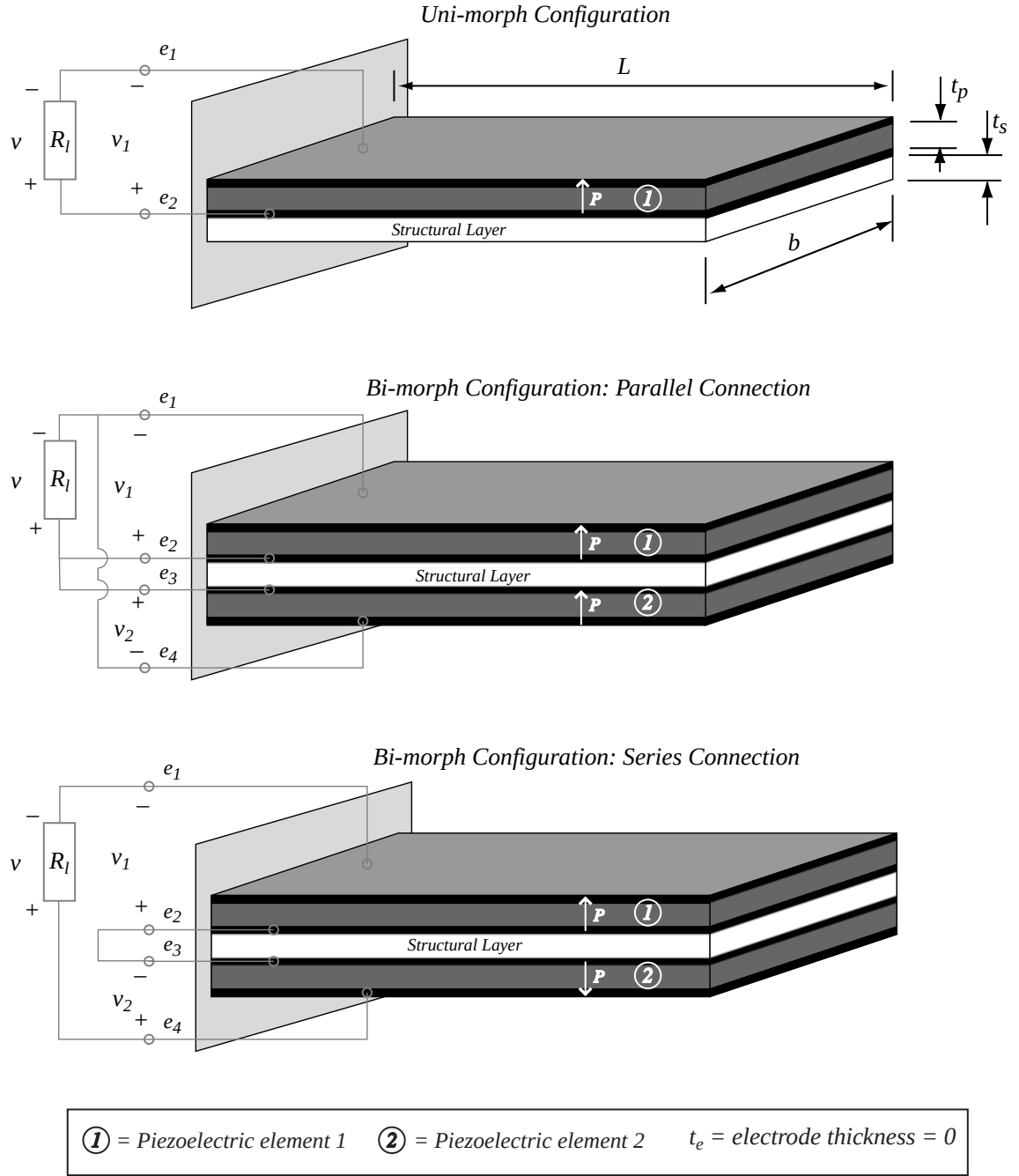


Figure C-1: Series and parallel connections for {3-1} cantilevered harvesters: (*top*) uni-morph configuration, (*middle*) bi-morph configuration with parallel connection, and (*bottom*) bi-morph configuration with series connection.

$$\Theta^t \mathbf{r} + \mathbf{C}_p \mathbf{v} = -\mathbf{q} \quad (\text{C.2})$$

Eq. (C.1) is known as the actuator equation and eq. (C.2) is the sensing equation.

