Polyhedral linkages synthesized using Cardan motion along radial axes

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Abstract—*Polyhedral linkages using Cardan motion along radial axes are synthesized. The resulting single degree-of-freedom linkages are compared with some existing designs. The classification and the transformation characteristics of the linkages are given.*

Keywords: deployable structures; polyhedral linkages; Cardan motion

I. Introduction

The Cardan motion is a special planar motion in which both the moving and fixed centrodes are circles with fixed centrode diameter twice that of the moving centrode (Fig. 1) [1]. In Cardan motion, the center of the moving centrode describes a circle, a point on the moving centrode a straight line and any other point describes an ellipse. Because of this, Cardan motion is also known as elliptical motion [1].

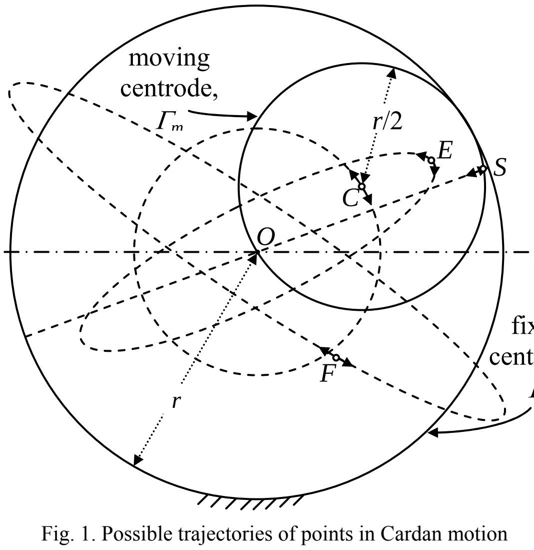

Fig. 1. Possible trajectories of points in Cardan motion

The Cardan motion can most simply be realized as the rolling of a circle inside another circle with twice the diameter. If these circles are gears, the planetary gear train obtained realizes the Cardan motion. Due to the straight line path of a point on the circumference of the moving centrode and the circular path of the center of the moving centrode circle, Cardan motion can also be realized by an isosceles slider-crank mechanism (Fig. 2) or a double slide along two intersecting lines (Fig. 3) [1]. Recently, it was shown that, multiple sliders can also be used to obtain the Cardan motion [2]. As illustrated in Fig. 4, if the isosceles slider cranks of the multiple slider are such that $|OQ_{ext}| = 2a_1 + \ldots + 2a_n = r$, then the coupler link QP has a fixed centrode with radius r and center at O and a moving centrode with radius $r/2$ and center on the perpendicular bisector of OQ with OC_θ making an angle θ with OQ for a configuration where the crank makes an angle θ with the slider axis. Then, any point on the moving centrode tracks a straight line. If the mirror image of such a linkage is implemented, the scissor mechanism obtained also realizes the Cardan motion. Also, the mechanism can be overconstrained with additional coupler points as shown in Fig. 5. Once the mechanism is overconstrained, the cranks and the intermediate links can be removed and only the so called "angulated elements" can be used to fulfill the task, as was done by Hoberman and You et. al. (Fig. 6) [3-4]. Although some certain special dimensions are possible in order for an angulated element pair to track two intersecting straight lines [4], the elements must be isosceles, the bend angle and the angle between the lines being tracked must be supplementary angles to obtain similarity (Fig. 7) [2].

