Consider a point robot located in the environment seen in the “sphere world” of Figure 1. The radius of the bounding sphere is $R_B = 10$. The two circular obstacles each have identical radius $R_O = 3$. The centers of both obstacles lie on the y-axis, and are each located a distance of 5 units from the bounding circle center. Assume that the point robot’s initial position is located at a distance of 7 units from the origin of the bounding sphere along the negative x-axis. Consider two different possible goal positions, $q_f_1$ and $q_f_2$. The first goal position is located along the positive x-axis a distance of 7 units from the center. The second goal position is located a distance of 7 units from the center, but is located along a line that makes a 60° angle with the x-axis.

The following problems are a sequence of programming exercises related to the potential field method.

**Problem 1:** Develop functions to determine the distance between a configuration, $q$, and the obstacles, the boundary circle, and the goal.

**Problem 2:** Plot (perhaps using Mathematica) the attractive potential $U_{attr}(q) = \frac{\xi}{2} d_{goal}^2(q)$ for the two different goal positions.
**Problem 3:** consider the repelling potential function

\[
U_{\text{rep}}^i(q) = \begin{cases} 
\frac{\eta}{2} \left( \frac{1}{d_i(q)} - \frac{1}{\rho_0} \right)^2 & \text{for } d_i(q) \leq \rho_0 \\
0 & \text{for } d_i(q) > \rho_0
\end{cases}
\]

where \(d_i(q)\) is the distance between \(q\) and the \(i^{th}\) obstacle. Plot the repelling potential made up of the sum of the boundary and obstacle repelling potentials: \(U_{\text{rep}}(q) = \sum_i U_{\text{rep}}^i\).

**Problem 4:** Plot the potential \(U(q) = U_{\text{attr}}(q) + U_{\text{rep}}(q)\) for the two different goal positions. Choose different constants \(\eta\) and \(\xi\).

**Problem 5:** For a given choice of \(\eta\) and \(\xi\) and for each of the goal positions, plot the path that results from solving the equation:

\[
\dot{q} = -\nabla U(q)
\]

or from the equation:

\[
m\ddot{q} = -\nabla U(q)
\]

where \(m\) is the “mass” of the virtual particle. If you choose the later approach, you may wish to add some “damping” to the equations as a crude guard against certain numerical roundoff errors. This can be done by adding a damping term \(-b\dot{q}\).

**Extra Credit:** Repeat the above exercises using the navigation functions of Rimon and Koditschek.