It should be noted that $\mathbf{r}$ is the generalized mechanical displacement vectors, which need to be multiplied by the mode shapes to obtain the lateral displacement (refer to eq. (3.28)). At the tip, the mode shape is normalized to a value of 2, not 1. When a single beam mode is considered, eq. (C.1) reduces to a scalar equation. The same is true for eq. (C.2) when the harvester consists of a single piezoelectric element (such as a uni-morph). However, for the bi-morph configuration, there are two piezoelectric elements as shown in Figure C-1, and eq. (C.2) has a dimension of 2. The equations are written out in full below for the bi-morph:

$$M\ddot{\mathbf{r}} + C\dot{\mathbf{r}} + K\mathbf{r} - (\theta_1 v_1 + \theta_2 v_2) = -B_f \ddot{w}_B \quad (\text{C.3})$$

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \mathbf{r} + \begin{bmatrix} C_{p1} & 0 \\ 0 & C_{p2} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = - \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \quad (\text{C.4})$$

The subscript of the coefficients in eq. (C.4) indicates the corresponding piezoelectric element (as illustrated in Figure C-1). It is convenient to have scalar equations of motion for the power optimization to be carried out. This is accomplished by defining an effective system with one electrode pair as follows: The two equations in eq. (C.4) can be added to obtain a scalar equation in terms of both element 1 and 2 parameters:

$$(\theta_1 + \theta_2)r + (C_{p1}v_1 + C_{p2}v_2) = -(q_1 + q_2) \quad (\text{C.5})$$

Eqs. (C.3) and (C.5) can now be analyzed, based on the specific interconnection and poling geometry for the device. The objective is to reduce eqs. (C.3) and (C.5) to equations with effective coefficients (indicated with a subscript e) and the load voltage (v) and/or the charge through the load (q):

$$M\ddot{\mathbf{r}} + C\dot{\mathbf{r}} + K\mathbf{r} - \theta_e v = -B_f \ddot{w}_B \quad (\text{C.6})$$

$$\theta_e r + C_{pe} v = -q \quad (\text{C.7})$$

The scalar governing equations (eqs. (C.6) and (C.7), with effective coefficients, will be employed to derive values for θ_e and C_{pe} for the three different harvester

configurations shown in Figure C-1.

C.2 Uni-morph Cantilevered Harvester

A uni-morph harvester consists of a single piezoelectric element, and the equations of motion reduce to scalar equations (when a single beam mode is considered). Referring to Figure C-1, writing the governing equations (eqs. (C.1) and (C.2)) in terms of the voltage across the load, v , the equations reduce to:

$$M\ddot{r} + C\dot{r} + Kr - \theta_1 v = -B_f \ddot{w}_B \quad (\text{C.8})$$

$$\theta_1 r + C_{p1} v = -q \quad (\text{C.9})$$

Current is related to the charge through $i = \frac{dq}{dt}$, which is related to the voltage across the load through $i = \frac{v}{R_l}$. Substituting these results back into eq. (C.9), the final form of the sensing equation is obtained:

$$\theta_1 \dot{r} + C_{p1} \dot{v} + \frac{1}{R_l} v = 0 \quad (\text{C.10})$$

Thus, by comparison to eqs. (C.6) and (C.7), the effective coefficients are $\theta_e = \theta_1$ and $C_{pe} = C_{p1}$, as expected.

C.3 Bi-morph Cantilevered Harvester

Two interconnection configurations are available for the bi-morph cantilevered harvester: the parallel and series connections. These connections *require* different poling configurations. The effect of poling direction on the element piezoelectric constants is discussed in Appendix B. First, the parallel connection will be described and implemented, before the series connection is considered (refer to Figure C-2).