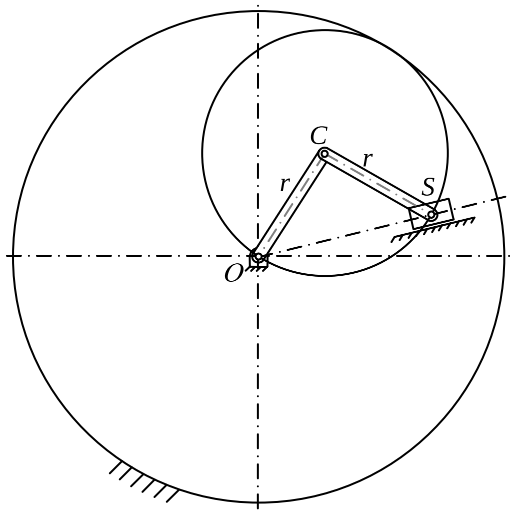

Fig. 2. An isosceles slider crank with coupler link in Cardan motion

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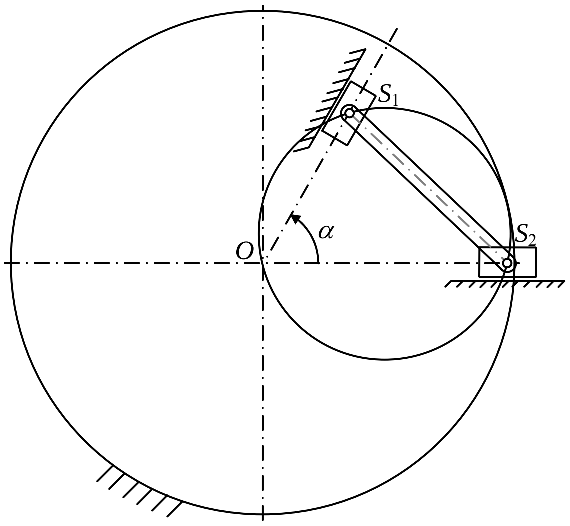


Fig. 3. A double slide in Cardan motion

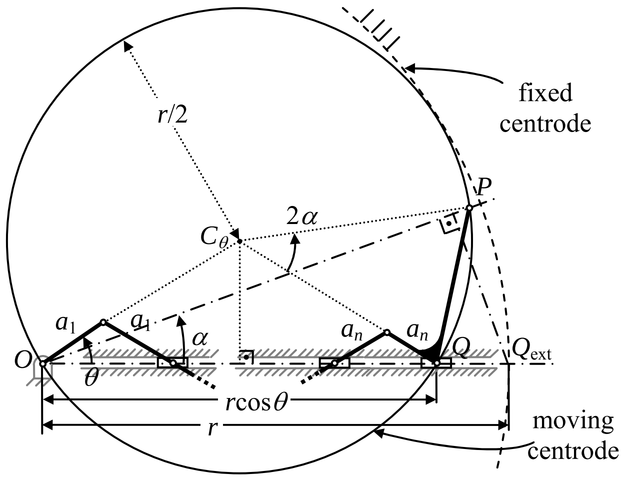


Fig. 4. The Cardan motion realized by using multiple sliders [2]

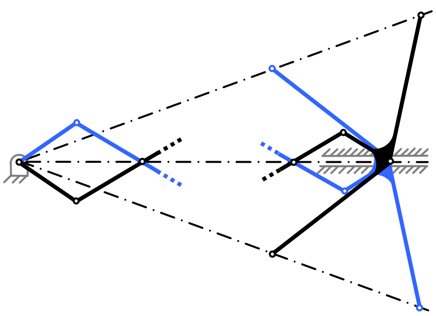


Fig. 5. A scissor mechanism designed to track two intersecting lines [2]

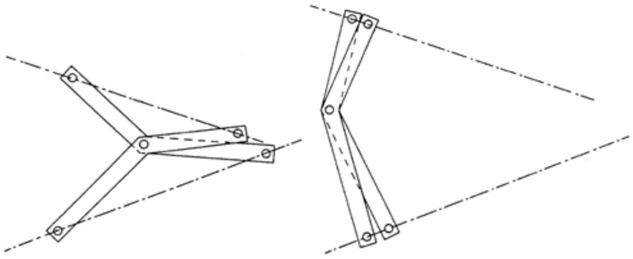


Fig. 6. A general angulated scissor element in two configurations [5]

