CDS 101/110: Lecture 7.2
Loop Analysis of Feedback Systems

November 11 2016

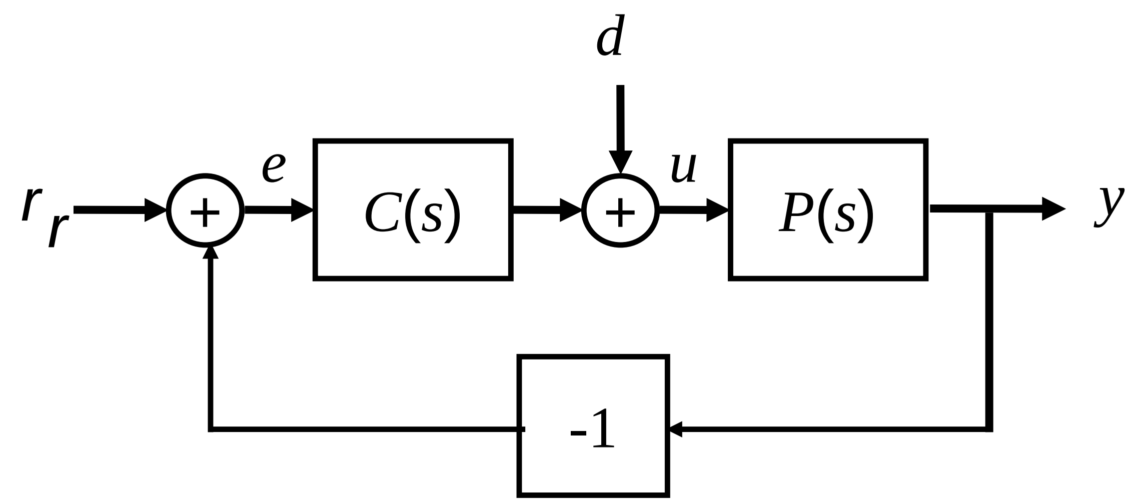
Goals:

- Why Nyquist Diagrams?
- *Gain margin* and *phase margin*
- Examples!

Reading:

- Åström and Murray, Feedback Systems, Chapter 10, Sections 10.1-10.4,

What can you do with a Nyquist Analysis?



Set Up (somewhat artificial)**:**

- **Given:** $P(s)$
  - (any unstable roots known)
- **Given:** $C(s)$
  - (any unstable roots known)
- **Q:** can negative output feedback stabilize the system (stable $G_{yr}(s)$)?

Possible Solutions:

$$G_{yr}(s) = \frac{PC}{1+PC} = \frac{n_p(s)n_c(s)}{d_p(s)d_c(s)+n_p(s)n_c(s)}$$

- Compute and check poles of G_{yr}
- Find another way to determine existence of unstable poles without computing roots of

$$d_P(s)d_C(s) + n_P(s)n_C(s)$$

The Nyquist plot *logic*

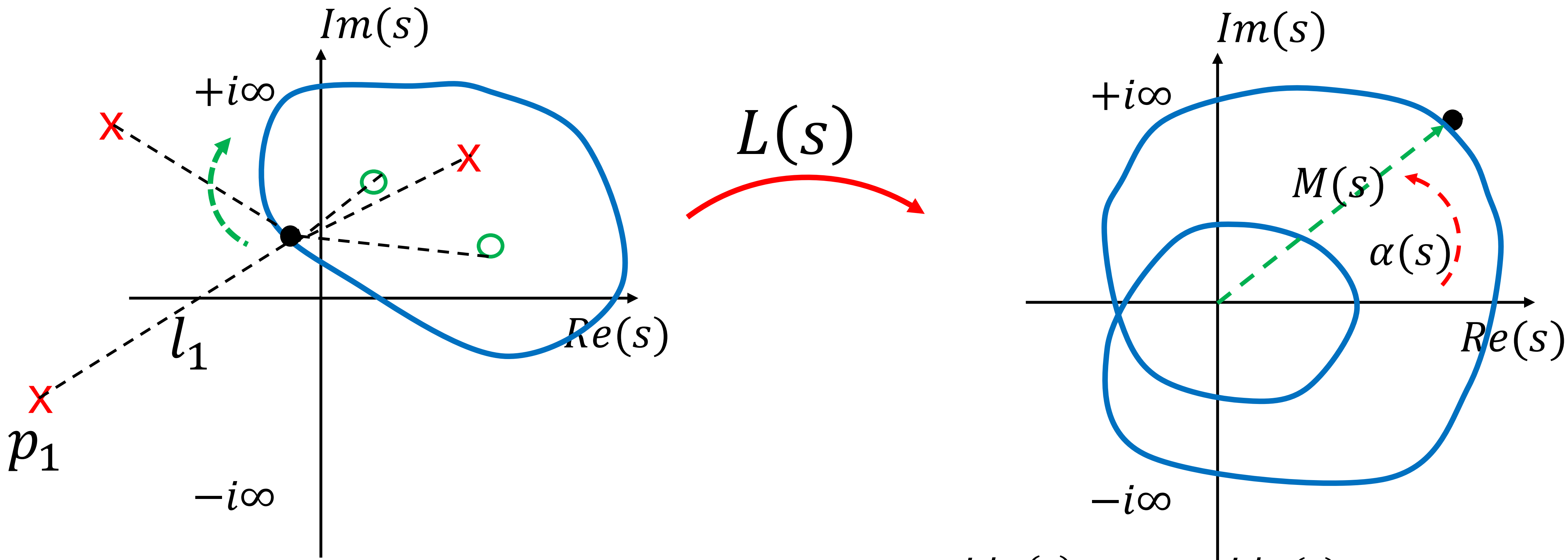
- Poles of $G_{yr}(s)$ are zeros of

$$1 + P(s)C(s) = \frac{d_P(s)d_C(s) + n_P(s)n_C(s)}{d_P(s)d_C(s)}$$

- If $G_{yr}(s)$ is unstable, then it has at least one pole in RHP
- An unstable pole of $G_{yr}(s)$ implies and unstable (RHP) zero of $1 + P(s)C(s)$
- Nyquist plot and Nyquist Criterion allow us to determine if $1 + PC$ has RHP zeros *without* polynomial solving.