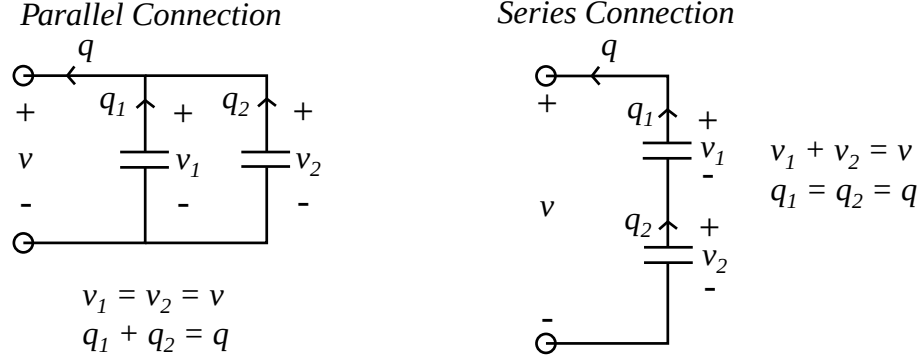


Figure C-2: Bi-morph configuration simplified effective electrical circuit for the (*left*) parallel and (*right*) series connections. The piezoelectric elements are represented as simple capacitors.

C.3.1 Parallel Connection

The parallel connection requires that both element 1 and 2 are poled in the same direction (refer to Figure C-1, middle) so that *voltages add* when the beam is bent (*i.e.*, compression on element 1 and tension on element 2 for positive tip displacement). Assuming both elements are positively poled (in the x_3 direction), the positive piezoelectric constants are used (refer to Appendix B). Since the strain above and below the neutral axis are equal and opposite (assuming symmetric beam bending), this will cause opposite electric fields in the elements. The bi-morph is connected to the electric load by connecting the center electrodes (e_2 and e_3) to the one side of the electrical load, and the outer electrodes (e_1 and e_4) to the other side. The effective connection obtained is illustrated in Figure C-2 (representing the piezoelectric elements as capacitors) and the following observations can be made:

$$v_1 = v_2 = v \quad \text{and} \quad q_1 + q_2 = q \quad (\text{C.11})$$

Using these relations, eqs. (C.3) and (C.5) are written in terms of the load voltage and the charge through the load:

$$M\ddot{r} + C\dot{r} + Kr - (\theta_1 + \theta_2)v = -B_f\ddot{w}_B \quad (\text{C.12})$$

$$(\theta_1 + \theta_2)r + (C_{p1} + C_{p2})v = -q \quad (\text{C.13})$$

Again, current through the load can be calculated from the charge through the load ($i = \frac{dq}{dt}$). This current is related to the voltage across the electrical load through $v = iR_l$. The sensing equation in its final form is obtained:

$$(\theta_1 + \theta_2)\dot{r} + (C_{p1} + C_{p2})\dot{v} + \frac{1}{R_l}v = 0 \quad (\text{C.14})$$

From here it is convenient to define the effective coefficients to be implemented in the equivalent single electrode system model.

$$\theta_e = \theta_1 + \theta_2 \quad \text{and} \quad C_{pe} = C_{p1} + C_{p2} \quad (\text{C.15})$$

This result is intuitive since the effective capacitance for two capacitors in parallel is simply the sum of the individual capacitances. Furthermore, the charges developed in each of the individual elements add to obtain the total charge through the load (θ relates the charge developed per strain). When the bi-morph is symmetric, $\theta_1 = \theta_2$ and $C_{p1} = C_{p2}$, resulting in effective parameters (in terms of element 1 parameters): $\theta_e = 2\theta_1$ and $C_{pe} = 2C_{p1}$.

C.3.2 Series Connection

For the series connection, the two elements are oppositely poled, such that the opposite strains above and below the neutral axis generate electric fields that are in the same direction (refer to Figure C-1, bottom). Referring to Figure C-1, the bottom piezoelectric element (2), is poled in the $-x_3$ direction. From Appendix B, it is seen that the negative of the piezoelectric constant should be used to determine the coupling term, θ_2 .

The device is connected to the electrical load by shorting the two center electrodes (e_2 and e_3), and connecting the top (e_1) and bottom electrodes (e_4) across the electrical load. The effective circuit is presented in Figure C-2 where the piezoelectric elements are represented as simple capacitors. From this circuit, the following

observations can be made:

$$v_1 + v_2 = v \quad \text{and} \quad q_1 = q_2 = q \quad (\text{C.16})$$

Using these relations, the sensing equation (eq. (C.5)) can be rewritten as:

$$(\theta_1 + \theta_2)r + (C_{p1}v_1 + C_{p2}v_2) = -2q \quad (\text{C.17})$$

Assuming that the bi-morph is symmetric: $\theta_1 = \theta_2$ and $C_{p1} = C_{p2}$, as well as, $v_1 = v_2$ and $q_1 = q_2$. Thus, the effective equations of motion (eqs. (C.6) and (C.7)) can be written in terms of element 1 parameters (v_1 , q_1 , θ_1 , and C_{p1}):

$$M\ddot{r} + C\dot{r} + Kr - 2\theta_1v_1 = -B_f\ddot{w}_B \quad (\text{C.18})$$

$$2\theta_1r + 2C_{p1}v_1 = -2q_1 \quad (\text{C.19})$$

Now, since the elements are connected in series, the charge (and current) through each element will be the same, $q = q_1$. The current is related to the charge through $i = \frac{dq}{dt}$. Thus, the derivative of eq. (C.19) with respect to time has to be taken. The voltages developed across each element will add ($v = v_1 + v_2 = 2v_1$). Also, the current is related to the voltage across the electrical load through $v = iR_l$, where R_l is the electrical load. Thus, the equations of motion become:

$$M\ddot{r} + C\dot{r} + Kr - \theta_1v = -B_f\ddot{w}_B \quad (\text{C.20})$$

$$\theta_1\dot{r} + \frac{1}{2}C_{p1}\dot{v} + \frac{1}{R_l}v = 0 \quad (\text{C.21})$$

By comparison to eqs. (C.6) and (C.7), the effective coupling is to the coupling of the individual elements, $\theta_e = \theta_1$. The effective capacitive term is one half the capacitance of the individual elements, $C_{pe} = \frac{1}{2}C_{p1}$, as expected for two identical capacitors connected in series. With these effective parameters, the equations of motion reduce to two scalar equations and the scheme can be used to determine the

performance of the system.

Appendix D

Experimental Results

Experimental results for Device 2 are presented in tabulated form. Refer to Section 4.3 for the procedure of the tests, and Section 5.4 for a graphical representation of the data. The signs of the base displacements accounts for the correction necessary to calculate the relative displacements, as discussed in Section 4.3.2.

Table D.1: Experimental results for 70 and 95 Hz tests for varying electrical loads.

Frequency = 70 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	0.074	0.179	0.515	0.594	0.621	0.629
Power $[\mu W]$	1.20	2.73	5.01	3.87	2.86	2.37
Absolute tip displacement $[\mu m]$	29.3	28.4	26.8	26.3	26.0	26.0
Base displacement[†] $[\mu m]$	-12.9	-12.9	-12.9	-12.9	-12.9	-12.9
Relative tip displacement $[\mu m]$	16.4	15.5	13.9	13.4	13.1	13.1
Frequency = 95 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	0.268	0.626	1.270	1.346	1.377	1.374
Power $[\mu W]$	15.62	33.28	30.47	19.90	14.03	11.31
Absolute tip displacement $[\mu m]$	49.1	46.8	36.9	35.2	34.8	36.4
Base displacement[†] $[\mu m]$	-7.02	-7.02	-7.02	-7.02	-7.02	-7.02
Relative tip displacement $[\mu m]$	42.1	39.8	29.9	28.2	27.8	29.4

[†] Refer to Section 4.3.2 for explanation of the sign base displacement values

Table D.2: Experimental results for 107, 113, 125, and 150 Hz tests for varying electrical loads.