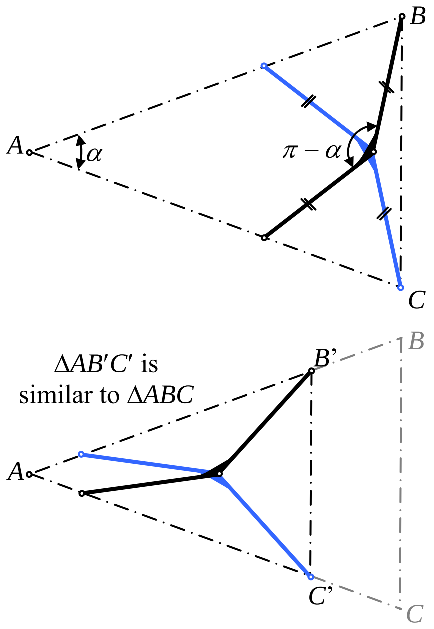


Fig. 7. An angulated element pair to magnify a triangle *ABC*

II. Using Cardan motion along radial axes

In [2], these mechanisms were used to magnify an isosceles triangle. Many such linkages are assembled to form cyclic polygons, which can finally assemble to form a polyhedral shape. In this paper, the triangle-magnifying linkages will be used to obtain a magnification of polyhedral shapes along radial axes.

Consider an inner point in a polyhedral shape. Connecting this point to the vertices, one obtains *e* many triangles, where *e* is the number of edges. These triangles, hence the whole polyhedral shape can be magnified using isosceles angulated elements. In order to avoid interference of skew revolute joint axes, an offset along the lines connecting the magnification center to the vertices must be introduced (Fig. 8). Depending on the design, the amount of offset can take any value.

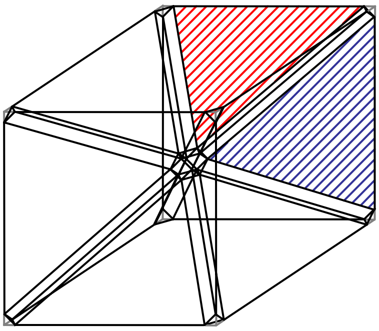
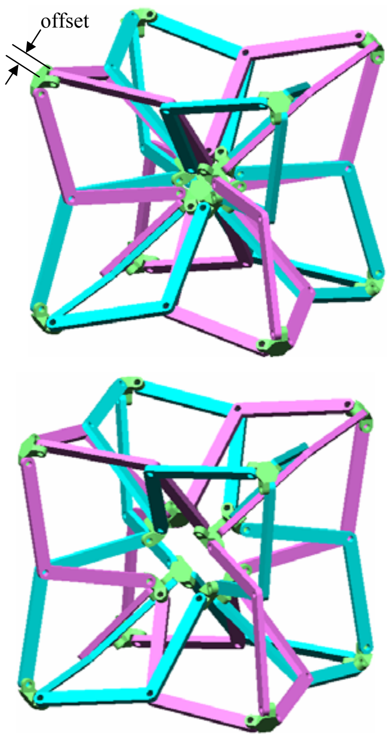