Argument Principle

(underlying Nyquist Criterion)



$$f(s) = \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = \frac{r_1(s)e^{i\psi_1(s)}\, r_2(s)e^{i\psi_2(s)} \ldots\, r_m(s)e^{i\psi\_m(s)}}{l_1(s)e^{i\phi_i(s)}\, l_2(s)e^{i\phi_2(s)} \ldots\, l_n(s)e^{i\phi_n(s)}}$$

$$= M(s)e^{i\alpha}$$

$$\alpha(s) = \sum_{i=1}^{m} \psi_i(s) - \sum_{j=1}^{n} \phi_j(s)$$

As s moves *clockwise* around Γ, $L(s)$ must rotate around the origin by 2π for each pole inside the contour, and by -2π for each zero inside the contour

P # RHP poles of open loop $L(s)$ (from $P(s), C(s)$ poles)

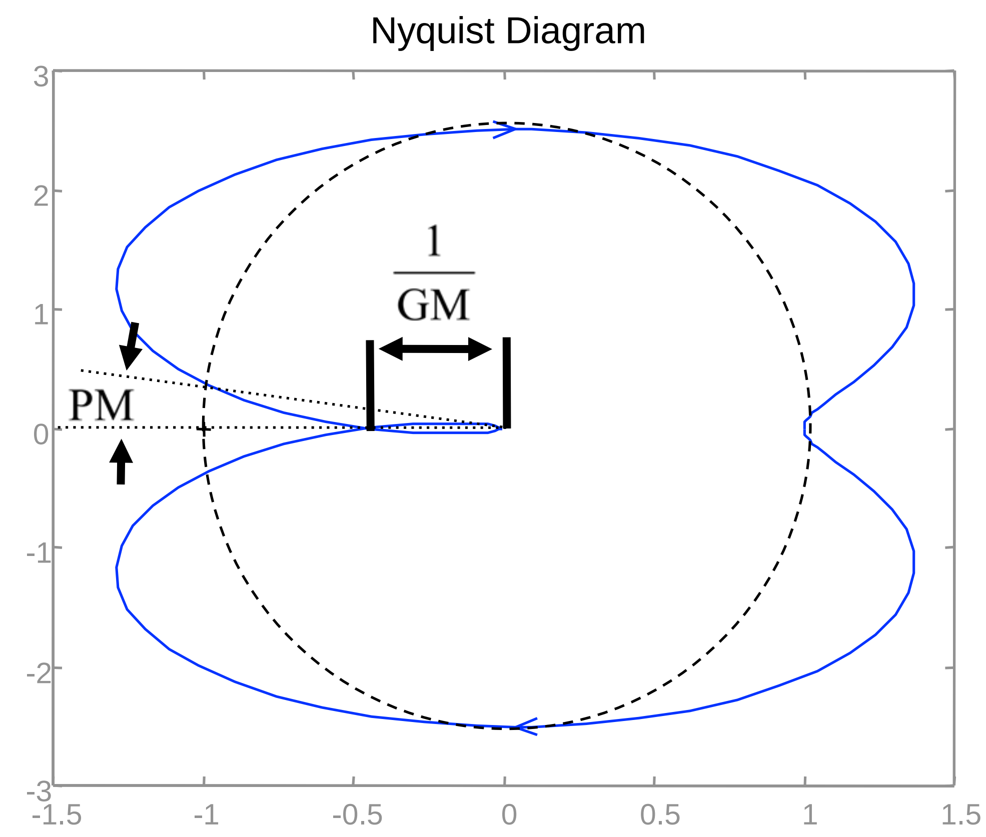
N # clockwise encirclements of -1 (from Nyquist plot)

Z # RHP zeros of $1 + L(s)$

Then $Z_{RHP} = N + P$

Robust stability: gain and phase margins

Nyquist plot tells us if closed loop is stable, but not how stable

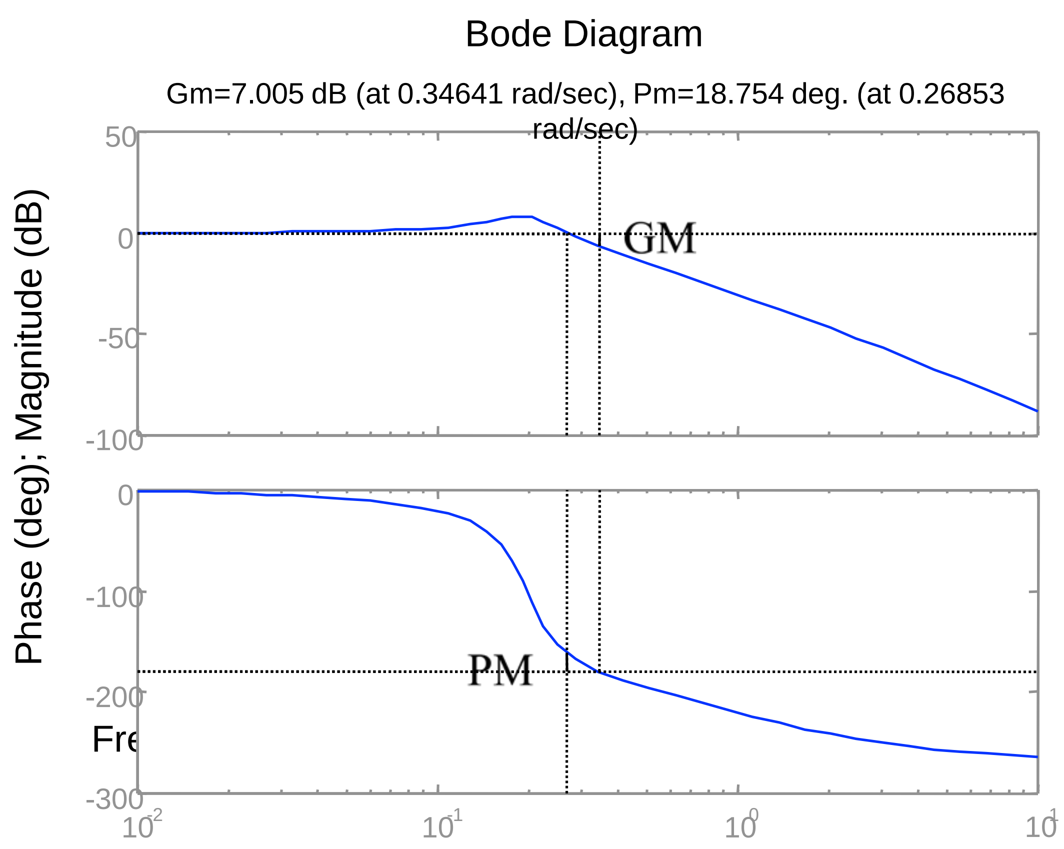


Gain margin

- How much we can modify the loop gain and still have the system be stable
- Determined by the location where the loop transfer function crosses 180˚ phase

Phase margin

- How much "phase delay" can be added while system remains stable
- Determined by the phase at which the loop transfer function has unity gain



Bode plot interpretation

- Look for gain = 1, 180˚ phase crossings
- MATLAB: margin(sys)