Resonance frequency = 107 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	1.514	2.627	3.428	3.492	3.572	3.565
Power $[\mu W]$	499.5	586.3	222.0	134.0	94.46	76.13
Absolute tip displacement $[\mu m]$	208.0	151.0	82.6	77.0	76.6	75.9
Base displacement $[\mu m]$	0.00	0.00	5.64	5.64	5.64	5.64
Relative tip displacement $[\mu m]$	208.0	151.0	88.2	82.6	82.2	81.5
Anti-resonance frequency = 113 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	0.533	1.339	4.992	7.039	8.435	9.093
Power $[\mu W]$	61.86	152.3	470.7	544.5	526.6	495.2
Absolute tip displacement $[\mu m]$	65.1	67.9	108.0	140.0	164.0	175.0
Base displacement $[\mu m]$	-4.96	-4.96	0.00	0.00	0.00	0.00
Relative tip displacement $[\mu m]$	60.1	62.9	108.0	140.0	164.0	175.0
Frequency = 125 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	0.208	0.514	1.448	1.667	1.749	1.762
Power $[\mu W]$	9.40	22.45	39.58	30.55	22.64	18.61
Absolute tip displacement $[\mu m]$	20.4	21.2	27.2	29.2	30.0	30.1
Base displacement $[\mu m]$	4.05	4.05	4.05	4.05	4.05	4.05
Relative tip displacement $[\mu m]$	24.5	25.3	31.3	33.3	34.1	34.2
Frequency = 150 Hz, $\ddot{w}_B = 2.5 \text{ m/s}^2$						
Resistance $[k\Omega]$	4.61	11.91	55.90	100.1	156.2	200.4
Voltage $[V]$	0.097	0.230	0.507	0.542	0.554	0.558
Power $[\mu W]$	2.07	4.51	4.85	3.23	2.27	1.87
Absolute tip displacement $[\mu m]$	6.50	6.69	7.47	7.59	7.64	7.65
Base displacement $[\mu m]$	2.81	2.81	2.81	2.81	2.81	2.81
Relative tip displacement $[\mu m]$	9.31	9.50	10.3	10.4	10.5	10.5

Appendix E

Coupled Electromechanical Model: Code

The coded implementation of the coupled electromechanical model, developed in Section 3.3, is given below for Device 2. Device 2 is a PZT-5A-based cantilevered bi-morph harvester which is poled for series connection operation. This code was used for the model validation/verification in Section 5.3, specifically for predicting the device response at the specified operating points (frequencies and electrical loads). The predicted and experimentally measured responses are compared in Figures 5-5 to 5-8.

```
clc;
clear all;
close all;
disp('PiezoSystems,Inc.:T226-A4-503X');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MATERIAL PROPERTIES
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Piezo material properties
Eps_0 = 8.854e-12;
Eps_T_33 = 1800*8.854e-12;
rho_p = 7800;
d31 = -190e-12;
cE11 = 66e9;
% Calculate material properties for current configuration
% Plate configuration
% cE11 = sE11/(sE11^2-sE12^2);
```

```

% e31 = d31/(sE11+sE12);
% Eps_S_33 = Eps_T_33-2*d31^2/(sE11+sE12);
% KS33 = Eps_S_33/Eps_0
% Beam Configuration
% cE11 = 1/sE11;
% e31 = d31/sE11;
e31 = -14; % fit to frequency spacing
% Eps_S_33 = Eps_T_33-d31^2/sE11;
Eps_S_33 = Eps_T_33-d31*e31;
KS33 = Eps_S_33/Eps_0;
% Parameter assignment
c_E = cE11;
e_m = e31;
Eps_S = Eps_S_33;
% Material properties for layers
E_pzt_p = c_E; % PZT plate stiffness
rho_pzt = rho_p;
% structure
E_s = 100e9; % calculated
rho_s = 7165; % measured
nu_s = 0; % beam configuration
E_s_p = E_s/(1-nu_s^2); % stiffness

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Define device parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L = 55.e-3; % length in m
b = 31.8e-3; % beam width in m
% Specify layer thicknesses
t_pzt = 270e-6;
t_s = 140e-6
% Define parameters for mass at tip, not applicable for this case
L0 = 0; % length of the mass == 0 (no mass)
H0 = 70e-6; % thickness of the mass
b0 = b; % width of the mass
rho_0 = 1200; % density of mass

% mechanical damping
Zeta_m = 0.0178; % measured
ddW_b = 2.5; % input vibration amplitude
% index of pzt element above the neutral axis
ind_pzt = 3;
% Specify geometry
t_l = [t_pzt t_s t_pzt];
E_l = [E_pzt_p E_s_p E_pzt_p];