Fig. 8. A triangular dissection for the radial magnification of a cube

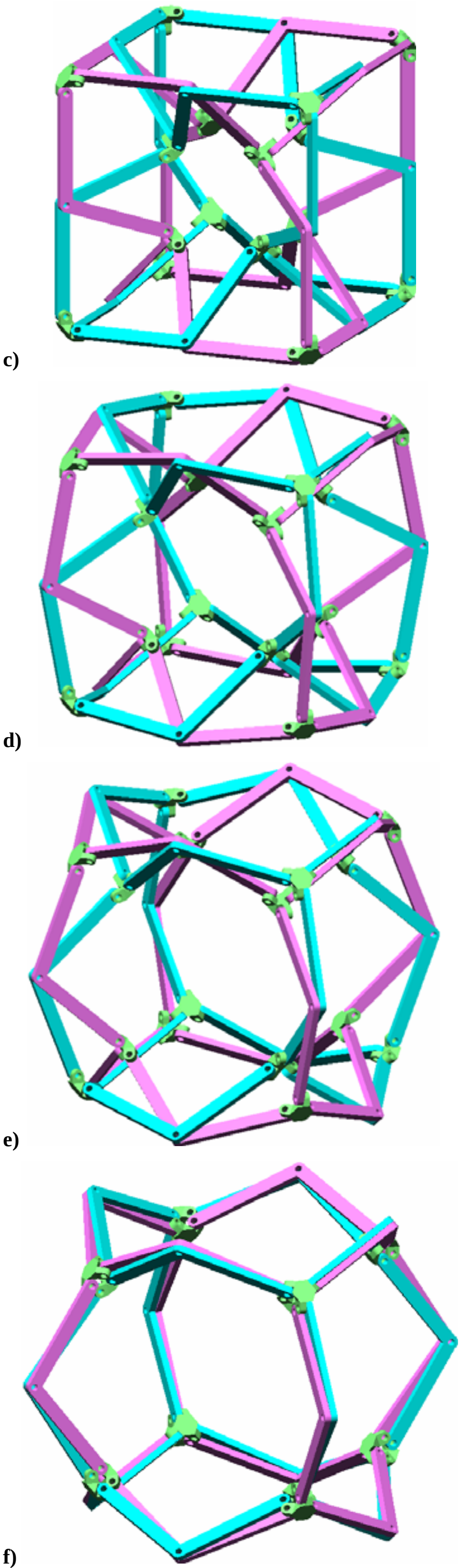
Since the triangular dissections must all be isosceles, all the vertices must be equidistant to the magnification center. Hence, only spherical polyhedra, i.e. the polyhedra with all vertices on a sphere, can be magnified using the linkages proposed. A cubic linkage synthesized accordingly is illustrated in Fig. 9.

Notice that, the total ratio of magnification is very small for single pair of angulated elements. One may add ordinary scissor elements in order to increase the ratio of magnification, as was done by Hoberman and Wohlhart (Figs. 10-11) [3, 6].



a)

b)



c)

d)

e)

f)

Fig. 9. **a-c)** A cube being magnified and **d-f)** further expansion of the linkage