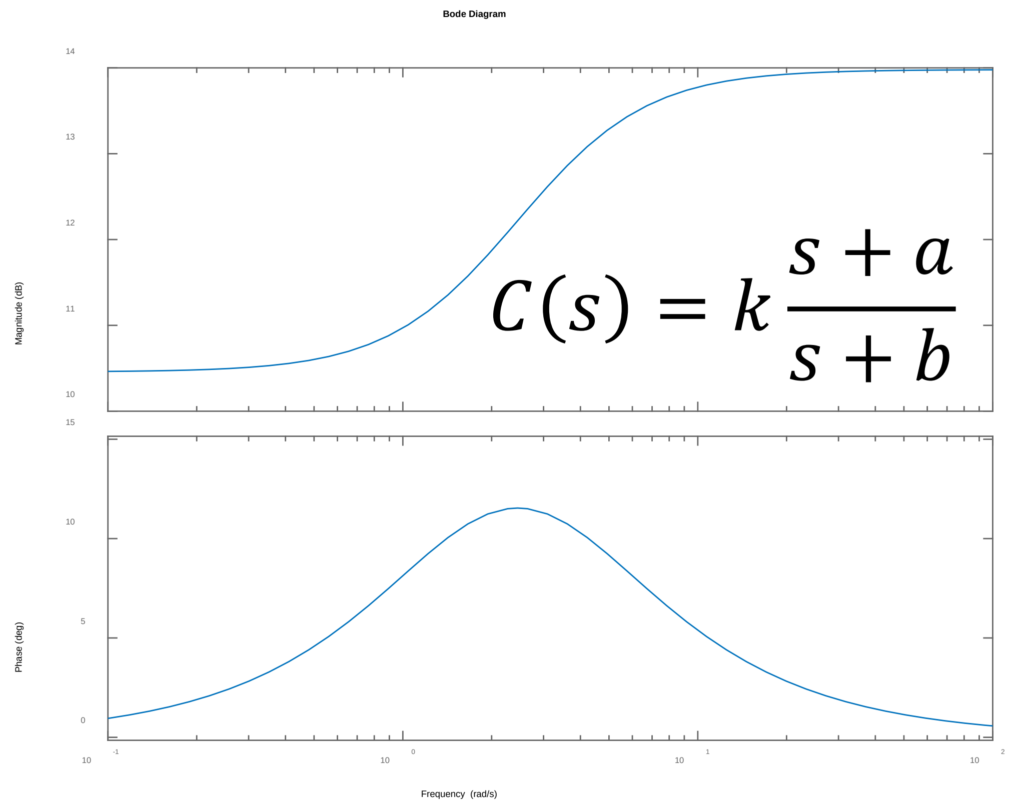
Nyquist Plot Example #1

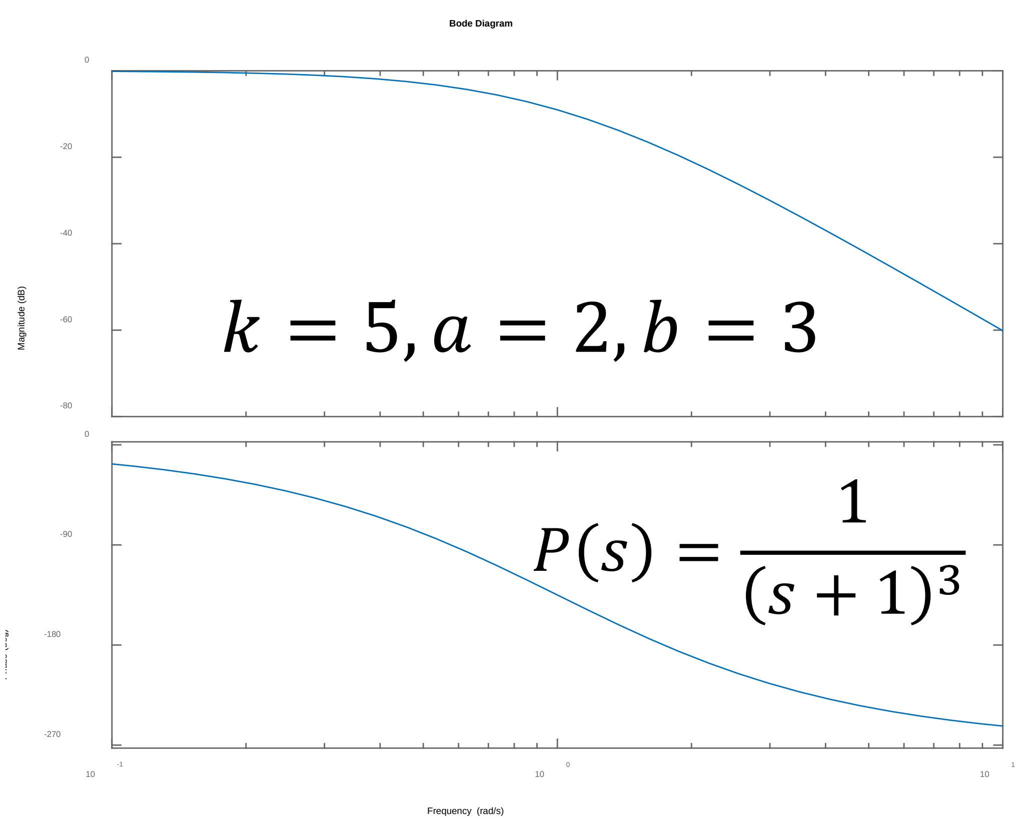
$$P(s) = \frac{1}{(s+1)^3} \qquad C(s) = k\frac{s+a}{s+b}$$