```

```

rho_l    =    [rho_pzt rho_s rho_pzt];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate neutral axis, I, EI, rho, etc.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Determine the neutral axis (NA)
for ii = 1: length(t_l)
    if ii == 1
        zb_l(ii)    =    t_l(ii)/2;
    else
        zb_l(ii)    =    zb_l(ii-1) + t_l(ii-1)/2 + t_l(ii)/2;
    end % if
    EAzb(ii)    =    zb_l(ii)*t_l(ii)*b*E_l(ii);
    EA(ii)    =    t_l(ii)*b*E_l(ii);
end % for
zb    =    sum(EAzb)/sum(EA);

% useful values for later calculation
zp_max    =    zb_l(ind_pzt)+t_pzt/2-zb; % max piezo distance from NA
zp_ave    =    abs(zb_l(ind_pzt)-zb); % distance: NA to piezo center
t_t    =    sum(t_l); % total modeled thickness

% determine the moment of inertia for each layer
for ii = 1:length(t_l)
    I_l(ii)    =    1/12*b*t_l(ii)^3 + t_l(ii)*b*(zb_l(ii)-zb)^2;
end % for ii
% Moment of area
I_s    =    sum(I_l);
% effective EI
EI_s    =    0;
for ii=1:length(t_l)
    EI_s    =    EI_s + E_l(ii)*I_l(ii);
end % for ii
% effective young's modulus
E_eff    =    EI_s/I_s;
% effective density
rho_c    =    0;
for ii    =    1:length(t_l)
    rho_c    =    rho_c +t_l(ii)*rho_l(ii);
end % for ii
rho_c    =    rho_c/t_t;
% mass per length
m    =    rho_c*b*t_t; % mass per length

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

% Calculate parameters for modal analysis
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Effect of non-point loaded proof mass
m0 = b0*(rho_c*t_t + rho_0*H0);
CG_m = (rho_c*t_t*b0*z_b + rho_0*H0*b0*(t_t+H0/2))/(rho_c*t_t+rho_0*H0);
ox = L0/2; % depends on the beam config
oy = CG_m-z_b; % distance from CG of mass to neutral axis

M0 = m0*L0;
M0_ = M0/(m*L);
% determine the static moment
S0 = M0*ox;
S0_ = S0/(m*L^2);
% determine the moment of inertia of the mass
for ii=1:length(t_l)
    Iz(ii) = rho_l(ii)*b0*(1/12*L0*t_l(ii)^3+...
        t_l(ii)*L0*(z_b_l(ii)-CG_m)^2);
    Ix(ii) = rho_l(ii)*b0*(1/12*t_l(ii)*L0^3);
end IzM = rho_0*b0*(1/12*L0*H0^3 + H0*L0*(t_t+H0/2-CG_m)^2);
IxM = rho_0*b0*(1/12*H0*L0^3);
Iyy = sum(Iz) + sum(Ix) + IzM + IxM;
I0 = Iyy+M0*(ox^2+oy^2);
I0_ = I0/(m*L^3);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Modal Analysis (for beam with non-point loaded proof mass)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the value for sigma_N*L - range and search increments
x = [1.87:0.0001:1.88]; % w/o mass
for ii=1:length(x)
    A11(ii) = (sinh(x(ii))+sin(x(ii)))+I0_*x(ii)^3*(-cosh(x(ii))+...
        cos(x(ii))) + S0_*x(ii)^2*(-sinh(x(ii))+sin(x(ii)));
    A12(ii) = (cosh(x(ii))+cos(x(ii)))+I0_*x(ii)^3*(-sinh(x(ii))-...
        sin(x(ii))) + S0_*x(ii)^2*(-cosh(x(ii))+cos(x(ii)));
    A21(ii) = (cosh(x(ii))+cos(x(ii)))+M0_*x(ii)^1*(sinh(x(ii))-...
        sin(x(ii))) + S0_*x(ii)^2*(cosh(x(ii))-cos(x(ii)));
    A22(ii) = (sinh(x(ii))-sin(x(ii)))+M0_*x(ii)^1*(cosh(x(ii))-...
        cos(x(ii))) + S0_*x(ii)^2*(sinh(x(ii))+sin(x(ii)));
    % Calculate determinant of matrix
    DET(ii) = A11(ii)*A22(ii)-A12(ii)*A21(ii);
end
% Find roots
if sign(DET(1)) == 1
    Ind = find(DET<0);
elseif sign(DET(1)) == -1