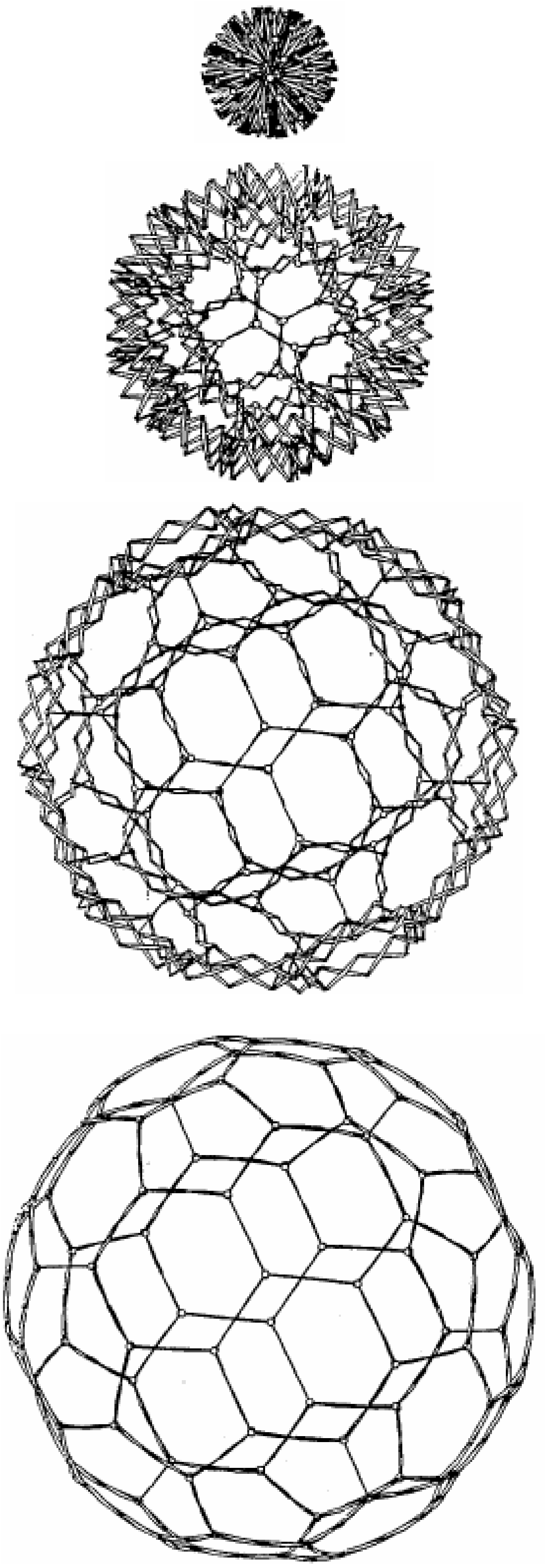


Fig. 10. Hoberman's truncated icosahedral linkage [3]

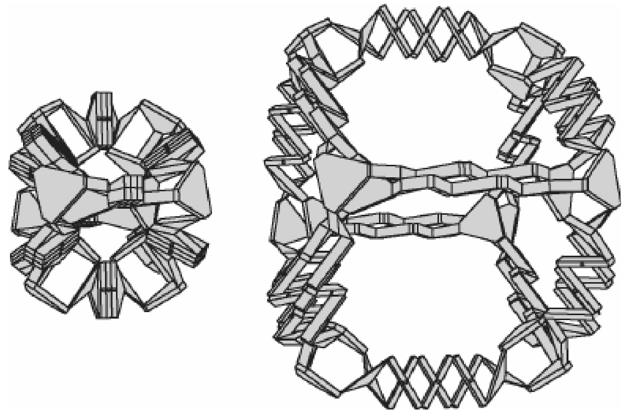


Fig. 11. Wohlhart's cubic Zig-Zag linkage [6]

III. Another possible dissection

Alternatively, one may dissect the polyhedral shape of interest such that the circumcenter of the polyhedron, a vertex and the circumcenter of a face form a triangle (Fig. 12 – triangles *OAA′*, *OBB′*, etc.). Notice that the triangular dissections will be intersecting each other along the line connecting the circumcenter of the polyhedron and the circumcenter of a face. Hence a constraint along an intersection line makes it possible to reduce the degree-of-freedom of the triangular linkage. So, binary links can be used instead of angulated elements, removing the need to have an isosceles triangle. Indeed, then these binary links move as double slides, which are also well known to realize the Cardan motion [1].

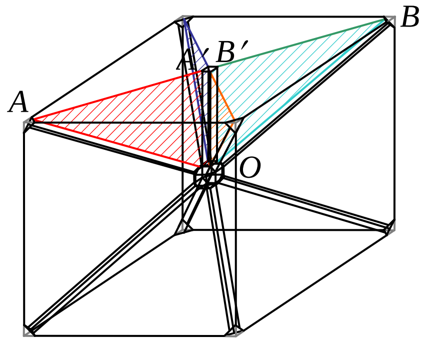
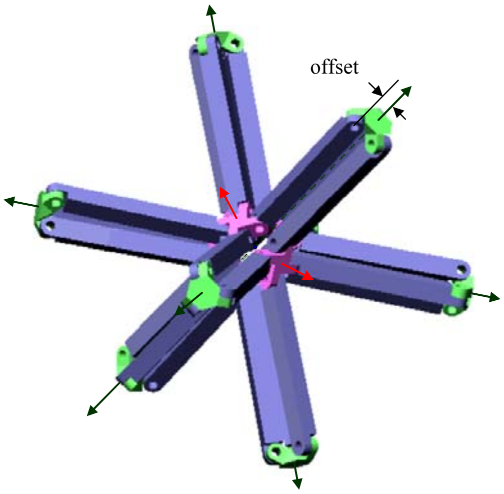
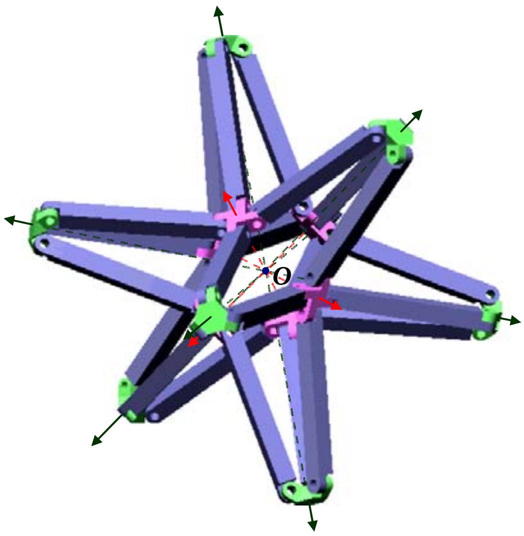