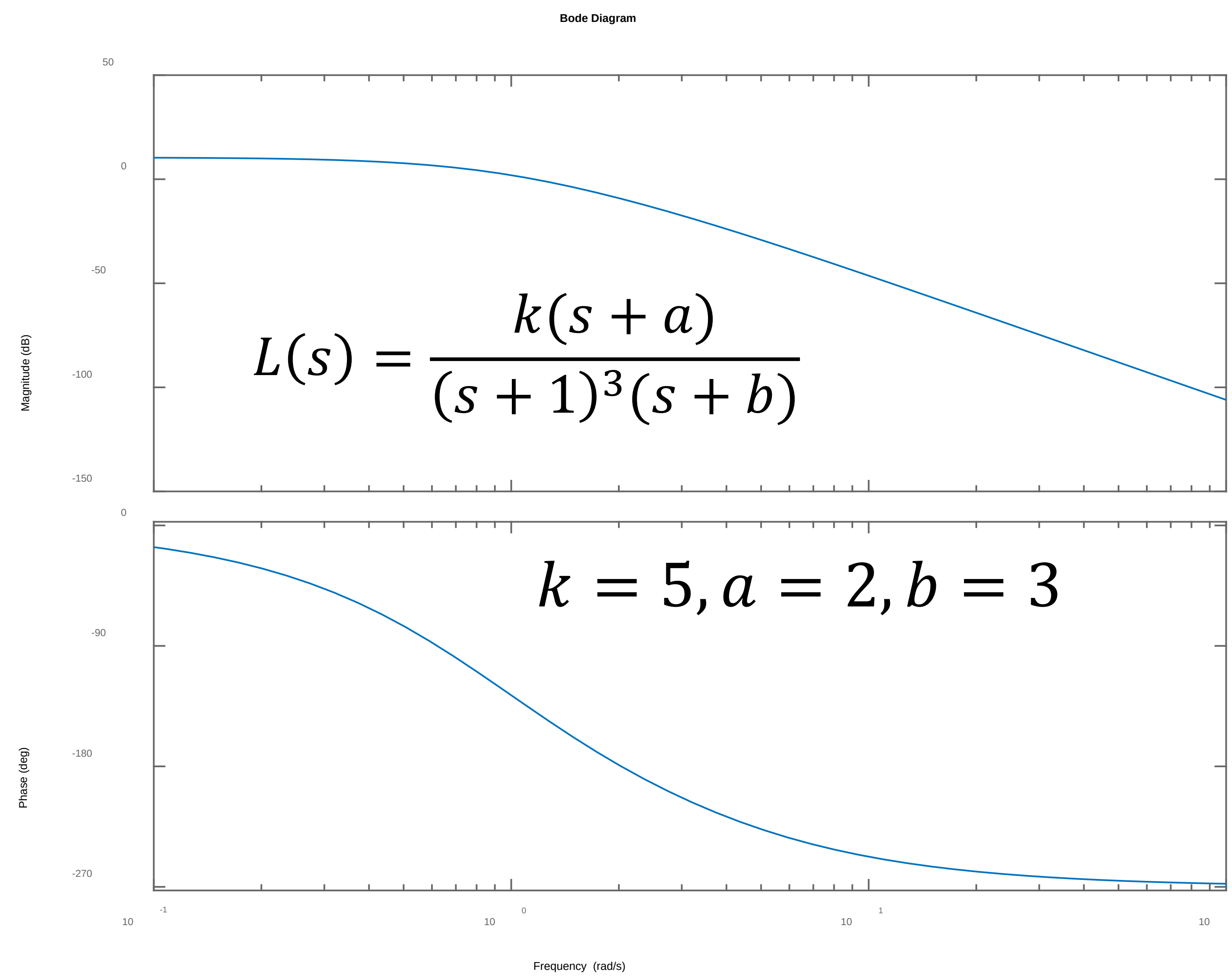
$$1 < a < b$$

Goal #1: Is closed loop system stable?

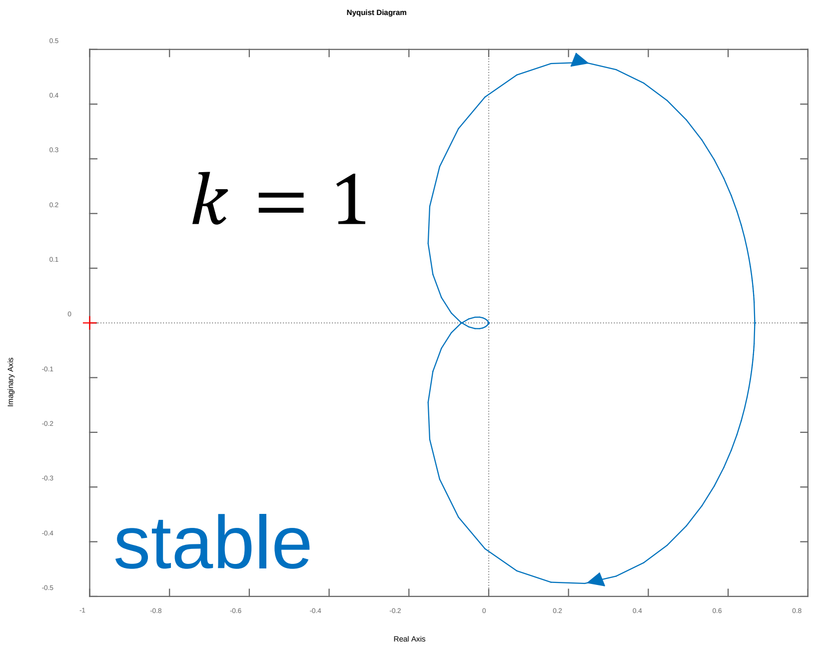
Goal #2: Does stability vary with gain?