```

```

    Ind = find(DET>0);
end
Ind = Ind(1);
B1_L = x(Ind); % lambda_N*L
clear x;
% calculate model parameters
B1 = B1_L/L; % lambda_N for current configuration
c1 = 1; % arbitrary constant - cancels
s1 = A12(Ind)/A11(Ind);
% natural freq from modal analysis
om_n_a = B1^2*sqrt(EI_s/m);
f = om_n_a/2/pi;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate lumped element model parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solving intergrals analytically with symbolic toolbox, following
% analytical expressions are obtained and solved for each case. Note
% these functions have been truncated!
xa = 0;
xb = L;
Int_Phi_2 = -1/8*c1^2*(-exp(-2*xb*B1)*s1^2*exp(4*xb*B1)+(truncated)
Int_ddPhi_2 = 1/8*c1^2*B1^3*exp(-2*xa*B1)+1/2*c1^2*B1^3*(truncated)
Int_Phi = c1*(sinh(xb*B1)-sin(xb*B1)-s1*cosh(xb*B1)-(truncated)
Int_ddPhi= c1*B1*sinh(xb*B1)+c1*B1*sin(xb*B1)-c1*B1*s1*(truncated)
clear xa xb;

% Mode shape value calculations
Phi_L = c1*((cosh(B1*L)-cos(B1*L))-s1*(sinh(B1*L)-sin(B1*L)));
dPhi_L = c1*B1*((sinh(B1*L)+sin(B1*L))-s1*(cosh(B1*L)-cos(B1*L)));
ddPhi_0 =c1*B1^2*((cosh(B1*0)+cos(B1*0))-s1*(sinh(B1*0)+sin(B1*0)));

% calculate coefficient matrices
% Mass matrix
for ii = 1:length(t_l)
    M_l(ii) = b*t_l(ii)*rho_l(ii)*Int_Phi_2;
end % for ii
M1 = sum(M_l) + M0*Phi_L^2 + 2*S0*Phi_L*dPhi_L + I0*dPhi_L^2;

% Stiffness
for ii = 1:length(t_l)
    K_l(ii) = E_l(ii)*I_l(ii)*Int_ddPhi_2;
end % for ii
K1 = sum(K_l);