Fig. 12. Another triangular dissection for the radial magnification of a cube

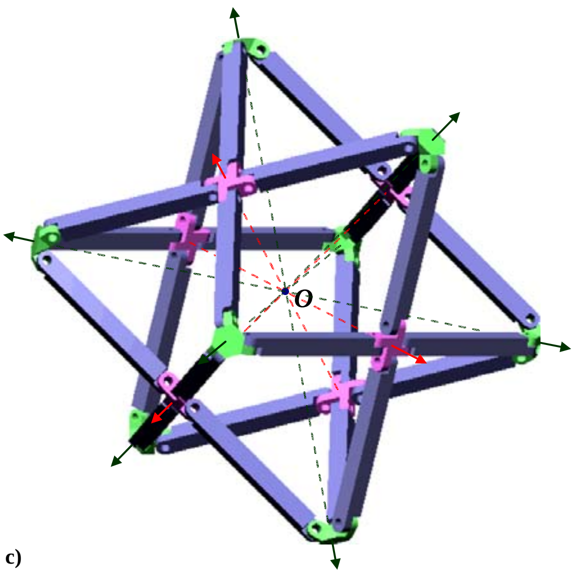
When proper connections are implemented between the binary links, it happens to be that the shape the linkage encloses varies between a family of polyhedra which belong to the same symmetry group. Because of this fact, there are two minimal and two maximal configurations, which are dual of each other (Dual polyhedra are obtained by interchanging faces and vertices). The motion of a cubic/octahedral linkage synthesized accordingly is illustrated in Fig. 13. This type of linkages was also synthesized by Wohlhart (Fig. 14) [7] and Hoberman (Fig. 15) [8].



a)



b)



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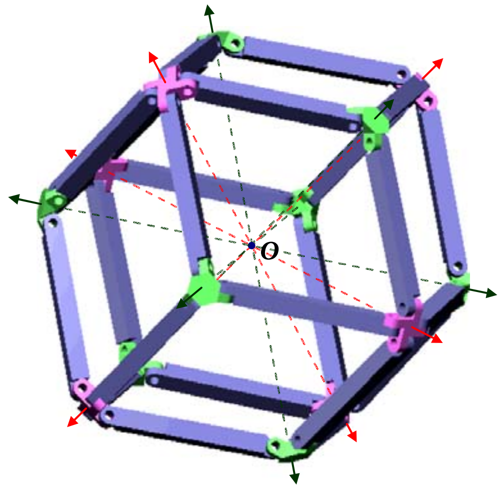

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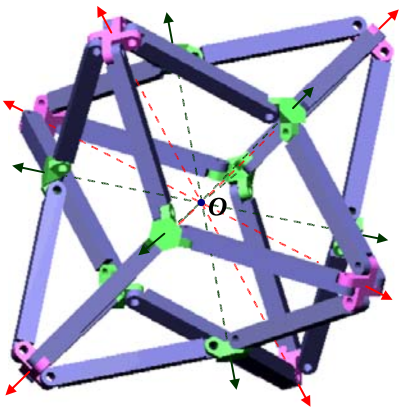