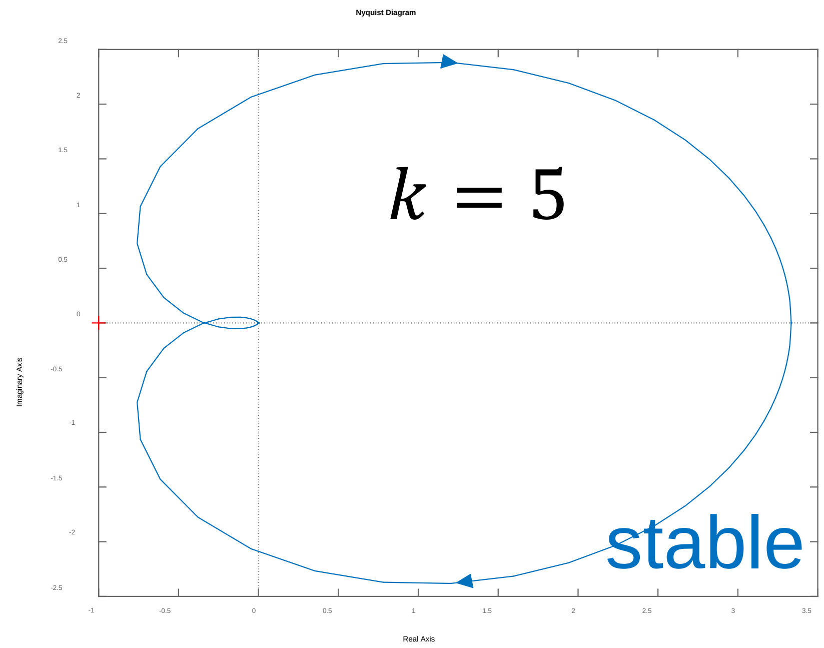


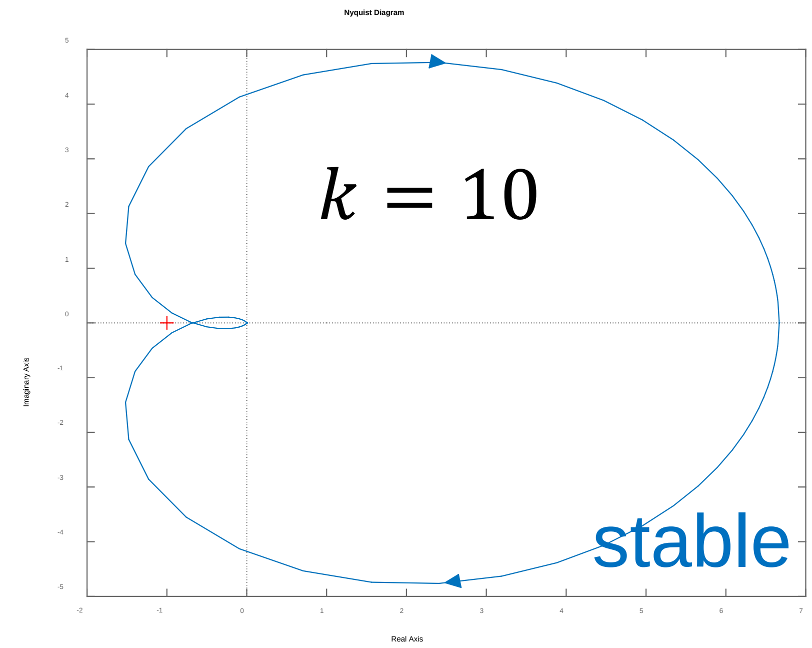




Nyquist Plot Example #1



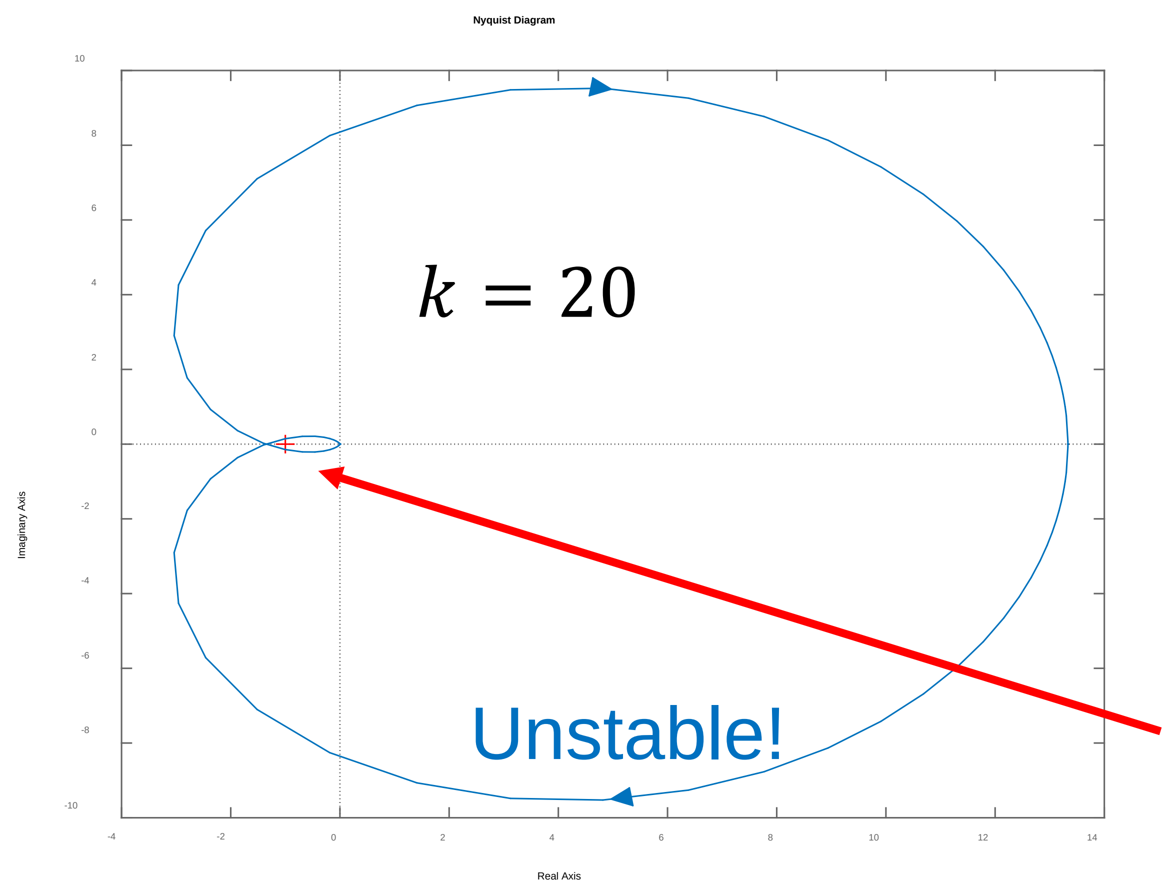




$$P(s) = \frac{1}{(s+1)^3}$$

$$C(s) = k\frac{s+a}{s+b}$$

$$1 < a < b$$



2 Encirclements of -1 point

Nyquist:

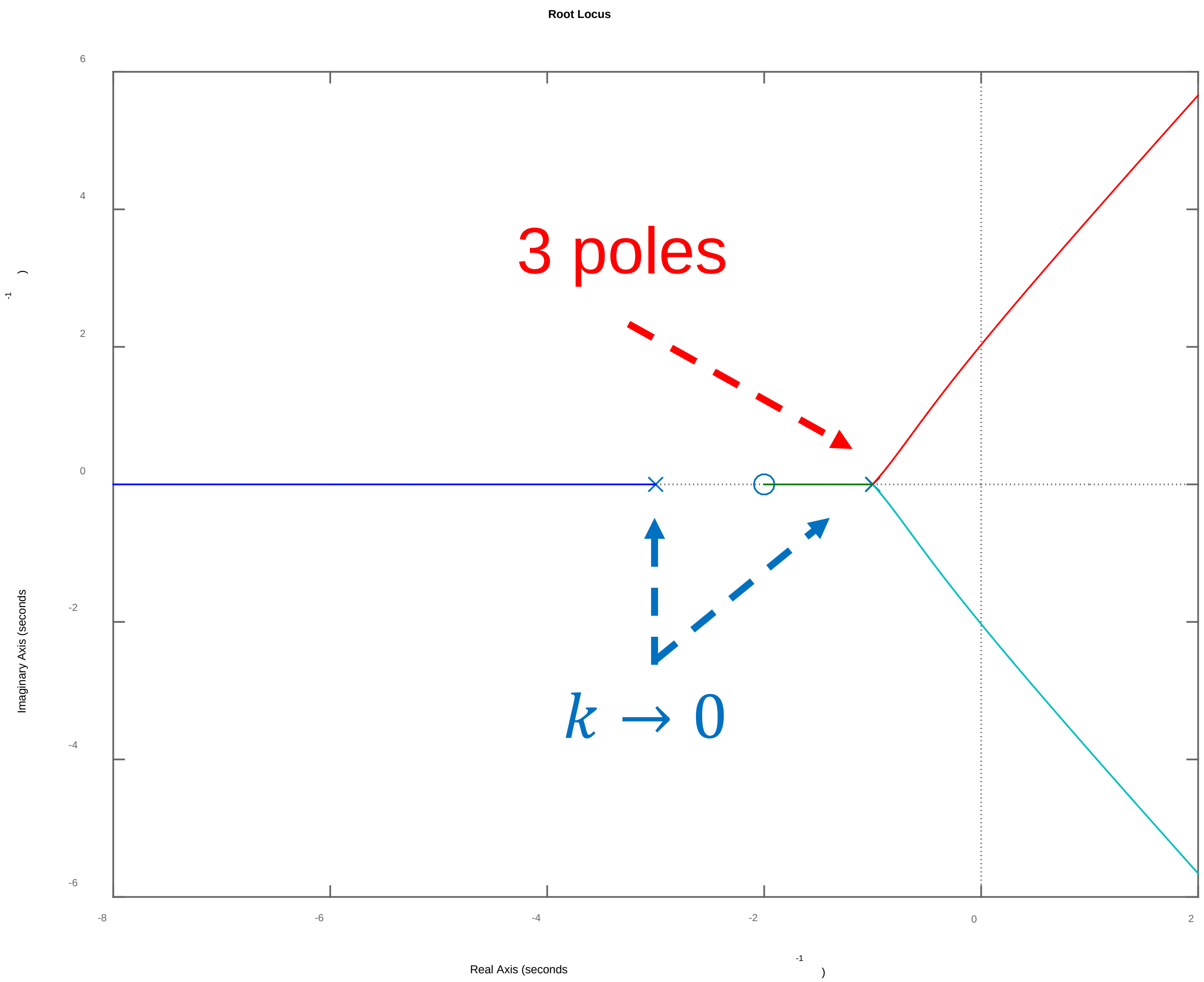
- $P = 0$
- $N = +2$
- $Z_{RHP} = 2$

Nyquist Plot Example #1

(alternative analysis without Nyquist)

$$L(s) = \frac{n_P(s)n_C(s)}{d_P(s)d_C(s)} = k\,\frac{n_P(s)n_{C'}(s)}{d_P(s)d_{C'}(s)} = k\,\frac{1}{(s+1)^3}\,\frac{s+a}{s+b}$$

$$G_{yr}(s) = \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{kP(s)C'(s)}{1+kP(s)C'(s)} = \frac{k\,n_P(s)n_{C'}(s)}{d_P(s)d_{C'}+k\,n_P(s)n_{C'}(s)}$$

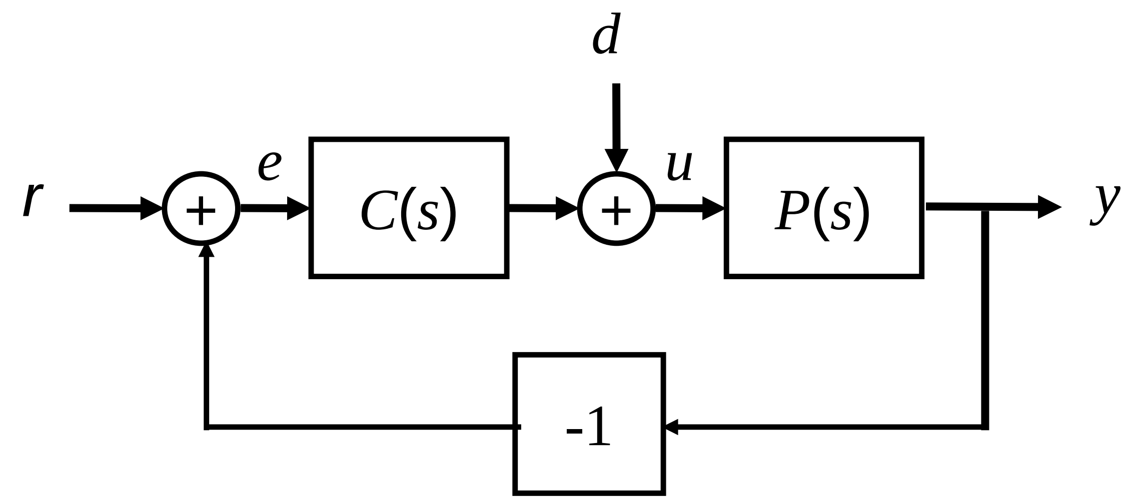


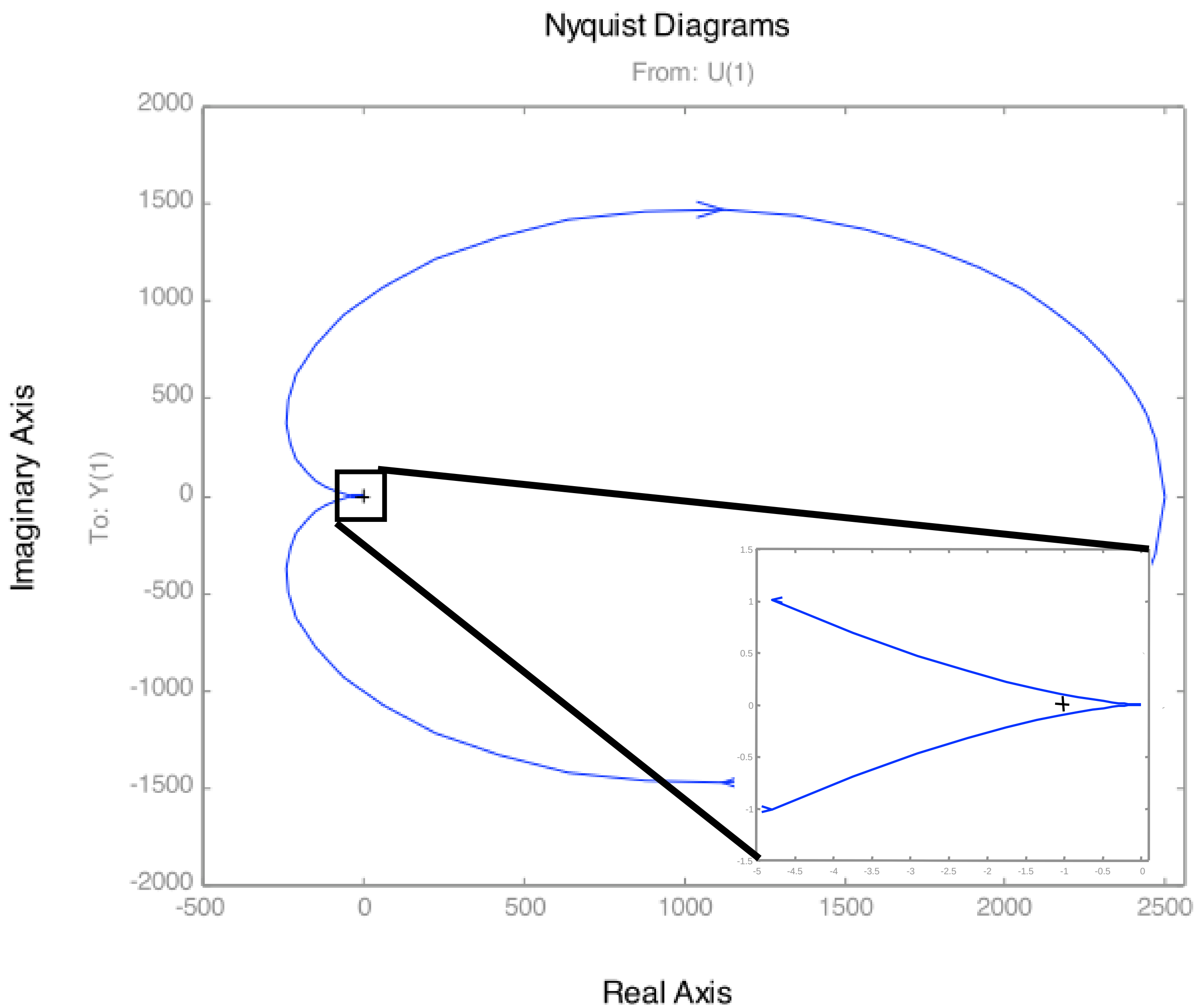
The ***Root Locus*** studies how the poles of $G_{yr}(s)$ vary with k