```

```

% Coupling matrix
Theta = b*zp_ave*e_m*dPhi_L; % for a single element
Theta = Theta; % effective coupling for series connection

% Capacitance
Cp = b*L*Eps_S/t_pzt; % For a single element
Cp = Cp/2; % effective capacitance for series connection
% System coupling, kappa
Ke_2 = Theta^2/(K1*Cp);
% Input matrix
Bf = m*Int_Tsi + M0*Phi_L + 1/2*M0*dPhi_L*(2*L+L0);
% estimate the natural frequency acc to lumped model
om_n = sqrt(K1/M1); % check to compare against modal analysis

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Specify frequencies and loads at which system response is analyzed
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
om = [70*2*pi 95*2*pi om_n om_n*sqrt(1+Ke_2) 125*2*pi 150*2*pi]';
OM = om/om_n; % normalize
Rl = [4.61 11.91 19.90 55.9 100.1 156.2 200.4].*1000;
Re = Rl*Cp*om_n; % dimensionless time constant, alpha

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate system response at these loads and frequencies
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate optimal resistances at OC and SC
OM_oc = sqrt(1+Ke_2); % Anti-resonance frequency
r_opt = (OM_oc^4 + (4*Zeta_m^2-2)*OM_oc^2 + 1)/(OM_oc^6 + ...
(4*Zeta_m^2-2*(1+Ke_2))*OM_oc^4 + (1+Ke_2)^2*OM_oc^2);
Re_opt_oc = sqrt(r_opt);
Rl_opt_oc = round(Re_opt_oc/(Cp*om_n));

OM_sc = 1; % Resonance frequency
r_opt = (OM_sc^4 + (4*Zeta_m^2-2)*OM_sc^2 + 1)/(OM_sc^6 + ...
(4*Zeta_m^2-2*(1+Ke_2))*OM_sc^4 + (1+Ke_2)^2*OM_sc^2);
Re_opt_sc = sqrt(r_opt);
Rl_opt_sc = round(Re_opt_sc/(Cp*om_n));
Rl_opt_df = Rl_opt_oc-Rl_opt_sc;

% Calculate the optimum frequency ratio for a given electrical load
factor_H = 4*Zeta_m^2 - 2*(1+Theta^2/(K1*Cp));
% calculate optimum frequency numerically
for ii=1:length(Re)
    OM_opt_roots = roots([2*Re(ii)^2 (1+factor_H*Re(ii)^2) 0 -1]);
    for jj=1:length(OM_opt_roots)

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        if real(OM_opt_roots(jj))>0
            OM_opt(ii) = sqrt(OM_opt_roots(jj));
        end
    end
end

% Use coefficients to calculate the response of the system
for ii=1:length(Re)
    for jj=1:length(OM)
        den = sqrt((1-(1+2*Zeta_m*Re(ii))*OM(jj)^2)^2+((2*Zeta_m...
            +(1+Theta^2/(K1*Cp))*Re(ii))*OM(jj)-Re(ii)*OM(jj)^3)^2);
        Z(ii,jj) = (Bf/K1*sqrt(1+Re(ii)^2*OM(jj)^2))*ddW_b/den;
        W_tip(ii,jj) = Phi_L*Z(ii,jj);
        Rot_tip(ii,jj) = dPhi_L*Z(ii,jj);
        Str_m(ii,jj)= -zp_max*ddPhi_0*Z(ii,jj);
        Vp(ii,jj) = Bf*Re(ii)*Ke_2*OM(jj)*ddW_b/abs(Theta)/den;
        Pout(ii,jj) = om_n/K1*Re(ii)*Ke_2*OM(jj)^2 / den^2;
        Ip(ii,jj) = Pout(ii,jj)/Vp(ii,jj);
    end
end

% Calculate response for power-optimal resistance at each frequency
for ii=1:length(OM)
    Re_opt_f(ii)=sqrt((OM(ii)^4+(4*Zeta_m^2-2)*OM(ii)^2+1)/(OM...
        (ii)^6+(4*Zeta_m^2-2*(1+Ke_2))*OM(ii)^4+(1+Ke_2)^2*OM(ii)^2));
    Rl_opt_f(ii) = Re_opt_f(ii)/(Cp*om_n);
    den_opt=sqrt((1-(1+2*Zeta_m*Re_opt_f(ii))*OM(ii)^2)^2+...
        ((2*Zeta_m+(1+Theta^2/(K1*Cp))*Re_opt_f(ii))*OM(ii)-...
        Re_opt_f(ii)*OM(ii)^3)^2);
    Z_opt(ii)= Bf/K1*sqrt(1+(Re_opt_f(ii)*OM(ii))^2)*ddW_b/den_opt;
    W_tip_opt(ii) = Phi_L*Z_opt(ii);
    Rot_tip_opt(ii) = dPhi_L*Z_opt(ii);
    Str_m_opt(ii) = -zp_max*ddPhi_0*Z_opt(ii);
    Vp_opt(ii) = Bf*Re_opt_f(ii)*Ke_2*OM(jj)*ddW_b/abs(Theta)/den;
    Pout_opt(ii)= om_n/K1*Re_opt_f(ii)*Ke_2*OM(jj)^2 / den^2;
    I_opt(ii) = Pout_opt(ii)/Vp_opt(ii);
end

% calculate mechanical response across length of structure
x_a = [0:L/100:L]';
for ii=1:length(x_a)
    den = sqrt((1-(1+2*Zeta_m*Re_opt_sc)*OM_sc^2)^2+((2*Zeta_m+...
        (1+Ke_2)*Re_opt_sc)*OM_sc-Re_opt_sc*OM_sc^3)^2);
    Z_1(ii) = Bf/K1*sqrt(1+(Re_opt_sc*OM_sc)^2)*ddW_b / den
    W_1(ii) = c1*((cosh(B1*x_a(ii))-cos(B1*x_a(ii)))-...

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        s1*(sinh(B1*x_a(ii))-sin(B1*x_a(ii))))*Z_1(ii);
    Str_1(ii) = -zp_max*(c1*B1^2*((cosh(B1*x_a(ii))+cos(B1*x_a...
(ii)))-s1*(sinh(B1*x_a(ii))+sin(B1*xv(ii)))))*Z_1(ii);
end

% calculate device power density
% operating volume
volume = (2*max(abs(W_tip_opt))+H0*cos(max(abs(Rot_tip_opt)))+...
    L0*sin(max(abs(Rot_tip_opt))))*(L+L0)*(max(b,b0))*1e6;
P_dens = (max(Pout_opt(:))/1e-6)/volume;

```