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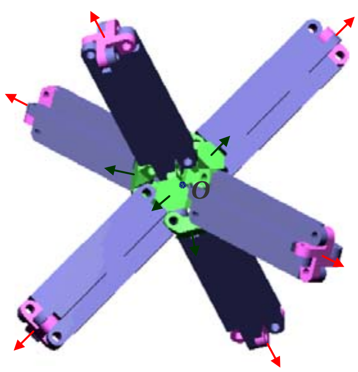


f)

Fig. 13. **a)** Cubic skeleton **c)** Cube **d)** Rhombic dodecahedron **e)** Octahedron **f)** Octahedral skeleton

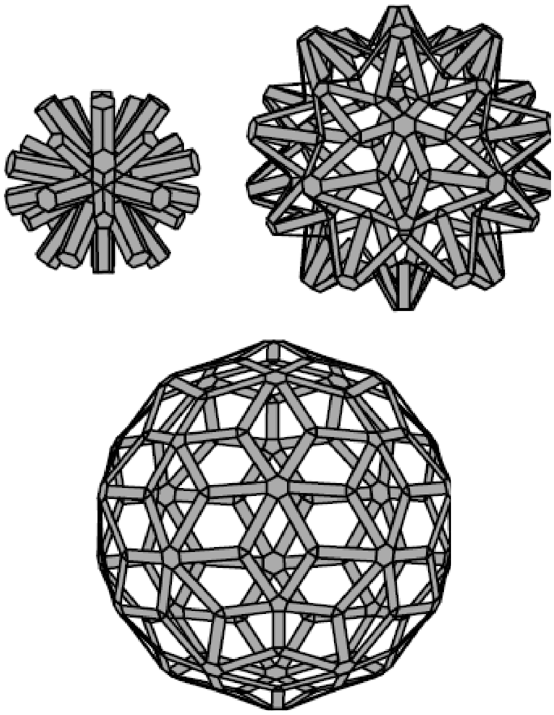
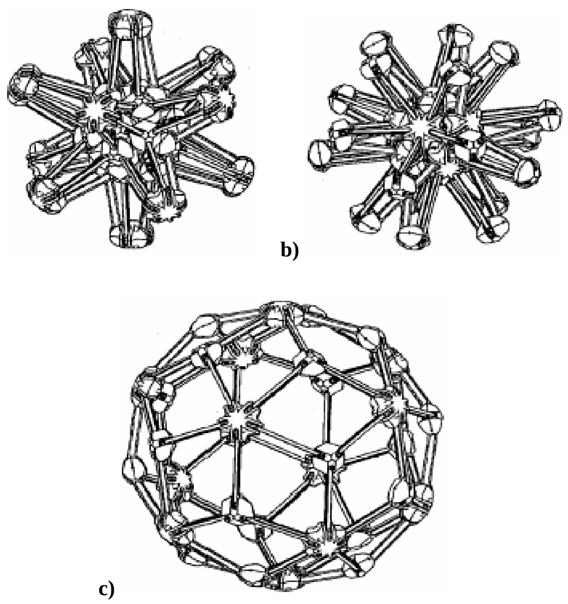


Fig. 14. Wohlhart's Polyhedral star-transformer for the truncated icosahedron [9]



a) b)

c)

Fig. 15. A design of Hoberman **a-b)** two possible minimal configurations **c)** an intermediate configuration [8]

IV. Discussion and conclusion

Cardan motion is used in magnification of isosceles triangles which radially assemble to form a spherical polyhedral shape. With this new synthesis method, the underlying theory of some past inventions is introduced. The Cardan motion is well known since 16th century; henceforth can be used by anyone in synthesis of polyhedral linkages. Although, for illustrative purposes, some simple examples are given in this paper, many new linkages can be synthesized using the theory explained. For instance, noticing that the links neighboring a vertex looks like an umbrella mechanism, one may use double-collapsible umbrella linkages for further magnification.

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