- When $k \to 0$, the poles of $G_{yr}(s)$ approach the poles of $L(s)$
- As $k \to \infty$, the poles of $G_{yr}(s)$ approach the zeros of $L(s)$ (or infinity)

For this problem, as k increases, two of the poles of $G_{yr}(s)$ become unstable.

Example: Proportional + Integral* speed controller





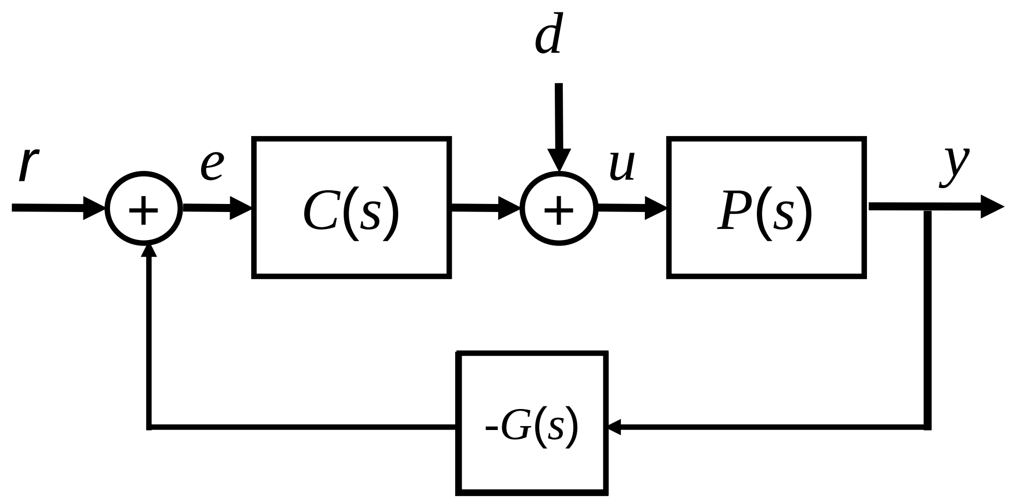
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

Remarks

- N = 0, P = 0 ⇒ Z = 0 (stable)
- Need to zoom in to make sure there are no net encirclements
- Note that we don't have to compute closed loop response

Example: cruise control



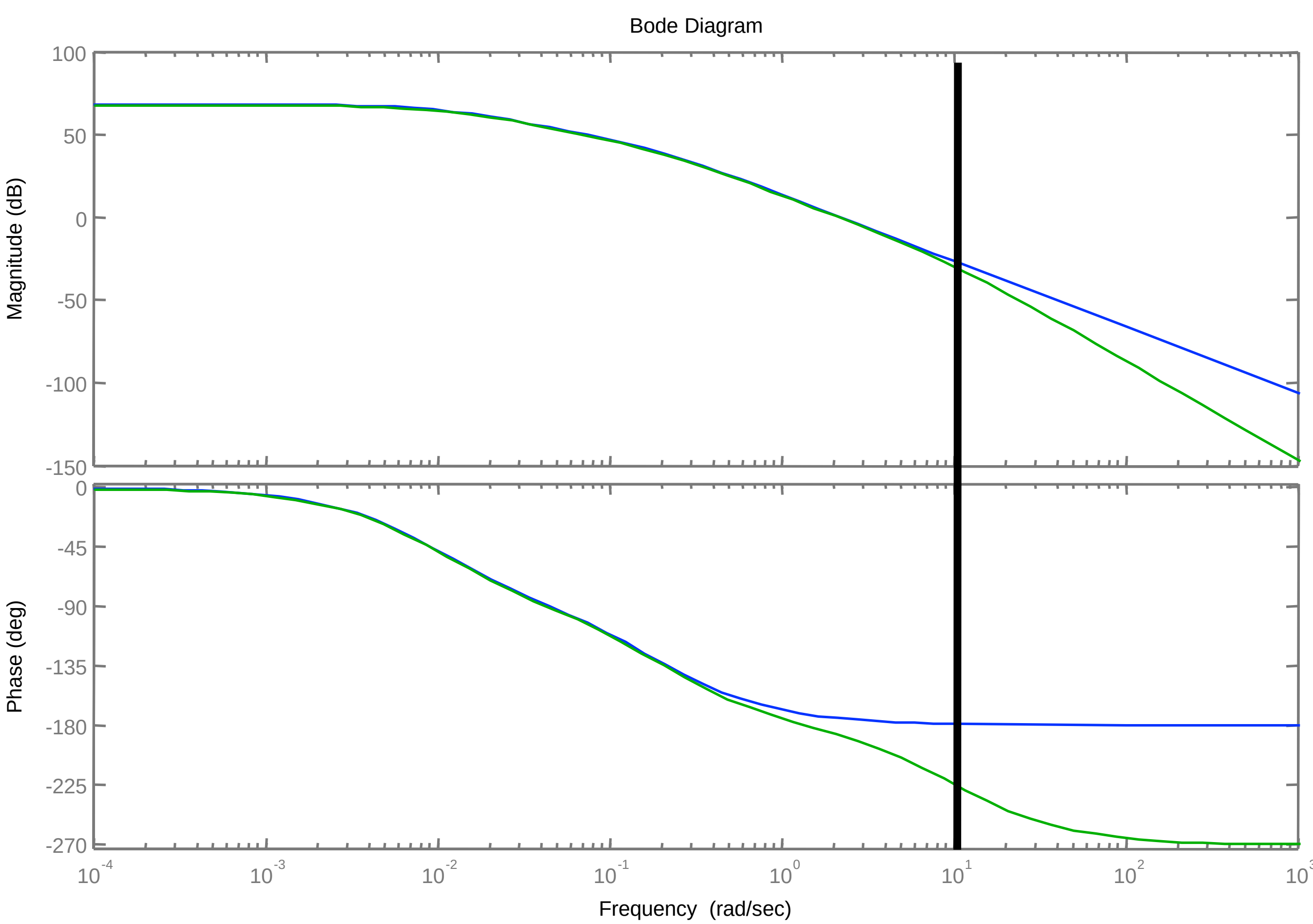
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s+a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

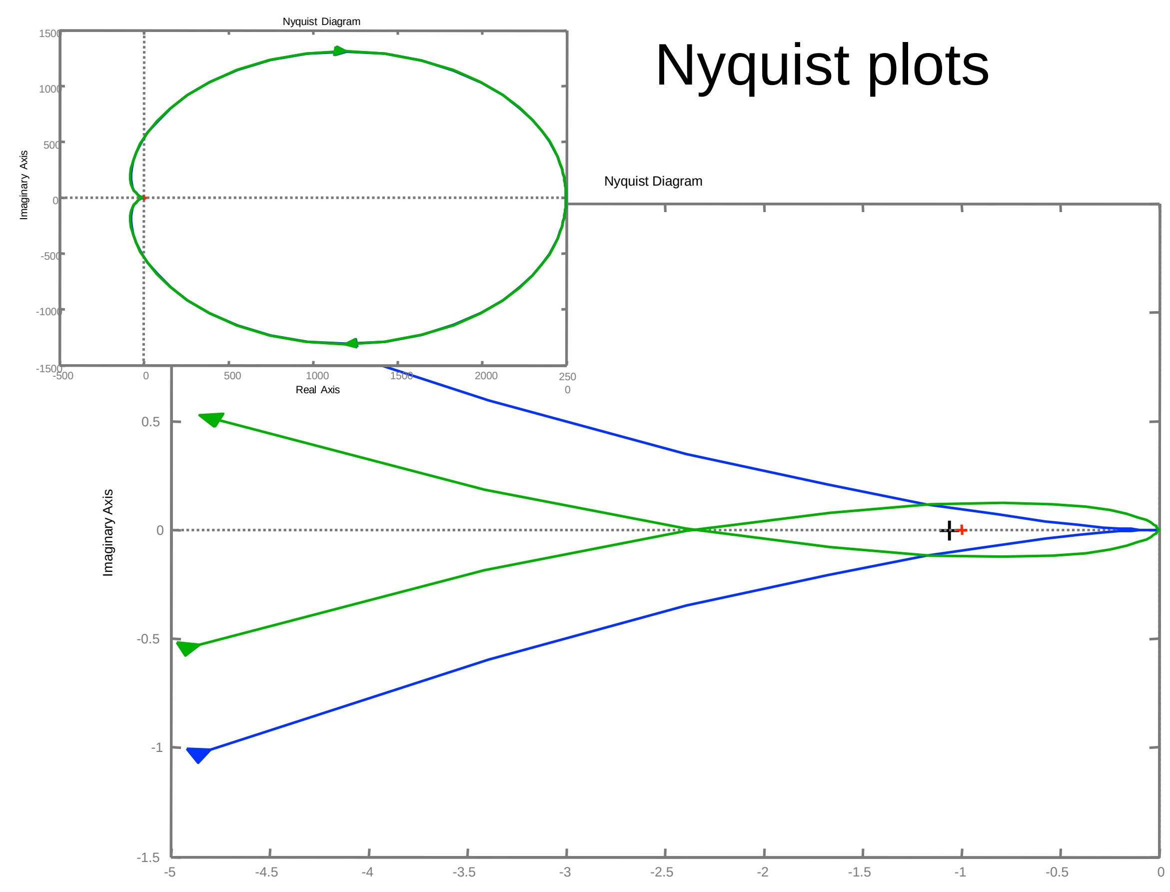
$$G(s) = \frac{10}{s+10}$$

Effect of additional sensor dynamics

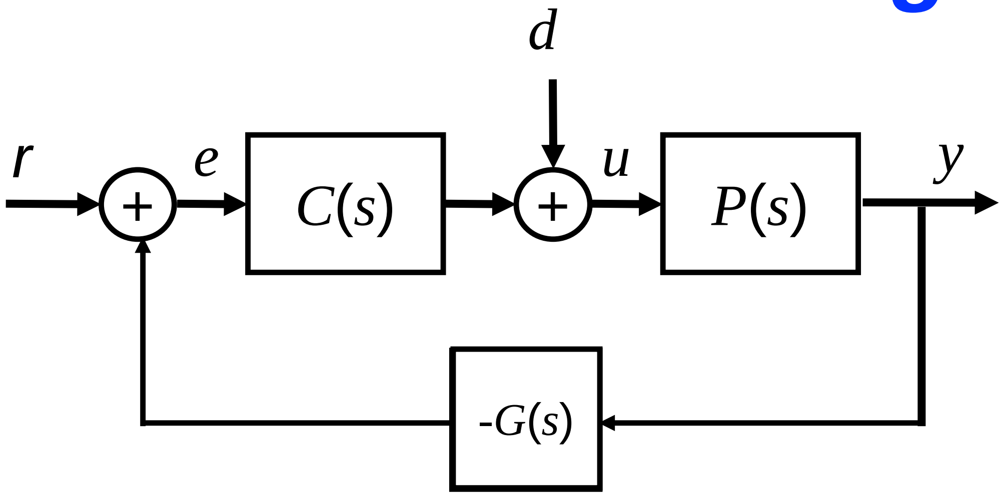
- New speedometer has pole at s = 10 (very fast); problems develop in the field
- What's the problem? A: insufficient phase margin in original design (not robust)



Nyquist plots



Preview: control *design*



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s+a}$$

$$C(s) = \alpha \left(K_p + \frac{K_i}{s + 0.01} \right)$$

$$G(s) = \frac{10}{s+10}$$

Approach: Increase phase margin

- Increase phase margin by reducing gain ⇒ can accommodate new sensor dynamics
- Tradeoff: lower gain at low frequencies ⇒ less bandwidth, larger steady state error